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THE COMPLETE ARITHMETIC

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Greenleaf's Mathematical Series.

THE

COMPLETE ARITHMETIC,

ORAL AND WRITTEN,

ON THE BASIS OF WORKS

By BENJAMIN GREENLEAF, A.M.

BOSTON:
ROBERT S. DAVIS AND COMPANY.

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Mar. 14, 1935

Edward H. Atherton

GREENLEAF'S

MATHEMATICAL SERIES.

INDUCTIVE COURSE.

FIRST LESSONS IN NUMBERS.

A BRIEF COURSE IN ARITHMETIC.

THE COMPLETE ARITHMETIC.

The BRIEF COURSE and the COMPLETE ARITHMETIC are each published with and without answers.

KEY TO THE COMPLETE ARITHMETIC, for Teachers only. (*In preparation.*)

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PREFACE.

THIS Arithmetic, undertaken at the suggestion of many educators of distinction, has been prepared with special reference to training for practical business, and to development of mind power through fixed habits of attention and lucid processes of reasoning.

To secure skill, rapidity, and accuracy in the use of numbers, required in common transactions, a large number of comparatively simple business questions has been provided, and prominence has been given to subjects of the most practical value.

That useful mental discipline may be attained the theory and principles of numbers have been clearly presented, and problems have been given requiring thought and discrimination.

The inductive plan has been followed throughout, principles have been developed from methods, rules derived from analyses, and oral and written exercises combined in a rational manner.

The greatest care has been observed to have the definitions brief, clear, and accurate, and the solutions simple, concise, and logical.

The methods employed are those which business experience, or test in the school-room, has shown to be the best.

Decimals to three places, and United States money, are simply treated at the beginning with integers.

The problems are abundant and varied, based on recent and reliable data, and drawn from the actual experiences of life. In commercial arithmetic the usages of the best business houses have been followed.

Several hundred examples which have been used in examinations by superintendents of public instruction in various cities and towns leading in educational matters, have been collected, and arranged as exercises for testing proficiency, and for supplementary practice, to be drawn from at the teacher's discretion.

Much matter formerly considered necessary in an arithmetic, but which modern progress has rendered useless or antiquated, has been omitted.

The Appendix contains tables for reference; information of a somewhat technical nature for the business man, the mechanic, and the farmer; subjects of minor importance to the majority of pupils; and rules and applications not needed in the body of the work.

Indebtedness is gratefully acknowledged to school superintendents of various cities and towns for examination papers and valuable suggestions; to the Metric Bureau for information and cuts; to Boston Custom-House officials for copies of invoices; to the Regents of the University of the State of New York, and to various college authorities, for entrance test-papers to be found in the Appendix.

Credit is due to all the able teachers who have aided, during the past three years, in the preparation of the Inductive Course, of which this book is a part. The larger share of this credit belongs to Mr. G. A. SOUTHWORTH, the successful and experienced Master of the Prescott Grammar School, Somerville, Mass., by whom much work has been done. His practical knowledge of what both teachers and pupils need in a text-book has been of great advantage.

The work is given to the public in the expectation that it will meet every reasonable requirement of our common schools and seminaries.

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THE

COMPLETE ARITHMETIC.

1. A **Unit** is a single thing, or one; as one hour, one dollar, one.

2. A **Number** is a unit, or a collection of units; as one month, four hours, seven.

3. **Arithmetic** treats of numbers and their use.

NOTATION AND NUMERATION.

4. **Notation** is writing numbers in figures.

5. **Numeration** is reading numbers written in figures.

6. **Figures** are characters used to express numbers.

Ten different figures are used; they are

0,	**1,**	**2,**	**3,**	**4,**	**5,**	**6,**	**7,**	**8,**	**9.**
Zero,	**One,**	**Two,**	**Three,**	**Four,**	**Five,**	**Six,**	**Seven,**	**Eight,**	**Nine.**

7. *Zero,* or *cipher,* used alone expresses *no units,* or *nothing.*

The other *nine figures* express the number of units shown by their names.

8. To express numbers larger than nine, two or more figures are written side by side.

9. A figure written *alone* has only a *simple name* and *value;* but when used with other figures it has also a *place-name* and *value.*

10. The **Place** of a figure is its position with reference to another figure in a number.

11. When two figures are written side by side, the figure at the right occupies the *first place,* has the place-name *ones,* and expresses *units of the first order.* The figure at the left occupies the *second place,* has the place-name *tens,* and expresses *units of the second order,* each of which is ten times larger than a unit of the first order. Thus,

80 is read 8 *tens,* 0 *ones,* or briefly *eighty.*
75 " 7 *tens,* 5 *ones,* " *seventy-five.*

By the use of the other possible combinations of two figures any number between *nine* and *one hundred* may be written.

Read the following:

1. 97
2. 45
3. 63
4. 84
5. 77
6. 38
7. 54
8. 91
9. 26
10. 48
11. 89
12. 98

Write in figures:

13. Eighty-six.
14. Sixty-four.
15. Seventy-nine.
16. Thirty-six.
17. Four *tens,* eight *ones.*
18. Ninety-two.
19. Eighty-one.
20. Twenty-nine.
21. Sixty-seven.

12. A figure at the left of the tens' figure occupies the *third place,* has the place-name *hundreds,* and expresses a *third order of units,* each ten times as large as one of those in the second place. Thus,

659 is read 6 *hundreds,* 5 *tens,* 9 *ones,* or briefly, *six hundred fifty-nine.* 508 is read 5 *hundreds,* 0 *tens,* 8 *ones,* or briefly, *five hundred eight.*

By using the different combinations of three figures any number between *ninety-nine* and *one thousand* may be expressed.

Read the following:

22. 789
23. 456
24. 804
25. 123
26. 984
27. 327
28. 656
29. 700
30. 608
31. 555
32. 390
33. 299

Write in figures:

34. Eight hundred forty-four.
35. Two *hundreds*, seven *ones*.
36. Seven hundred fifty-three.
37. Nine hundred ninety.
38. Five hundred eighty-eight.
39. Seven hundred.
40. Six hundred sixty-six.
41. Nine hundred four.

13. A figure at the left of the hundreds' figure occupies the *fourth place*, has the place-name *thousands*, and expresses the *fourth order of units*, each ten times as large as one in the third place. Thus,

8976 is read 8 *thousands*, 9 *hundreds*, 7 *tens*, 6 *ones*, or briefly, eight thousand nine hundred seventy-six.

In this way the various combinations of four figures may be used to express the numbers between *nine hundred ninety-nine* and *ten thousand*.

Read the following:

42. 7382
43. 8641
44. 6047
45. 9308
46. 4629
47. 5432
48. 1333
49. 3004
50. 2908

Write in figures:

51. Six *thousands*, no *hundreds*, five *tens*, seven *ones*.
52. Eight *thousands*, two *hundreds*, no *tens*, no *ones*.
53. How may the preceding number be read more briefly?
54. Three thousand eight hundred forty-six.
55. Nine thousand two hundred seven.

56. Five thousand seven hundred forty.
57. Two thousand eight.
58. Four *thousands*, four *tens*, four *ones*.
59. Eight *ones*, three *tens*, nine *hundreds*, seven *thousands*.

14. On the same principle larger numbers are written by using five, six, seven, eight or more figures, each additional figure filling a *new place*, named successively towards the left, *ten-thousands, hundred-thousands, millions, ten-millions,* and so on, and forming a new order of units, each of which is ten times as large as the next smaller.

15. The *place-names* and *values* of figures are shown in the following

TABLE.

Place.	*Place-name.*	*Order of Units.*
First,	Ones,	1st,
Second,	Tens,	2d,
Third,	Hundreds,	3d,
Fourth,	Thousands,	4th,
Fifth,	Ten-thousands,	5th,
Sixth,	Hundred-thousands,	6th,
Seventh,	Millions,	7th,
Eighth,	Ten-millions,	8th,
Ninth,	Hundred-millions,	9th,
Tenth.	Billions.	10th.

16. For convenience in reading and writing numbers, each three places in a number, beginning with the ones, form a *group*. Each group is named from its right-hand order of units.

The *first* group is the group of *ones*.
" *second* " " " *thousands*.
" *third* " " " *millions*.
" *fourth* " " " *billions*.

The comma is used to mark off the groups.

Thus, in the number 109,876,543,210,

the *first* group is 210 *ones,*
the *second* group is 543 *thousands,*
the *third* group is 876 *millions,*
the *fourth* group is 109 *billions.*

17. Each complete group contains *hundreds, tens,* and *ones* of its group-name. Thus,

847,298,341, instead of being read 8 hundred-millions, 4 ten-millions, 7 millions, etc., is more briefly read, eight hundred forty-seven *millions,* two hundred ninety-eight *thousands,* three hundred forty-one.

EXERCISES.

60. In the number 321,654,987, what is the name of the first group? Of the third? Of the second?

61. Give the place name of the 1. Of the 2. Of the 3. Of the 5. Of the 9. Of the 6. Of the 4.

62. What figure expresses the 4th order of units? The 7th? The 5th? The 6th? The 8th?

18. The method of writing numbers in figures is based on the following

Principles of Notation.

1. *Ten units of any order equal one unit of the order next larger.*

2. *Each removal of a figure one place toward the left increases its value ten times, and each removal of a figure toward the right diminishes its value ten times.*

19. Our system of Numbers is called a *Decimal System,* from the Latin *decem,* ten, because of the uniform ten-fold increase of units from any order to the next larger.

The successive orders of units in a number form a

Scale; and, where ten units of any order always make a unit of the next larger, the scale is *ten* or *decimal.*

20. A period (.), called the *decimal point,* may be written at the right of the *ones'* figure to mark the ones' place.

21. A number at the left of the decimal point is an **Integer,** or *whole number,* because it is a collection of *entire ones.*

22. Figures may be written at the right of ones, if separated from them by the decimal point.

23. The first figure at the right of ones is in the *first decimal place,* has the place-name *tenths,* and expresses the *first order of decimal units,* each being one tenth of a one.

A figure at the right of tenths is in the *second decimal place,* has the place-name *hundredths,* and expresses the *second order of decimal units,* each being one tenth of a tenth.

A figure at the right of hundredths is in the *third decimal place,* has the place-name *thousandths,* and expresses the *third order of decimal units,* each being one tenth of a hundredth. Thus,

6.378 expresses six ones, and three *tenths,* seven *hundredths,* eight *thousandths,* or briefly six, and three hundred seventy-eight *thousandths.*

NOTE. — In reading numbers use the conjunction *and* only after the *ones* to show the place of the decimal point.

24. A number at the right of the decimal point is a **Decimal,** or a collection of tenths, hundredths, thousandths, or other decimal parts of a *one.*

NOTE. — In writing a decimal without an integer the ones' place may be filled with a zero. Thus,

Three hundred forty-one thousandths may be written 0.341.

25. The method of writing numbers, and the names of units, places, and groups are shown in the following

TABLE.

	Integer.													*Decimal.*		
ORDERS OF UNITS.	12th,	11th,	10th,	9th,	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,		1st,	2d,	3d.
PLACE-NAMES.	Hundred-billions	Ten-billions	*Billions*	Hundred-millions	Ten-millions	*Millions*	Hundred-thousands	Ten-thousands	*Thousands*	Hundreds	Tens	*Ones*	Decimal Point	Tenths	Hundredths	*Thousandths*
FIGURES.	3	6	1,	5	4	0,	7	9	3,	1	5	4	.	6	3	8
GROUPS.	4th,			3d,			2d,			1st,			1st Decimal,			
GROUP-NAMES.	Billions,			Millions,			Thousands,			Ones,			Thousandths.			

The number in the table is, three hundred sixty-one *billions,* five hundred forty *millions,* seven hundred ninety-three *thousands,* one hundred fifty-four, and six hundred thirty-eight *thousandths.*

NOTE 1. — Group-names above billions are trillions, quadrillions, quintillions, etc.; below thousandths, millionths, billionths, etc.

NOTE 2. — The left-hand group of the integer, or the right-hand group of the decimal may contain only one or two figures. The name of a partial decimal group is the same as the place-name of its right-hand figure. Thus,

16.84 is read sixteen and eighty-four *hundredths.*

EXERCISES IN WRITING NUMBERS.

63. Write in figures, thirty-five million six hundred twenty-seven thousand two hundred three, and sixteen hun- [illegible]

35,627,203.16

Solution. — Writing 35 for the millions, 627 for the thousands, 203 for the ones, and .16 for the hundredths, we have as the result required, 35,627,203.16.

64. Write in figures, three hundred twenty-nine thousand four hundred fifteen.

65. Write in figures, nine thousand seven hundred fifty-two, and one hundred one thousandths.

26. Rule for writing Numbers.

Beginning at the left, write the figures of each group in their order, filling vacant places and groups with ciphers.

Write in figures:

66. Six thousand five hundred nine.

67. Eleven thousand nine hundred eleven.

68. Thirty-seven thousand four hundred eighty-nine.

69. Ninety thousand four hundred forty-four.

70. One hundred sixty-three thousand.

71. Two hundred twenty thousand two hundred sixty-two.

72. Five hundred seventy-four thousand three hundred thirty-five.

73. Seven hundred fifty-three thousand seven hundred fifty.

74. Five thousand four hundred eighty-nine.

75. Four hundred eighty thousand eight.

76. One million one hundred thirteen.

77. Three million three thousand thirty.

78. Nine hundred seven million eight hundred five thousand seventy-four.

79. Fifty-seven billion forty-four million ninety-three thousand eighty-three.

80. Thirteen million six hundred thousand one hundred seventy.

81. Five hundred six billion two million four [illegible] one.

82. One thousand seven hundred sixty-two, and five tenths.

83. Nine hundred ninety-four, and thirty-eight hundredths.

84. One hundred nineteen thousand, and one hundred nineteen thousandths.

85. One million four hundred thousand three hundred, and thirty-three hundredths.

86. Sixteen thousand seven hundred forty-five, and eighty-six thousandths.

87. Three billions three millions two thousands seven hundreds, and three thousandths.

EXERCISES IN READING NUMBERS.

88. Write and read 25362795.

25,362,795

Solution. — 25362795 marked off into groups becomes 25,362,795. The first group from the right expresses 795 ones, or 795; the next group expresses 362 thousands; and the third expresses twenty-five millions. The whole is read: twenty-five million three hundred sixty-two thousand seven hundred ninety-five.

89. Write and read 3257893.

90. Write and read 64058.109.

64,058.109

Solution. — 64058.109 marked off into groups becomes 64,058.109, which is read sixty-four thousand fifty-eight, and one hundred nine thousandths.

91. Write and read 1673405.75.

92. Write and read 906005.805.

27. Rule for Reading Numbers.

Beginning at the ones' place, mark off the numbers by commas, into as many groups as possible of three figures each.

Begin at the left and read each group as if it stood alone, giving the group-name except to the ones' group.

If there is a decimal, read it as if it were at the left of the point, and add the place-name of the last figure.

Copy and read :

93. 486
94. 2385
95. 7275
96. 12361
97. 62004
98. 199
99. 78382
100. 160405
101. 6082
102. 19009
103. 163404
104. 789685
105. 99999
106. 1634562
107. 25401300
108. 312407981
109. 0.31
110. 117.03
111. .064
112. 118646.5
113. 2567.02
114. 88.999
115. 1158834.55
116. 100600.789

QUESTIONS.

Art. 1. What is a unit ? 2. What is a number ? 3. What is arithmetic ?

4. What is notation ? 5. What is numeration ? 6. What are figures ? 7. What does zero, or cipher, express ? What do the other nine figures express ?

9. When has a figure a simple name and value ? When has it a place-name and value ?

10. What is the place of a figure ? 11. If in the first place, what place-name has a figure ? What units does it express ? If in the second place, what place-name has a figure ? What units does it express ?

12. What place-name has a figure at the left of tens ? What units does it express ?

13. What place-name has a figure at the left of hundreds ? What units does it express ?

15. Give the place-names beginning with ones. Give the corresponding orders of units.

16. What form a group for convenience in reading and writing numbers ? How is each group named ? Name groups beginning with ones. What is used to mark off the groups ? 17. What does each complete group contain ?

18. What are principles of notation ? 19. What is our system called ? What do the successive orders of units form ?

20. What is the decimal point ? For what is it written ? 21. What is the number at the left of the point ?

26. How are numbers written in figures ? 27. How are numbers written in figures read ?

ADDITION.

28. 1. James has 9 apples and Henry has 3. How many apples have both?

2. Mary had 7 oranges and her brother gave her 5. How many had she then?

3. A father gave to one of his children 8 cents, to another 5 cents, and to a third 6 cents. How many cents did he give them in all?

4. How many are 9 and 3? 7 and 5? 8, 5, and 6?

5. How many dollars are 8 dollars, 7 dollars, and 3 dollars?

6. What is the unit of 8 dollars, 7 dollars, and 3 dollars?

29. The **Unit of a number** is one of that number. Thus,

One pound is the unit of 8 pounds, one quart is the unit of 27 quarts.

30. **Like numbers** are numbers having the same unit. Thus 3, 5, 7; and 6 cents, 4 cents and 5 cents, are like numbers.

31. **Addition** is finding a number equal to two or more given numbers.

32. The **sum**, or **amount**, is the result of an addition.

33. The **sign of addition** is +, named *plus.* It means *more,* and is generally read *and.* Thus,

$4 + 5 + 6$ is read four and five and six.

34. The **sign of equality** is =. It means *equal,* or *equal to,* and is often read *are.* Thus,

$7 + 8 = 15$ is read seven and eight are fifteen.

35. The sign, $, written before a number, means *dollars.* Thus,

$10 is read ten dollars.

36. *Cents,* expressed in figures, may occupy two decimal places, *tenths* and *hundredths,* and *mills* one place, *thousandths.* Thus,

$0.53, or $.53, is read fifty-three cents, and $28.005 is read twenty-eight dollars and five mills.

ORAL EXERCISES.

7. How many dollars are $ 7, $ 6, and $ 2?

8. If you pay 10 cents for a slate, 9 cents for paper, and 3 cents for a pencil, how much do you pay for the whole?

9. 12 boys are at play in one place, 6 in another, and 5 in another. How many boys are at play in all?

10. John one day caught 8 trout, another day 9, and a third day 4. How many did he catch in the three days?

How many are:

11. $5 + 3$
12. $8 + 5$
13. $7 + 2$
14. $6 + 6$
15. $9 + 8$
16. $11 + 7$
17. $2 + 3 + 7$
18. $1 + 5 + 3$
19. $3 + 5 + 7$
20. $5 + 2 + 6$
21. $4 + 1 + 8$
22. $6 + 0 + 9$
23. $8 + 3 + 7$
24. $9 + 1 + 8$
25. $7 + 6 + 5$
26. $8 + 2 + 3$
27. $3 + 6 + 7$
28. $9 + 7 + 8$

29. Add by 2's from 0 to 24, naming only results.

Solution. — 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

Add:

30. By 2's from 1 to 25.
31. By 3's from 2 to 29.
32. By 3's from 3 to 30.
33. By 3's from 4 to 31.
34. By 4's from 0 to 32.
35. By 4's from 1 to 33.
36. By 4's from 2 to 34.
37. By 4's from 3 to 35.
38. By 5's from 1 to 31.
39. By 5's from 2 to 32.
40. By 5's from 3 to 33.
41. By 5's from 4 to 34.

42. How many are 8 + 6 + 5? 5 + 6 + 8? 8 + 5 + 6? 6 + 8 + 5?

37. Principles of Addition.

1. *Only like numbers, and units of the same order can be added.*

2. *The sum is the same in whatever order the numbers are added.*

WRITTEN EXERCISES.

43. What is the sum of 142, 16, 201, and 410?

$$\begin{array}{r} 142 \\ 16 \\ 201 \\ \underline{410} \\ 769 \end{array}$$

Solution.—We write the numbers so that units of the same order may be in the same column.

Beginning with ones, we add, naming results, thus: 0, 1, 7, 9, and writing the 9 beneath in ones' place.

Adding the tens, 1, 0, 2, 6, the sum, 6 tens, we write beneath in tens' place.

Adding the hundreds, 4, 6, 7, the sum, 7 hundreds, we write beneath in hundreds' place.

The sum, then, is 7 hundreds 6 tens 9 ones, or 769.

To test or prove the work we add the columns downward, and have, as before, the sum 769.

44. What is the sum of 121, 516, 361, and 11?

45. What is the sum of 231 + 114 + 324?

46. How many are 235 + 321 + 142?

47. Sold 416 bushels of corn to one man, 301 to another, and 42 to a third. How many bushels were sold in all?

48. Paid for labor $104, for boards $530, for timber $243, and for hardware $112. How much was paid for all?

49. Bought a horse for $150, a carriage for $200, a harness for $45, and hay and grain for $104. What was the cost of the whole?

50. Mr. Smith, in his will, gave to his son Arthur $500; to his son John $365; to his daughters $475, and to his brother $125. How much did he give them all?

ORAL EXERCISES.

51. In one basket there are 20 apples, in another 9, and in a third 7. How many are there in all?

Add:

52. By 5's from 5 to 35.
53. By 6's from 0 to 36.
54. By 6's from 1 to 37.
55. By 6's from 3 to 39.
56. By 6's from 4 to 40.
57. By 6's from 5 to 41.
58. By 7's from 1 to 43.
59. By 7's from 2 to 44.
60. By 8's from 3 to 43.
61. By 8's from 4 to 44.
62. By 9's from 1 to 46.
63. By 9's from 2 to 47.

64. Add by 7's from 4 to 39; from 5 to 40; from 6 to 41.

65. Add by 8's from 1 to 41; from 2 to 42; from 5 to 45; from 6 to 46; from 7 to 47.

66. Add by 9's from 3 to 48; from 4 to 49; from 5 to 50; from 6 to 51; from 7 to 52.

67. Add by 7's from 0 to 42; by 8's from 0 to 48; by 9's from 0 to 54.

How many are:

68. $17 + 2 + 5 + 6$?
69. $21 + 5 + 8 + 2$?
70. $13 + 2 + 7 + 4$?
71. $32 + 6 + 3 + 9$?

72. What is the sum of 67 and 25?

73. If you should travel one day 20 miles and the next day 24 miles, how far would you travel in the two days?

74. A man earned in one week $11, in another $16, and in a third $13. How much did he earn in all?

75. A father gave to one of his sons 30 cents, to another 14 cents, and to his daughter 12 cents. How much did he give them in all?

76. In one field there are 55 acres, in another 40, and in the third 17. How many acres are there in the three fields?

77. Johnson keeps on one farm 60 cows, on another 19, and on a third 21. How many does he keep on all?

WRITTEN EXERCISES.

78. What is the sum of 595, 961, and 23.

$$\begin{array}{r}595\\961\\23\\\hline 1579\end{array}$$

Solution.—We write the numbers so that units of the same order may be in the same column.

Beginning with ones, we add, thus: 3, 4, 9; sum, 9 ones, which we write beneath in ones' place.

We add the tens, thus: 2, 8, 17; sum, 17 tens, which are 170, or 1 hundred 7 tens. We write the 7 tens beneath in tens' place, and add the 1 hundred with the hundreds in the next column.

We add the hundreds, thus: 1, 10, 15; sum, 15 hundreds, which are 1500, or 1 thousand 5 hundreds. We write the 5 hundreds beneath in hundreds' place, and the 1 thousand in thousands' place.

The sum is 1 thousand 5 hundred 7 tens 9 ones, or 1579.

To prove the work, we add the columns downward, and have, as before, the sum 1579.

79. Add 626, 317, 529, and 12.

80. Add 368, 689, 73, and 19.

81. Add $ 21.642, $ 0.763, $ 3.05, and $ 5.90.

$$\begin{array}{r}\$\,21.642\\0.763\\3.05\;\\5.90\;\\\hline \$\,31.355\end{array}$$

Solution.—Beginning with thousandths, we add, thus: 3, 5; sum, 5 thousandths, which we write beneath in thousandths' place.

We add the hundredths, thus: 5, 11, 15; sum, 15 hundredths, or 1 tenth 5 hundredths. We write the 5 hundredths beneath in hundredths' place, and add the 1 tenth with the tenths of the next column.

We add the tenths, thus: 1, 10, 17, 23; sum, 23 tenths, or 2 ones 3 tenths. We write the 3 tenths beneath in the place of tenths, and add the 2 ones with the ones of the next column.

We add the ones, thus: 2, 7, 10, 11; the sum, 11 ones, or 1 ten 1 one. We write the 1 one beneath in the ones' place, and add the 1 ten with the tens of the next column.

We add the tens, thus: 1, 3; sum, 3 tens, which we write beneath in tens' place.

The sum is $ 31.355, or 31 dollars and 35 cents 5 mills.

82. What is the sum of $31.067, $8.091, $0.46, and $0.31?

83. What is the sum of $13.615, $24.25, $6.14, and $17.66?

84. What is the sum of $91.55, $82.35, $63, and $80.50?

38. Rule for Addition.

Write the numbers so that units of the same order may be in the same column.

Add the right-hand column, writing the units of the sum beneath, and adding the tens, if any, to the next column.

So proceed with all the columns, writing the entire sum of the last column.

39. Proof. — Add the numbers a second time and in a different order (Art. 37).

85.	**86.**	**87.**	**88.**
346	800	1123	5050
275	455	578	7825
62	619	642	1367
6	104	53	1243

89.	**90.**	**91.**	**92.**
315	6541	841.5	13.14
640	1809	302.6	21.65
179	723	427.5	82.91
106	480	122.4	11.78
812	235	324.7	14.43

93.	**94.**	**95.**	**96.**
$53.19	$93.47	$103.64	$111.625
61.25	6.80	205.75	41.55
16.87	0.39	83.91	76.875

97. Find the sum of 406, 781, 918, 846, and 67.
98. Find the sum of 1081, 686, 423, 925, and 328.
99. Find the sum of 2642, 3417, 506, 689, and 482.
100. Find the sum of 391, 638, 402, 83, 493, and 16.
101. Find the sum of 555, 64, 380, 75, 878, and 74.
102. Find the sum of 159, 363, 4682, 8405, and 3377.

How many are:

103. 62.48 + 31.41 + 8.95 + 73 + 0.56?
104. 8.15 + 31.68 + 9.60 + 18.53 + 85.93?
105. 106 + 57.25 + 46.17 + 9.36 + 62.94?
106. 3.19 + 11.93 + 61.85 + 376 + 4781?
107. $16.43 + $24.77 + $75.35 + $9.10?
108. $342 + $164 + $4.95 + $0.74?
109. $78.05 + $72.09 + $8.11 + $9.83?
110. $415 + $88.24 + $13.08 + $16.08?

111. On a farm 105 trees bear pears, 492 bear peaches, 85 bear cherries, and 316 bear apples. How many trees are there in all?

112. An army consisted of 2358 infantry, 868 cavalry, and 1165 artillery. What was the number of the army?

113. A man bought a house for $8750. He paid $363.55 for repairs, $95.63 for painting, and $106.50 for taxes. For how much must he sell it to gain $350?

114. A farmer has four fat oxen. The first weighs 1463 pounds, the second 1385 pounds, the third 1507 pounds, and the fourth 1264 pounds. What is the weight of them all?

115. Bought a suit of clothes for $42.50, an overcoat for $22.75, a pair of boots for $5.25, and a hat for $4.63. What was the cost of the whole?

116. The area of France is 204091 square miles and that of Italy 114290. What is the area of the two countries?

117. What is the sum of $103, $0.06, $15.05, $19.75, and $7.31?

118. Gave $ 73 for a watch, $ 15.50 for a carriage robe, $ 250 for a horse, and sold them so that I gained $ 21.50. What did I sell them for?

119. A merchant began business with goods worth $ 6750; a store worth $ 5700; fixtures worth $ 555.25; and gained $ 1165.45. What was he then worth?

120.	**121.**	**122.**	**123.**
321	8106	31.46	93.045
406	7334	42.47	10.304
718	5570	60.05	7.105
304	2344	53.06	16.321
818	648	71.85	4.554
103	102	40.25	17.056
145	341	14.40	2.005

124. Mount Everest is 13270 feet higher than Mont Blanc, Mont Blanc 4832 feet higher than Etna, and Mount Etna rises 10900 feet above the level of the sea. What is the height of Mount Everest above the sea level?

125. The area of the United States consists of territory ceded by Great Britain as the result of the Revolution, 815615 square miles; acquired from France, 930928 square miles; from Spain, 59268 square miles; by admission of Texas, 237504 square miles; Oregon by treaty, 280425 square miles; from Mexico, 677262 square miles; and from Russia, 577390 square miles. What is the total area of the United States?

126. In building a cottage, the excavating cost $ 34; the cellar walls, $ 110.50; chimneys, $ 18.46; the plastering, $ 73.42; the frame, $ 64.50; the boarding, $ 33.50; the siding, $ 25; the roof boards, $ 20.67; the shingling, $ 62.80; the gutters and hardware, $ 42; the truss work, $ 13; the water tank, $ 15; the cornices, $ 46; the windows, $ 98; the doors, $ 126; the flooring, $ 45; the stairs, $ 20; the base, $ 34.40; the sink, $ 9.50; the cistern, $ 25; the painting, $ 80.12; and incidentals, $ 50. What was the entire cost?

127. Bought a house for $ 23650 and land for $ 73640. Paid $ 4500 for repairs and taxes. For house and land how much must I receive to gain the cost of the land?

128. The first of four numbers is 8437, the second 9325, the third is the sum of the first two, and the fourth is the sum of the second and third. Find the result if the numbers are put together.

129. A, B, C, and D go into business. A puts in $7430, B $ 3200 more, C as much as A and B together, and D as much as A and C together. What is the capital of the firm?

130. In 1880 the Boston and Providence R. R. received from passengers $ 776362.87, from freight $ 486724.85, from rents $ 19395.08, from express companies $ 30202.34, and from mails $ 11240.49. What was the total income for the year?

131. Find the sum of the numbers in Art. 26, Exercises 66 to 74 inclusive.

132. In Exercises 73 to 81 inclusive.

133. In Exercises 81 to 87 inclusive.

134. In Art. 27, Exercises 93 to 100 inclusive.

135. In Exercises 101 to 108 inclusive.

136. In Exercises 109 to 116 inclusive.

137. What are my sales for the week if my daily sales are as follows: Monday $ 347.19, Tuesday $ 847.62, Wednesday $ 9643.27, Thursday $ 9876.50, Friday $ 843.91, Saturday $ 10986.75?

138. What is the income of a gentleman who receives annually from rents $ 2465.29, from mining profits $ 3462, from other business $ 9478.50, and from interest of U. S. Bonds $ 8600?

QUESTIONS.

29. What is the unit of a number? 30. What are like numbers? 31. What is addition? 32. What is the sum?

33. What is the sign of addition? 34. The sign of equality? 35. The dollar sign?

37. What are principles of addition? How do you add? What is the proof?

SUBTRACTION.

40. 1. John has 8 marbles, and his brother 5. How many more marbles has John than his brother?

2. Thomas had 9 apples, and gave 4 of them to Peter How many had he left?

3. How much more are 11 cents than 6 cents?

4. How many dollars are $13 less $7?

5. Ella is 14 years old, and Mary is 9 years old. How many years older is Ella than Mary?

6. Sold a knife for 15 cents and a ball for 7 cents. How much more did I get for the knife than for the ball?

41. **Subtraction** is taking one of two like numbers from the other.

42. The **Difference** is the result of a subtraction.

43. The **Minuend** is the number subtracted from.

44. The **Subtrahend** is the number subtracted.

45. The **Sign of Subtraction** is –, named *minus*. It means *less*. Thus,

$15 - 9 = 6$ is read fifteen less nine are six.

ORAL EXERCISES.

7. A boy raised 13 melons and sold 6. How many had he left?

8. Arthur had 11 cents and gave away 7. How many had he left?

9. In a nest there were 14 eggs, but 5 have been taken away. How many remain in the nest?

10. In a pool were 13 lilies, and 8 have been carried away. How many remain?

11. If of 16 peaches 8 should be eaten, how many would be left?

How many are:

12. 16 − 9
13. 15 − 7
14. 11 − 5
15. 13 − 8
16. 14 − 6
17. 27 − 9
18. 19 − 10
19. 26 − 8
20. 17 − 3
21. 12 − 7
22. 18 − 9
23. 25 − 6
24. 19 − 9
25. 23 − 8
26. 21 − 5
27. 24 − 6
28. 20 − 7
29. 28 − 9

30. Subtract by 2's from 40 back to 20, naming only results.

31. By 3's from 38 to 24.
32. By 4's from 37 to 21.
33. By 5's from 51 to 26.
34. By 6's from 49 to 31.
35. By 7's from 50 to 29.
36. By 8's from 48 to 32.
37. By 5's from 58 to 28.
38. By 4's from 56 to 40.
39. By 7's from 57 to 36.
40. By 6's from 54 to 30.
41. By 8's from 53 to 31.
42. By 9's from 49 to 22.

43. How many are $16 less $9? $7 and how many dollars are $16?

46. Principles of Subtraction.

1. *Only like numbers and units of the same order can be subtracted one from the other.*

2. *The difference and subtrahend together must equal the minuend.*

WRITTEN EXERCISES.

44. Find the difference between 865 and 242.

Minuend	865
Subtrahend	242
Difference	623
Proof	865

Solution. — For convenience, we write the subtrahend under the minuend, so that units of the same order may be in the same column.

2 ones from 5 ones leave 3 ones, which we write beneath in ones' place; 4 tens from 6 tens leave 2 tens, which we write beneath in tens' place; and 2 hundreds from 8 hundreds leave 6 hundreds, which we write beneath in hundreds' place.

The difference is 6 hundreds 2 tens 3 ones, or 623.

To test or prove the work, we add together the difference and subtrahend, and the sum equals the minuend (Art. 46).

Subtract:

45. 235 from 567.
46. 763 from 986.
47. 684 from 889.
48. 462 from 684.
49. 1425 from 2539.
50. 6523 from 9735.
51. 5651 from 7864.
52. 2746 from 3864.

53. A farmer raised 1896 bushels of wheat, and sold 675. How many bushels had he left?

54. A man bought a house for $ 2550, and sold it for $ 2875. How much did he gain?

ORAL EXERCISES.

55. Henry gave 13 cents to John and 9 to James. How many more cents did John receive than James?

56. William is 17 years old and Albert is 9. What is the difference in their ages?

57. A boy had 25 cents, and spent 9. How many cents had he left?

58. Ellen had 23 roses, and gave her teacher 4 and her playmates 6. How many had she left?

59. Gave $ 25 for an overcoat, and $ 8 for a vest. How much more did the one cost than the other?

60. John had 33 peaches and gave 9 away. How many had he left?

How many are:

61. 31 + 7 − 9?
62. 28 + 5 − 7?
63. 35 + 7 − 8?
64. 17 + 7 − 3?
65. 27 + 9 − 6?
66. 30 + 8 − 5?
67. 19 − 8 + 6?
68. 29 − 5 + 9?
69. 37 − 9 + 10?

70. How many are 52 − 37? 63 − 49? 84 − 29?

71. Sold a horse which cost me $ 100 for $ 80. How much did I lose?

72. I have a book of 56 pages, and have read 39 of the pages. How many pages remain to be read?

73. John travels 35 miles a day and Edwin 60 miles. How many more miles does the one travel than the other?

74. A boy had 51 cents and spent 38. How many cents had he left?

WRITTEN EXERCISES.

75. What is the difference between 1624 and 342?

Minuend	1624
Subtrahend	342
Difference	1282
Proof	1624

Solution. — 2 ones from 4 ones leave 2 ones, which we write beneath in the place of ones.

4 tens cannot be taken from 2 tens; we therefore take 1 hundred, or 10 tens, from the 6 hundreds of the minuend, leaving 5 hundreds, and adding the 10 tens to the 2 tens we have 12 tens; 4 tens from 12 tens leave 8 tens, which we write beneath in the place of tens.

3 hundreds from 5 hundreds leave 2 hundreds, which we write in hundreds' place; no thousands from 1 thousand leaves 1 thousand, which we write in thousands' place.

The difference is 1282. This we prove by adding the difference and subtrahend, and finding the sum to be equal to the minuend.

76. Find the difference between 167 and 476.

77. Find the difference between 389 and 581.

78. What number and 1543 make 1735?

79. Subtract 32.15 from 78.8.

Minuend	78.80
Subtrahend	32.15
Difference	46.65

Solution. — We make the decimal places the same in the two numbers by filling the place of hundredths in the minuend by 0.

5 hundredths cannot be taken from 0 hundredths; we therefore take 1 tenth, or 10 hundredths, from the 8 tenths, leaving 7 tenths; 5 hundredths from the 10 hundredths leave 5 hundredths, which we write beneath in hundredths' place.

1 tenth from 7 tenths leaves 6 tenths, which we write beneath in tenths' place; and subtracting the ones and tens, and writing the result beneath, we have 46.65 as the result required.

80. Subtract 167.807 from 970.96.

81. Take $158.55 from $549.60.

82. How much more than $91.97 is $147.11?

83. How much and $6711.45 make $7050.25?

47. Rule for Subtraction.

Write the subtrahend under the minuend, placing units of the same order in the same column.

Begin with the units of the lowest order to subtract, and proceed to the highest, writing the result beneath.

If any order of the minuend has less units than the same order of the subtrahend, increase its units by ten, and subtract; consider the units of the next minuend order one less, and proceed as before.

48. PROOF. Add the subtrahend and the difference together; the sum should equal the minuend (Art. 46).

	84.	**85.**	**86.**	**87.**
From	8647	5375	6365	9406
Take	3451	406	4506	8350

	88.	**89.**	**90.**	**91.**
From	31867	12805	148.48	63.859
Take	1905	9264	92.09	49.608

	92.	**93.**	**94.**	**95.**
From	$63.95	$85.69	$96.70	$182.05
	57.68	1.73	0.89	152.06

96. Find the difference between 34000 and 21345.

(3)(9)(9)(10)
34000
21345
12655

Solution. — There being 0 ones, 0 tens, 0 hundreds in the minuend, we take 1 of the 4 thousands (leaving 3 thousands), or 10 hundreds; then taking 1 of the 10 hundreds (leaving 9 hundreds), or 10 tens; and 1 of the 10 tens (leaving 9 tens), or 10 ones; the minuend may then be considered 3 ten-thousands 3 thousands 9 hundreds 9 tens and 10 ones.

Taking from the changed minuend the 2 ten-thousands 1 thousand 3 hundreds 4 tens 5 ones of the subtrahend, we have as the difference required, 12655.

Subtract:

97. 5544 from 40000.
98. 180 from 98000.
99. 12453 from 35421.
100. 97 from 10000.
101. 58346 from 67500.
102. 9999 from 10000.
103. 67.055 from 73.607.
104. 19.55 from 831.50.
105. 444.5 from 10060.
106. 921.56 from 1000.
107. $13.63 from $500.20.
108. $127 from $1963.75.

109. What is the value of 83956 — 78415?

110. What is the value of 60440 — 33457?

111. What is the value of 109800 — 98799?

112. From one hundred nine take one, and nine hundredths?

113. America was discovered in 1492; how many years from that date to 1881?

114. A man began business with $1760, and after two years had $2500.75. How much had he gained?

115. The number of regular soldiers furnished by the several States in the war of the Revolution was 231771; of these a single State furnished 67907. How many were furnished by other States?

116. A merchant bought goods to the amount of $7563.56, and sold them for $11630.50. How much did he gain?

117. The Atlantic slope contains 967576 square miles, and the Mississippi valley 1237111 square miles. How much does the latter exceed the former?

118. The population of New York City in 1870 was 942192, and in 1880 was 1209561. How much was the gain?

119. The product of gold in a certain year was from Nevada $19546516, and from California $17760679. How much greater was the product from Nevada than from California?

MISCELLANEOUS EXERCISES.

120. A man owing $767.50 paid one time $190, at another $131, and at a third $155.25. How much did he then owe?

121. James Dow's real estate is valued at $3769, and his personal estate at $2648.75. He owes Job Smith $1728 and Abraham Tyler $1161.93. How much is he worth when these debts are paid?

122. Modern notation in arithmetic was introduced from Arabia into Europe in the year 991, algebra from the same country in 431 years later, and decimal fractions were invented in 1602. Required the number of years from each to 1882.

123. Sydney has $178.50, Albert $75.75 more than Sydney, and Charles has as much as Sydney and Albert less $80.93. How much more has Charles than Sydney?

124. What is the value of 1645 + 635 + 416 − 1314?

125. Two vessels 4563 miles apart start to meet each other. When one of them has made 1575 miles of the distance and the other 1658 miles, how far are they apart?

126. A man whose property was $50675 gave his son Edwin $8555.50, his son Robert $7000, his daughter Mary $9563.75, his wife $20000, and a public library the remainder. How much was given the public library?

127. The population, in 1880, of Philadelphia was 847542; of Boston, 362535; and of Providence, 104760. How much greater was the population of Philadelphia than that of Boston and Providence?

128. A man who had $1250 in a savings bank took out $51.75 at one time, $84.93 at another, and $267 at another. He then put in $185. How much then had he in the bank?

QUESTIONS.

41. What is subtraction? 42. What is the difference? 43. The minuend? 44. The subtrahend? 45. The sign of subtraction?

46. What are principles of subtraction? 47. How do you subtract? If any order of the minuend has less units than the same order of the subtrahend, what is done? 48. What is the proof of tion?

MULTIPLICATION.

49. 1. If a man can earn 8 dollars in a week, how many dollars can he earn in 4 weeks? How many dollars are 4 times 8 dollars?

2. What will 5 quarts of cherries cost at 10 cents a quart?

3. William has 9 apples and Alfred 7 times as many. How many has Alfred? 7 times 9 are how many ones?

50. **Multiplication** is taking one number as many times as there are ones in another.

51. The **Multiplicand** is the number to be taken or multiplied.

52. The **Multiplier** is the number that shows how many times the multiplicand is to be taken.

53. The **Product** is the result of a multiplication; and the *factors* of *a product* are the numbers multiplied together to produce it.

54. The **Sign** of Multiplication is ×. It means *multiplied by*, or *times*. Thus,

7 × 5 may be read seven multiplied by five, or seven times five.

55. A **Concrete Number** is a number in which some kind of unit is named. Thus,

2 books, 3 days, $ 7, are concrete numbers.

56. An **Abstract Number** is a number in which no particular kind of unit is named. Thus,

2, 5, 7 are abstract numbers.

57. MULTIPLICATION TABLES.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

ORAL EXERCISES.

4. How many wings have 8 doves?

5. How much can you earn in 3 days if you earn 10 cents a day?

6. At 9 cents each what will a half-dozen writing-books cost?

7. How many horns have 8 yoke of oxen?

8. How many days in 12 weeks? In 9 weeks?

9. How far can you ride in 7 hours at the rate of 8 miles an hour?

10. How many desks in a school-room having 8 rows with 7 desks in a row?

11. How many legs have 6 span of horses?

12. At 7 cents each what cost a dozen pencils?

13. A ten-foot pole is how many inches long?

How many are:

14.
7 × 9
8 × 8
9 × 6
7 × 7
8 × 3

15.
5 × 12
8 × 10
6 × 11
9 × 12
8 × 10

16.
2 × 7
10 × 12
8 × 7
9 × 4
11 × 11

17.
6 × 7
12 × 12
11 × 8
9 × 7
4 × 7

18.
12 × 9 − 8 × 12
6 × 6 − 4 × 7
7 × 8 − 5 × 9
6 × 7 − 6 × 6
9 × 0 × 8 × 4

19.
6 × 9 − 7 × 7
8 × 6 − 4 × 10
7 × 12 − 9 − 9
12 × 11 − 10 × 10
12 × 6 − 8 × 8

20. Give the products by 2, from 2 × 2 to 2 × 12.

Solution. — 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

21. Give the products by 4, from 4 × 0 to 4 × 12.

22. Count by 7's from 0 to 84.

23. Subtract by 9's from 108 to 0.

24. Name all the products of which 12 is a factor to 144.

25. What cost 9 six-cent books ? 12 ? 8 ?

26. What will 12 dozen eggs cost at a cent apiece ?

27. How many days in a 9 weeks' vacation ?

28. How many corners have 11 squares ?

29. I pay 7 cents daily for milk. What is my milk bill for a week ?

30. How many inches in 8 feet ? In 9 ? In 11 ?

31. How many feet in 12 yards of ribbon ?

32. What cost 7 lbs. at 5 cents a pound, and 5 quarts at 4 cents a quart ?

33. What numbers multiplied make 36 ? 56 ? 81 ? 96 ?

34. What factors produce 48 ? 72 ? 121 ? 63 ? 49 ?

35. At $8 a barrel, how many times $8 will 9 barrels of flour cost ?

36. How many are 9 × $8 ? 8 × $9 ? 3 × 2 × 2 ? 2 × 3 × 2 ? 2 × 2 × 3 ?

58. Principles of Multiplication.

1. *The multiplier is always considered an abstract number.*

2. *The product and the multiplicand are always like numbers.*

3. *The product is the same whatever the order of the factors.*

WRITTEN EXERCISES.

37. Find the product of 274 multiplied by 7.

Factors	Multiplicand	274
	Multiplier	7
	Product	1918

Solution. — We write the multiplier, 7 ones, under the ones' figure of the multiplicand.

Beginning with the ones, we multiply : 7 times 4 ones are 28 ones, or 2 tens 8 ones. We write the 8 ones beneath in the place of ones, and reserve the 2 tens.

7 times 7 tens are 49 tens, and 49 tens plus the 2 tens reserved are 51 tens, or 5 hundreds 1 ten. We write the 1 ten beneath in tens' place, and reserve the 5 hundreds.

7 times 2 hundreds are 14 hundreds, and 14 hundreds plus 5 hundreds reserved are 19 hundreds, or 1 thousand and 9 hundreds. We write the 9 hundreds beneath in hundreds' place, and the 1 thousand in thousands' place.

The product is 1 thousand 9 hundreds 1 ten 8 ones, or 1918.

	38.	**39.**	**40.**	**41.**
Multiplicand	756	4567	1109	6201
Multiplier	4	3	5	8

	42.	**43.**	**44.**	**45.**
Multiplicand	3416	2608	12345	24301
Multiplier	6	2	7	9

46. What is the product of 64.25 by 5 ?

Multiplicand	64.25
Multiplier	5
Product	321.25

Solution. — 5 times 5 hundredths are 25 hundredths, or two tenths and 5 hundredths. We write the 5 hundredths beneath in hundredths' place and reserve the 2 tenths.

5 times 2 tenths are 10 tenths, and 10 tenths plus the 2 tenths reserved are 12 tenths, or 1 one and 2 tenths. We write the 2 tenths beneath in tenths' place, and reserve the 1 one.

5 times 4 ones are 20 ones, and 20 ones plus the 1 one reserved are 21 ones, or 2 tens and 1 one. We write the 1 one beneath in ones' place, and reserve the 2 tens; and so on. The multiplier being an integer, the product has as many decimal places as the multiplicand.

	47.	**48.**	**49.**	**50.**
Multiply	35.07	6.135	1635.9	7128.53
By	8	5	2	7

	51.	**52.**	**53.**	**54.**
Multiply	\$ 124.35	\$ 192.547	\$ 823.50	\$ 8537.64
By	4	6	8	9

64. Write in figures, three hundred twenty-nine thousand four hundred fifteen.

65. Write in figures, nine thousand seven hundred fifty-two, and one hundred one thousandths.

26. Rule for writing Numbers.

Beginning at the left, write the figures of each group in their order, filling vacant places and groups with ciphers.

Write in figures:

66. Six thousand five hundred nine.

67. Eleven thousand nine hundred eleven.

68. Thirty-seven thousand four hundred eighty-nine.

69. Ninety thousand four hundred forty-four.

70. One hundred sixty-three thousand.

71. Two hundred twenty thousand two hundred sixty-two.

72. Five hundred seventy-four thousand three hundred thirty-five.

73. Seven hundred fifty-three thousand seven hundred fifty.

74. Five thousand four hundred eighty-nine.

75. Four hundred eighty thousand eight.

76. One million one hundred thirteen.

77. Three million three thousand thirty.

78. Nine hundred seven million eight hundred five thousand seventy-four.

79. Fifty-seven billion forty-four million ninety-three thousand eighty-three.

80. Thirteen million six hundred thousand one hundred seventy.

81. Five hundred six billion two million four [illegible] one.

82. One thousand seven hundred sixty-two, and five tenths.

83. Nine hundred ninety-four, and thirty-eight hundredths.

84. One hundred nineteen thousand, and one hundred nineteen thousandths.

85. One million four hundred thousand three hundred, and thirty-three hundredths.

86. Sixteen thousand seven hundred forty-five, and eighty-six thousandths.

87. Three billions three millions two thousands seven hundreds, and three thousandths.

EXERCISES IN READING NUMBERS.

88. Write and read 25362795.

25,362,795

Solution. — 25362795 marked off into groups becomes 25,362,795. The first group from the right expresses 795 ones, or 795; the next group expresses 362 thousands; and the third expresses twenty-five millions. The whole is read: twenty-five million three hundred sixty-two thousand seven hundred ninety-five.

89. Write and read 3257893.

90. Write and read 64058.109.

64,058.109

Solution. — 64058.109 marked off into groups becomes 64,058.109, which is read sixty-four thousand fifty-eight, and one hundred nine thousandths.

91. Write and read 1673405.75.

92. Write and read 906005.805.

27. Rule for Reading Numbers.

Beginning at the ones' place, mark off the numbers by commas, into as many groups as possible of three figures each.

Begin at the left and read each group as if it stood alone, giving the group-name except to the ones' group.

If there is a decimal, read it as if it were at the left of the point, and add the place-name of the last figure.

Copy and read :

93. 486
94. 2385
95. 7275
96. 12361
97. 62004
98. 199
99. 78382
100. 160405
101. 6082
102. 19009
103. 163404
104. 789685
105. 99999
106. 1634562
107. 25401300
108. 312407981
109. 0.31
110. 117.03
111. .064
112. 118646.5
113. 2567.02
114. 88.999
115. 1158834.55
116. 100600.789

QUESTIONS.

Art. 1. What is a unit? 2. What is a number? 3. What is arithmetic?

4. What is notation? 5. What is numeration? 6. What are figures? 7. What does zero, or cipher, express? What do the other nine figures express?

9. When has a figure a simple name and value? When has it a place-name and value?

10. What is the place of a figure? 11. If in the first place, what place-name has a figure? What units does it express? If in the second place, what place-name has a figure? What units does it express?

12. What place-name has a figure at the left of tens? What units does it express?

13. What place-name has a figure at the left of hundreds? What units does it express?

15. Give the place-names beginning with ones. Give the corresponding orders of units.

16. What form a group for convenience in reading and writing numbers? How is each group named? Name groups beginning with ones. What is used to mark off the groups? 17. What does each complete group contain?

18. What are principles of notation? 19. What is our system called? What do the successive orders of units form?

20. What is the decimal point? For what is it written? 21. What is the number at the left of the point?

26. How are numbers written in figures? 27. How are numbers written in figures read?

ADDITION.

28. 1. James has 9 apples and Henry has 3. How many apples have both?

2. Mary had 7 oranges and her brother gave her 5. How many had she then?

3. A father gave to one of his children 8 cents, to another 5 cents, and to a third 6 cents. How many cents did he give them in all?

4. How many are 9 and 3? 7 and 5? 8, 5, and 6?

5. How many dollars are 8 dollars, 7 dollars, and 3 dollars?

6. What is the unit of 8 dollars, 7 dollars, and 3 dollars?

29. The **Unit of a number** is one of that number. Thus,

One pound is the unit of 8 pounds, one quart is the unit of 27 quarts.

30. **Like numbers** are numbers having the same unit.

Thus 3, 5, 7; and 6 cents, 4 cents and 5 cents, are like numbers.

31. **Addition** is finding a number equal to two or more given numbers.

32. The **sum**, or **amount**, is the result of an addition.

33. The **sign of addition** is +, named *plus*. It means *more*, and is generally read *and*. Thus,

$4 + 5 + 6$ is read four and five and six.

34. The **sign of equality** is =. It means *equal*, or *equal to*, and is often read *are*. Thus,

$7 + 8 = 15$ is read seven and eight are fifteen.

35. The sign, $, written before a number, means *dollars.* Thus,

$10 is read ten dollars.

36. *Cents,* expressed in figures, may occupy two decimal places, *tenths* and *hundredths,* and *mills* one place, *thousandths.* Thus,

$0.53, or $.53, is read fifty-three cents, and $28.005 is read twenty-eight dollars and five mills.

ORAL EXERCISES.

7. How many dollars are $7, $6, and $2?

8. If you pay 10 cents for a slate, 9 cents for paper, and 3 cents for a pencil, how much do you pay for the whole?

9. 12 boys are at play in one place, 6 in another, and 5 in another. How many boys are at play in all?

10. John one day caught 8 trout, another day 9, and a third day 4. How many did he catch in the three days?

How many are:

11. 5 + 3
12. 8 + 5
13. 7 + 2
14. 6 + 6
15. 9 + 8
16. 11 + 7
17. 2 + 3 + 7
18. 1 + 5 + 3
19. 3 + 5 + 7
20. 5 + 2 + 6
21. 4 + 1 + 8
22. 6 + 0 + 9
23. 8 + 3 + 7
24. 9 + 1 + 8
25. 7 + 6 + 5
26. 8 + 2 + 3
27. 3 + 6 + 7
28. 9 + 7 + 8

29. Add by 2's from 0 to 24, naming only results.

Solution. — 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

Add:

30. By 2's from 1 to 25.
31. By 3's from 2 to 29.
32. By 3's from 3 to 30.
33. By 3's from 4 to 31.
34. By 4's from 0 to 32.
35. By 4's from 1 to 33.
36. By 4's from 2 to 34.
37. By 4's from 3 to 35.
38. By 5's from 1 to 31.
39. By 5's from 2 to 32.
40. By 5's from 3 to 33.
41. By 5's from 4 to 34.

42. How many are 8 + 6 + 5? 5 + 6 + 8? 8 + 5 + 6? 6 + 8 + 5?

37. Principles of Addition.

1. *Only like numbers, and units of the same order can be added.*

2. *The sum is the same in whatever order the numbers are added.*

WRITTEN EXERCISES.

43. What is the sum of 142, 16, 201, and 410?

$$\begin{array}{r} 142 \\ 16 \\ 201 \\ \underline{410} \\ 769 \end{array}$$

Solution.—We write the numbers so that units of the same order may be in the same column.

Beginning with ones, we add, naming results, thus: 0, 1, 7, 9, and writing the 9 beneath in ones' place.

Adding the tens, 1, 0, 2, 6, the sum, 6 tens, we write beneath in tens' place.

Adding the hundreds, 4, 6, 7, the sum, 7 hundreds, we write beneath in hundreds' place.

The sum, then, is 7 hundreds 6 tens 9 ones, or 769.

To test or prove the work we add the columns downward, and have, as before, the sum 769.

44. What is the sum of 121, 516, 361, and 11?

45. What is the sum of 231 + 114 + 324?

46. How many are 235 + 321 + 142?

47. Sold 416 bushels of corn to one man, 301 to another, and 42 to a third. How many bushels were sold in all?

48. Paid for labor $104, for boards $530, for timber $243, and for hardware $112. How much was paid for all?

49. Bought a horse for $150, a carriage for $200, a harness for $45, and hay and grain for $104. What was the cost of the whole?

50. Mr. Smith, in his will, gave to his son Arthur $500; to his son John $365; to his daughters $475, and to his brother $125. How much did he give them all?

ORAL EXERCISES.

51. In one basket there are 20 apples, in another 9, and in a third 7. How many are there in all?

Add:

52. By 5's from 5 to 35.
53. By 6's from 0 to 36.
54. By 6's from 1 to 37.
55. By 6's from 3 to 39.
56. By 6's from 4 to 40.
57. By 6's from 5 to 41.
58. By 7's from 1 to 43.
59. By 7's from 2 to 44.
60. By 8's from 3 to 43.
61. By 8's from 4 to 44.
62. By 9's from 1 to 46.
63. By 9's from 2 to 47.

64. Add by 7's from 4 to 39; from 5 to 40; from 6 to 41.

65. Add by 8's from 1 to 41; from 2 to 42; from 5 to 45; from 6 to 46; from 7 to 47.

66. Add by 9's from 3 to 48; from 4 to 49; from 5 to 50; from 6 to 51; from 7 to 52.

67. Add by 7's from 0 to 42; by 8's from 0 to 48; by 9's from 0 to 54.

How many are:

68. $17 + 2 + 5 + 6$?
69. $21 + 5 + 8 + 2$?
70. $13 + 2 + 7 + 4$?
71. $32 + 6 + 3 + 9$?

72. What is the sum of 67 and 25?

73. If you should travel one day 20 miles and the next day 24 miles, how far would you travel in the two days?

74. A man earned in one week $11, in another $16, and in a third $13. How much did he earn in all?

75. A father gave to one of his sons 30 cents, to another 14 cents, and to his daughter 12 cents. How much did he give them in all?

76. In one field there are 55 acres, in another 40, and in the third 17. How many acres are there in the three fields?

77. Johnson keeps on one farm 60 cows, on another 19, and on a third 21. How many does he keep on all?

WRITTEN EXERCISES.

78. What is the sum of 595, 961, and 23.

$$\begin{array}{r}595\\961\\23\\\hline 1579\end{array}$$

Solution. — We write the numbers so that units of the same order may be in the same column.

Beginning with ones, we add, thus: 3, 4, 9; sum, 9 ones, which we write beneath in ones' place.

We add the tens, thus: 2, 8, 17; sum, 17 tens, which are 170, or 1 hundred 7 tens. We write the 7 tens beneath in tens' place, and add the 1 hundred with the hundreds in the next column.

We add the hundreds, thus: 1, 10, 15; sum, 15 hundreds, which are 1500, or 1 thousand 5 hundreds. We write the 5 hundreds beneath in hundreds' place, and the 1 thousand in thousands' place.

The sum is 1 thousand 5 hundred 7 tens 9 ones, or 1579.

To prove the work, we add the columns downward, and have, as before, the sum 1579.

79. Add 626, 317, 529, and 12.

80. Add 368, 689, 73, and 19.

81. Add \$21.642, \$0.763, \$3.05, and \$5.90.

$$\begin{array}{r}\$21.642\\0.763\\3.05\\5.90\\\hline \$31.355\end{array}$$

Solution. — Beginning with thousandths, we add, thus: 3, 5; sum, 5 thousandths, which we write beneath in thousandths' place.

We add the hundredths, thus: 5, 11, 15; sum, 15 hundredths, or 1 tenth 5 hundredths. We write the 5 hundredths beneath in hundredths' place, and add the 1 tenth with the tenths of the next column.

We add the tenths, thus: 1, 10, 17, 23; sum, 23 tenths, or 2 ones 3 tenths. We write the 3 tenths beneath in the place of tenths, and add the 2 ones with the ones of the next column.

We add the ones, thus: 2, 7, 10, 11; the sum, 11 ones, or 1 ten 1 one. We write the 1 one beneath in the ones' place, and add the 1 ten with the tens of the next column.

We add the tens, thus: 1, 3; sum, 3 tens, which we write beneath in tens' place.

The sum is \$31.355, or 31 dollars and 35 cents 5 mills.

82. What is the sum of $31.067, $8.091, $0.46, and $0.31?

83. What is the sum of $13.615, $24.25, $6.14, and $17.66?

84. What is the sum of $91.55, $82.35, $63, and $80.50?

38. Rule for Addition.

Write the numbers so that units of the same order may be in the same column.

Add the right-hand column, writing the units of the sum beneath, and adding the tens, if any, to the next column.

So proceed with all the columns, writing the entire sum of the last column.

39. Proof. — Add the numbers a second time and in a different order (Art. 37).

85.	**86.**	**87.**	**88.**
346	800	1123	5050
275	455	578	7825
62	619	642	1367
6	104	53	1243

89.	**90.**	**91.**	**92.**
315	6541	841.5	13.14
640	1809	302.6	21.65
179	723	427.5	82.91
106	480	122.4	11.78
812	235	324.7	14.43

93.	**94.**	**95.**	**96.**
$ 53.19	$ 93.47	$ 103.64	$ 111.625
61.25	6.80	205.75	41.55
16.87	0.39	83.91	76.875

97. Find the sum of 406, 781, 918, 846, and 67.
98. Find the sum of 1081, 686, 423, 925, and 328.
99. Find the sum of 2642, 3417, 506, 689, and 482.
100. Find the sum of 391, 638, 402, 83, 493, and 16.
101. Find the sum of 555, 64, 380, 75, 878, and 74.
102. Find the sum of 159, 363, 4682, 8405, and 3377.

How many are:

103. 62.48 + 31.41 + 8.95 + 73 + 0.56?
104. 8.15 + 31.68 + 9.60 + 18.53 + 85.93?
105. 106 + 57.25 + 46.17 + 9.36 + 62.94?
106. 3.19 + 11.93 + 61.85 + 376 + 4781?
107. $16.43 + $24.77 + $75.35 + $9.10?
108. $342 + $164 + $4.95 + $0.74?
109. $78.05 + $72.09 + $8.11 + $9.83?
110. $415 + $88.24 + $13.08 + $16.08?

111. On a farm 105 trees bear pears, 492 bear peaches, 85 bear cherries, and 316 bear apples. How many trees are there in all?

112. An army consisted of 2358 infantry, 868 cavalry, and 1165 artillery. What was the number of the army?

113. A man bought a house for $8750. He paid $363.55 for repairs, $95.63 for painting, and $106.50 for taxes. For how much must he sell it to gain $350?

114. A farmer has four fat oxen. The first weighs 1463 pounds, the second 1385 pounds, the third 1507 pounds, and the fourth 1264 pounds. What is the weight of them all?

115. Bought a suit of clothes for $42.50, an overcoat for $22.75, a pair of boots for $5.25, and a hat for $4.63. What was the cost of the whole?

116. The area of France is 204091 square miles and that of Italy 114290. What is the area of the two countries?

117. What is the sum of $103, $0.06, $15.05, $19.75, and $7.31?

118. Gave $73 for a watch, $15.50 for a carriage robe, $250 for a horse, and sold them so that I gained $21.50. What did I sell them for?

119. A merchant began business with goods worth $6750; a store worth $5700; fixtures worth $555.25; and gained $1165.45. What was he then worth?

120.	**121.**	**122.**	**123.**
321	8106	31.46	93.045
406	7334	42.47	10.304
718	5570	60.05	7.105
304	2344	53.06	16.321
818	648	71.85	4.554
103	102	40.25	17.056
145	341	14.40	2.005

124. Mount Everest is 13270 feet higher than Mont Blanc, Mont Blanc 4832 feet higher than Etna, and Mount Etna rises 10900 feet above the level of the sea. What is the height of Mount Everest above the sea level?

125. The area of the United States consists of territory ceded by Great Britain as the result of the Revolution, 815615 square miles; acquired from France, 930928 square miles; from Spain, 59268 square miles; by admission of Texas, 237504 square miles; Oregon by treaty, 280425 square miles; from Mexico, 677262 square miles; and from Russia, 577390 square miles. What is the total area of the United States?

126. In building a cottage, the excavating cost $34; the cellar walls, $110.50; chimneys, $18.46; the plastering, $73.42; the frame, $64.50; the boarding, $33.50; the siding, $25; the roof boards, $20.67; the shingling, $62.80; the gutters and hardware, $42; the truss work, $13; the water tank, $15; the cornices, $46; the windows, $98; the doors, $126; the flooring, $45; the stairs, $20; the base, $34.40; the sink, $9.50; the cistern, $25; the painting, $80.12; and incidentals, $50. What was the entire cost?

127. Bought a house for $ 23650 and land for $ 73640. Paid $ 4500 for repairs and taxes. For house and land how much must I receive to gain the cost of the land?

128. The first of four numbers is 8437, the second 9325, the third is the sum of the first two, and the fourth is the sum of the second and third. Find the result if the numbers are put together.

129. A, B, C, and D go into business. A puts in $7430, B $ 3200 more, C as much as A and B together, and D as much as A and C together. What is the capital of the firm?

130. In 1880 the Boston and Providence R. R. received from passengers $ 776362.87, from freight $ 486724.85, from rents $ 19395.08, from express companies $ 30202.34, and from mails $ 11240.49. What was the total income for the year?

131. Find the sum of the numbers in Art. 26, Exercises 66 to 74 inclusive.

132. In Exercises 73 to 81 inclusive.

133. In Exercises 81 to 87 inclusive.

134. In Art. 27, Exercises 93 to 100 inclusive.

135. In Exercises 101 to 108 inclusive.

136. In Exercises 109 to 116 inclusive.

137. What are my sales for the week if my daily sales are as follows: Monday $ 347.19, Tuesday $ 847.62, Wednesday $ 9643.27, Thursday $ 9876.50, Friday $ 843.91, Saturday $ 10986.75?

138. What is the income of a gentleman who receives annually from rents $ 2465.29, from mining profits $ 3462, from other business $ 9478.50, and from interest of U. S. Bonds $ 8600?

QUESTIONS.

29. What is the unit of a number? 30. What are like numbers? 31. What is addition? 32. What is the sum?

33. What is the sign of addition? 34. The sign of equality? 35. The dollar sign?

37. What are principles of addition? How do you add? What is the proof?

SUBTRACTION.

40. 1. John has 8 marbles, and his brother 5. How many more marbles has John than his brother?

2. Thomas had 9 apples, and gave 4 of them to Peter How many had he left?

3. How much more are 11 cents than 6 cents?

4. How many dollars are $ 13 less $ 7?

5. Ella is 14 years old, and Mary is 9 years old. How many years older is Ella than Mary?

6. Sold a knife for 15 cents and a ball for 7 cents. How much more did I get for the knife than for the ball?

41. **Subtraction** is taking one of two like numbers from the other.

42. The **Difference** is the result of a subtraction.

43. The **Minuend** is the number subtracted from.

44. The **Subtrahend** is the number subtracted.

45. The **Sign of Subtraction** is $-$, named *minus*. It means *less*. Thus,

$15 - 9 = 6$ is read fifteen less nine are six.

ORAL EXERCISES.

7. A boy raised 13 melons and sold 6. How many had he left?

8. Arthur had 11 cents and gave away 7. How many had he left?

9. In a nest there were 14 eggs, but 5 have been taken away. How many remain in the nest?

10. In a pool were 13 lilies, and 8 have been carried away. How many remain?

11. If of 16 peaches 8 should be eaten, how many would be left?

How many are:

12. 16 – 9
13. 15 – 7
14. 11 – 5
15. 13 – 8
16. 14 – 6
17. 27 – 9
18. 19 – 10
19. 26 – 8
20. 17 – 3
21. 12 – 7
22. 18 – 9
23. 25 – 6
24. 19 – 9
25. 23 – 8
26. 21 – 5
27. 24 – 6
28. 20 – 7
29. 28 – 9

30. Subtract by 2's from 40 back to 20, naming only results.

31. By 3's from 38 to 24.
32. By 4's from 37 to 21.
33. By 5's from 51 to 26.
34. By 6's from 49 to 31.
35. By 7's from 50 to 29.
36. By 8's from 48 to 32.
37. By 5's from 58 to 28.
38. By 4's from 56 to 40.
39. By 7's from 57 to 36.
40. By 6's from 54 to 30.
41. By 8's from 53 to 31.
42. By 9's from 49 to 22.

43. How many are $16 less $9? $7 and how many dollars are $16?

46. Principles of Subtraction.

1. *Only like numbers and units of the same order can be subtracted one from the other.*

2. *The difference and subtrahend together must equal the minuend.*

WRITTEN EXERCISES.

44. Find the difference between 865 and 242.

Minuend	865
Subtrahend	242
Difference	623
Proof	865

Solution. — For convenience, we write the subtrahend under the minuend, so that units of the same order may be in the same column.

2 ones from 5 ones leave 3 ones, which we write beneath in ones' place; 4 tens from 6 tens leave 2 tens, which we write beneath in tens' place; and 2 hundreds from 8 hundreds leave 6 hundreds, which we write beneath in hundreds' place.

The difference is 6 hundreds 2 tens 3 ones, or 623.

To test or prove the work, we add together the difference and subtrahend, and the sum equals the minuend (Art. 46).

Subtract:

45. 235 from 567.
46. 763 from 986.
47. 684 from 889.
48. 462 from 684.
49. 1425 from 2539.
50. 6523 from 9735.
51. 5651 from 7864.
52. 2746 from 3864.

53. A farmer raised 1896 bushels of wheat, and sold 675. How many bushels had he left?

54. A man bought a house for $ 2550, and sold it for $ 2875. How much did he gain?

ORAL EXERCISES.

55. Henry gave 13 cents to John and 9 to James. How many more cents did John receive than James?

56. William is 17 years old and Albert is 9. What is the difference in their ages?

57. A boy had 25 cents, and spent 9. How many cents had he left?

58. Ellen had 23 roses, and gave her teacher 4 and her playmates 6. How many had she left?

59. Gave $ 25 for an overcoat, and $ 8 for a vest. How much more did the one cost than the other?

60. John had 33 peaches and gave 9 away. How many had he left?

How many are:

61. 31 + 7 − 9?
62. 28 + 5 − 7?
63. 35 + 7 − 8?
64. 17 + 7 − 3?
65. 27 + 9 − 6?
66. 30 + 8 − 5?
67. 19 − 8 + 6?
68. 29 − 5 + 9?
69. 37 − 9 + 10?

70. How many are 52 − 37? 63 − 49? 84 − 29?

71. Sold a horse which cost me $ 100 for $ 80. How much did I lose?

72. I have a book of 56 pages, and have read 39 of the pages. How many pages remain to be read?

73. John travels 35 miles a day and Edwin 60 miles. How many more miles does the one travel than the other?

74. A boy had 51 cents and spent 38. How many cents had he left?

WRITTEN EXERCISES.

75. What is the difference between 1624 and 342?

Minuend	1624
Subtrahend	342
Difference	1282
Proof	1624

Solution. — 2 ones from 4 ones leave 2 ones, which we write beneath in the place of ones.

4 tens cannot be taken from 2 tens; we therefore take 1 hundred, or 10 tens, from the 6 hundreds of the minuend, leaving 5 hundreds, and adding the 10 tens to the 2 tens we have 12 tens; 4 tens from 12 tens leave 8 tens, which we write beneath in the place of tens.

3 hundreds from 5 hundreds leave 2 hundreds, which we write in hundreds' place; no thousands from 1 thousand leaves 1 thousand, which we write in thousands' place.

The difference is 1282. This we prove by adding the difference and subtrahend, and finding the sum to be equal to the minuend.

76. Find the difference between 167 and 476.

77. Find the difference between 389 and 581.

78. What number and 1543 make 1735?

79. Subtract 32.15 from 78.8.

Minuend	78.80
Subtrahend	32.15
Difference	46.65

Solution. — We make the decimal places the same in the two numbers by filling the place of hundredths in the minuend by 0.

5 hundredths cannot be taken from 0 hundredths; we therefore take 1 tenth, or 10 hundredths, from the 8 tenths, leaving 7 tenths; 5 hundredths from the 10 hundredths leave 5 hundredths, which we write beneath in hundredths' place.

1 tenth from 7 tenths leaves 6 tenths, which we write beneath in tenths' place; and subtracting the ones and tens, and writing the result beneath, we have 46.65 as the result required.

80. Subtract 167.807 from 970.96.

81. Take $158.55 from $549.60.

82. How much more than $91.97 is $147.11 ?

83. How much and $6711.45 make $7050.25 ?

47. Rule for Subtraction.

Write the subtrahend under the minuend, placing units of the same order in the same column.

Begin with the units of the lowest order to subtract, and proceed to the highest, writing the result beneath.

If any order of the minuend has less units than the same order of the subtrahend, increase its units by ten, and subtract; consider the units of the next minuend order one less, and proceed as before.

48. PROOF. Add the subtrahend and the difference together; the sum should equal the minuend (Art. 46).

	84.	**85.**	**86.**	**87.**
From	8647	5375	6365	9406
Take	3451	406	4506	8350

	88.	**89.**	**90.**	**91.**
From	31867	12805	148.48	63.859
Take	1905	9264	92.09	49.608

	92.	**93.**	**94.**	**95.**
From	$63.95	$85.69	$96.70	$182.05
	57.68	1.73	0.89	152.06

96. Find the difference between 34000 and 21345.

(3)(9)(9)(10)
3 4 0 0 0
2 1 3 4 5
1 2 6 5 5

Solution. — There being 0 ones, 0 tens, 0 hundreds in the minuend, we take 1 of the 4 thousands (leaving 3 thousands), or 10 hundreds; then taking 1 of the 10 hundreds (leaving 9 hundreds), or 10 tens; and 1 of the 10 tens (leaving 9 tens), or 10 ones; the minuend may then be considered 3 ten-thousands 3 thousands 9 hundreds 9 tens and 10 ones.

Taking from the changed minuend the 2 ten-thousands 1 thousand 3 hundreds 4 tens 5 ones of the subtrahend, we have as the difference required, 12655.

Subtract:

97. 5544 from 40000.
98. 180 from 98000.
99. 12453 from 35421.
100. 97 from 10000.
101. 58346 from 67500.
102. 9999 from 10000.
103. 67.055 from 73.607.
104. 19.55 from 831.50.
105. 444.5 from 10060.
106. 921.56 from 1000.
107. $13.63 from $500.20.
108. $127 from $1963.75.

109. What is the value of 83956 — 78415 ?

110. What is the value of 60440 — 33457 ?

111. What is the value of 109800 — 98799 ?

112. From one hundred nine take one, and nine hundredths?

113. America was discovered in 1492; how many years from that date to 1881 ?

114. A man began business with $1760, and after two years had $2500.75. How much had he gained ?

115. The number of regular soldiers furnished by the several States in the war of the Revolution was 231771; of these a single State furnished 67907. How many were furnished by other States ?

116. A merchant bought goods to the amount of $7563.56, and sold them for $11630.50. How much did he gain ?

117. The Atlantic slope contains 967576 square miles, and the Mississippi valley 1237111 square miles. How much does the latter exceed the former ?

118. The population of New York City in 1870 was 942192, and in 1880 was 1209561. How much was the gain ?

119. The product of gold in a certain year was from Nevada $19546516, and from California $17760679. How much greater was the product from Nevada than from California ?

MISCELLANEOUS EXERCISES.

120. A man owing $ 767.50 paid one time $ 190, at another $ 131, and at a third $ 155.25. How much did he then owe?

121. James Dow's real estate is valued at $ 3769, and his personal estate at $ 2648.75. He owes Job Smith $ 1728 and Abraham Tyler $ 1161.93. How much is he worth when these debts are paid?

122. Modern notation in arithmetic was introduced from Arabia into Europe in the year 991, algebra from the same country in 431 years later, and decimal fractions were invented in 1602. Required the number of years from each to 1882.

123. Sydney has $ 178.50, Albert $ 75.75 more than Sydney, and Charles has as much as Sydney and Albert less $ 80.93. How much more has Charles than Sydney?

124. What is the value of 1645 + 635 + 416 − 1314?

125. Two vessels 4563 miles apart start to meet each other. When one of them has made 1575 miles of the distance and the other 1658 miles, how far are they apart?

126. A man whose property was $ 50675 gave his son Edwin $ 8555.50, his son Robert $ 7000, his daughter Mary $ 9563.75, his wife $ 20000, and a public library the remainder. How much was given the public library?

127. The population, in 1880, of Philadelphia was 847542; of Boston, 362535; and of Providence, 104760. How much greater was the population of Philadelphia than that of Boston and Providence?

128. A man who had $ 1250 in a savings bank took out $ 51.75 at one time, $ 84.93 at another, and $ 267 at another. He then put in $ 185. How much then had he in the bank?

QUESTIONS.

41. What is subtraction? 42. What is the difference? 43. The minuend? 44. The subtrahend? 45. The sign of subtraction?

46. What are principles of subtraction? 47. How do you subtract? If any order of the minuend has less units than the same order of the subtrahend, what is done? 48. What is the proof of subtraction?

MULTIPLICATION.

49. 1. If a man can earn 8 dollars in a week, how many dollars can he earn in 4 weeks? How many dollars are 4 times 8 dollars?

2. What will 5 quarts of cherries cost at 10 cents a quart?

3. William has 9 apples and Alfred 7 times as many. How many has Alfred? 7 times 9 are how many ones?

50. **Multiplication** is taking one number as many times as there are ones in another.

51. The **Multiplicand** is the number to be taken or multiplied.

52. The **Multiplier** is the number that shows how many times the multiplicand is to be taken.

53. The **Product** is the result of a multiplication; and the *factors* of *a product* are the numbers multiplied together to produce it.

54. The **Sign** of Multiplication is ×. It means *multiplied by*, or *times*. Thus,

7 × 5 may be read seven multiplied by five, or seven times five.

55. A **Concrete Number** is a number in which some kind of unit is named. Thus,

2 books, 3 days, $ 7, are concrete numbers.

56. An **Abstract Number** is a number in which no particular kind of unit is named. Thus,

2, 5, 7 are abstract numbers.

57. MULTIPLICATION TABLES.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

ORAL EXERCISES.

4. How many wings have 8 doves?

5. How much can you earn in 3 days if you earn 10 cents a day?

6. At 9 cents each what will a half-dozen writing-books cost?

7. How many horns have 8 yoke of oxen?

8. How many days in 12 weeks? In 9 weeks?

9. How far can you ride in 7 hours at the rate of 8 miles an hour?

10. How many desks in a school-room having 8 rows with 7 desks in a row?

11. How many legs have 6 span of horses?

12. At 7 cents each what cost a dozen pencils?

13. A ten-foot pole is how many inches long?

How many are:

14.	15.	16.	17.
7 × 9	5 × 12	2 × 7	6 × 7
8 × 8	8 × 10	10 × 12	12 × 12
9 × 6	6 × 11	8 × 7	11 × 8
7 × 7	9 × 12	9 × 4	9 × 7
8 × 3	8 × 10	11 × 11	4 × 7

18.	19.
12 × 9 − 8 × 12	6 × 9 − 7 × 7
6 × 6 − 4 × 7	8 × 6 − 4 × 10
7 × 8 − 5 × 9	7 × 12 − 9 − 9
6 × 7 − 6 × 6	12 × 11 − 10 × 10
9 × 0 × 8 × 4	12 × 6 − 8 × 8

20. Give the products by 2, from 2 × 2 to 2 × 12.

Solution. — 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

21. Give the products by 4, from 4 × 0 to 4 × 12.

22. Count by 7's from 0 to 84.

23. Subtract by 9's from 108 to 0.

24. Name all the products of which 12 is a factor to 144.

25. What cost 9 six-cent books? 12? 8?

26. What will 12 dozen eggs cost at a cent apiece?

27. How many days in a 9 weeks' vacation?

28. How many corners have 11 squares?

29. I pay 7 cents daily for milk. What is my milk bill for a week?

30. How many inches in 8 feet? In 9? In 11?

31. How many feet in 12 yards of ribbon?

32. What cost 7 lbs. at 5 cents a pound, and 5 quarts at 4 cents a quart?

33. What numbers multiplied make 36? 56? 81? 96?

34. What factors produce 48? 72? 121? 63? 49?

35. At $8 a barrel, how many times $8 will 9 barrels of flour cost?

36. How many are 9 × $8? 8 × $9? 3 × 2 × 2? 2 × 3 × 2? 2 × 2 × 3?

58. Principles of Multiplication.

1. *The multiplier is always considered an abstract number.*
2. *The product and the multiplicand are always like numbers.*
3. *The product is the same whatever the order of the factors.*

WRITTEN EXERCISES.

37. Find the product of 274 multiplied by 7.

Factors	Multiplicand	274
	Multiplier	7
	Product	1918

Solution. — We write the multiplier, 7 ones, under the ones' figure of the multiplicand.

Beginning with the ones, we multiply: 7 times 4 ones are 28 ones, or 2 tens 8 ones. We write the 8 ones beneath in the place of ones, and reserve the 2 tens.

7 times 7 tens are 49 tens, and 49 tens plus the 2 tens reserved are 51 tens, or 5 hundreds 1 ten. We write the 1 ten beneath in tens' place, and reserve the 5 hundreds.

7 times 2 hundreds are 14 hundreds, and 14 hundreds plus 5 hundreds reserved are 19 hundreds, or 1 thousand and 9 hundreds. We write the 9 hundreds beneath in hundreds' place, and the 1 thousand in thousands' place.

The product is 1 thousand 9 hundreds 1 ten 8 ones, or 1918.

	38.	**39.**	**40.**	**41.**
Multiplicand	756	4567	1109	6201
Multiplier	4	3	5	8

	42.	**43.**	**44.**	**45.**
Multiplicand	3416	2608	12345	24301
Multiplier	6	2	7	9

46. What is the product of 64.25 by 5 ?

Multiplicand	64.25
Multiplier	5
Product	321.25

Solution. — 5 times 5 hundredths are 25 hundredths, or two tenths and 5 hundredths. We write the 5 hundredths beneath in hundredths' place and reserve the 2 tenths.

5 times 2 tenths are 10 tenths, and 10 tenths plus the 2 tenths reserved are 12 tenths, or 1 one and 2 tenths. We write the 2 tenths beneath in tenths' place, and reserve the 1 one.

5 times 4 ones are 20 ones, and 20 ones plus the 1 one reserved are 21 ones, or 2 tens and 1 one. We write the 1 one beneath in ones' place, and reserve the 2 tens; and so on. The multiplier being an integer, the product has as many decimal places as the multiplicand.

	47.	**48.**	**49.**	**50.**
Multiply	35.07	6.135	1635.9	7128.53
By	8	5	2	7

	51.	**52.**	**53.**	**54.**
Multiply	$ 124.35	$ 192.547	$ 823.50	$ 8537.64
By	4	6	8	9

ORAL EXERCISES.

55. How much will 12 coats cost at \$ 9 each ?

56. When plows are \$ 11 each, what will eight cost ?

57. If 7 men can do a piece of work in 9 days, in what time will 1 man do the same work ?

58. If a vessel sails at the rate of 12 miles an hour, how far will it sail in 12 hours ?

59. What will 11 barrels of flour cost at \$ 10 each ?

60. How many horses will consume in one day as many bushels of oats as 11 horses consume in 12 days ?

What is the value of

61. $4 \times 3 \times 6$?

62. $8 \times 7 + 9$?

63. $6 \times 4 - 10$?

64. $9 \times 5 - 8$?

65. $6 \times 5 + 12$?

66. $3 \times 4 \times 10$?

67. $12 \times 10 + 9$?

68. $11 \times 11 - 12$?

69. $11 \times 6 - 10$?

70. $11 \times 10 - 12$?

71. $10 \times 7 - 11$?

72. $12 \times 8 + 10$?

73. Give the product of the following numbers multiplied by 12 ; by 11 ; by 5 ; by 4.

9, 11, 3, 5, 12, 8, 2, 7, 10, 6, 4.

74. Multiply the numbers of the preceding exercise by 8 ; by 9 ; by 7 ; by 6, and add 3 to each product.

75. How many are 6 times 24 ?

Solution. — 24 is $20 + 4$; 6 times 20 are 120; 6 times 4 are 24; hence 6 times $20 + 4$, or 24, are $120 + 24$, or 144.

76. At 26 cents a yard, how many cents will 7 yards of cloth cost ?

77. At the rate of 23 miles an hour, how many miles will a train of cars move in 8 hours ?

78. What will 9 suits of clothes cost at \$ 30 a suit ?

79. What will 5 sewing-machines cost at \$ 50 each ?

WRITTEN EXERCISES.

80. What is the product of 534 and 242?

534	... Multiplicand ...	242
242	... Multiplier	534
1068	Partial Products	968
2136		726
1068		1210
129228	... Product ...	129228

Solution.—We write the factors so that the right-hand figure of each shall stand in the same column.

Multiplying by the 2 ones, we have 1068 ones as the first partial product; multiplying by the 4 tens, we have 2136 tens for the second partial product, which we write so that its right-hand figure shall come in the tens' column; multiplying by the 2 hundreds, we have 1068 hundreds for the third partial product, which we write so that units of the same order shall come in the same column; adding the three partial products, we have 129228 as the product required.

To prove the work, since the product is the same whatever the order of the factors, we multiply the 242by the 534, and have, as before, 129228.

	81.	**82.**	**83.**	**84.**
Multiply	763	1345	406	1621
By	37	45	25	34

	85.	**86.**	**87.**	**88.**
Multiply	134.7	17.58	3.049	$25.75
By	86	285	329	703

NOTE.—In solution of Ex. 88, there being 0 tens in the multiplier, we pass to the hundreds of the multiplier.

59. Rule for Multiplication.

Write the multiplier under the multiplicand, with a line beneath.

Beginning at the right, multiply each figure of the multiplicand by each figure of the multiplier successively, placing the *right-hand figure of each partial product under the figure of the multiplier that produced it.*

Add the partial products, and from the right of the result point off as many decimal places as are found in both factors.

60. Proof. See if the same result is obtained by multiplying the multiplier by the multiplicand.

Multiply:

89. 347 by 769.
90. 826 by 243.
91. 90.4 by 85.
92. $ 32.13 by 91.
93. 456.7 by 68.
94. 8.901 by 542.
95. $ 23.45 by 397.
96. 6789.0 by 645.
97. $ 198.06 by 805.
98. 45.32 by 907.
99. 982.4 by 3004.
100. $ 60.51 by 768.
101. 87.35 by 94.
102. $ 80.42 by 832.
103. 30.69 by 907.
104. $ 817.42 by 358.
105. 8.439 by 125.
106. 86491 by 683.
107. 49382 by 294.
108. $ 887.95 by 761.
109. 4963 by 845.
110. $ 28.59 by 927.
111. 938.42 by 347.
112. 61904 by 869.
113. $329.87 by 35.
114. 42935 by 942.
115. $ 864.23 by 346.
116. 84917 by 809.
117. $ 73.24 by 935.
118. $ 98.983 by 871.

119. On board of a steamer there are 163 barrels of sugar, each weighing 295 pounds. What is the weight of the whole ?

120. The factors of a product are 1468 and 87 ; what is the product ?

121. The factors of a product are 681, 507, and 12 ; what is the product ?

122. The multiplicand is 804.51, and the multiplier 63 ; what is the product ?

123. What number = 914.08 × 64 ?

124. At $ 82.50 an acre, what is the value of 25 acres of land ?

125. What is the product of five thousand four hundred fourteen, and fifteen thousandths, multiplied by 38 ?

126. 8304.5 × 77 = what ?
127. 7038.61 × 126 = what ?
128. 824.84 × 424 = what ?
129. $ 62.005 × 91 = what ?
130. $ 47.168 × 208 = what ?
131. $ 617.43 × 355 = what ?

132. How many bushels of wheat can be raised on 5634 acres at the rate of 47 bushels per acre ?

133. Multiply nine hundred sixty-five, and thirteen hundredths, by three thousand seven hundred five ?

The multiplier a number of tens, hundreds, etc.

134. Multiply 48 by 10 ; by 100.

$$\begin{array}{r} 48 \\ \underline{10} \\ 480 \end{array} \qquad \begin{array}{r} 48 \\ \underline{100} \\ 4800 \end{array}$$

Solution. — 10 forty-eights is the same as 48 tens (Art. 58), or 480.

Also, 100 forty-eights is the same as 48 hundreds, or 4800.

135. Multiply 427 by 10, by 100, and by 1000, and add the results.

136. How many are a thousand times 7854 ? 138 ? What is the product of $ 55.56 multiplied by 100 ?

$$\begin{array}{r} \$\,55.56 \\ \underline{100} \\ \$\,5556.00 \end{array}$$

Solution. — The removal of a figure one place to the left in a number increases the value expressed ten-fold (Art. 18). Hence, we multiply $ 55.56 by 10 by removing the decimal point one place to the right, and by 10 × 10, or by 100, by removing the point two places to the right.

137. Multiply 97 by 600.

$$\begin{array}{r} 97 \\ \underline{600} \\ 58200 \end{array}$$

Solution. — 600 is 100 times 6 ; 600 times 97 is the same as 100 times 6 times 97. 6 times 97 are 582, and 100 times 6 times 97 are 100 times 582, or 58200. That is, —

61. To multiply by a number of tens, hundreds, etc.

Multiply without regard to the ciphers at the right of the multiplier, annex that number of ciphers to the product, and give it as many decimal figures as the multiplicand has.

Multiply:

138. 814 by 1000.
139. 3921 by 70.
140. 54.78 by 9000.
141. 1342 by 450.
142. $ 611.31 by 110.
143. 3450.9 by 800.
144. $ 1604.05 by 2000.
145. 446.008 by 4600.
146. 33300 by 820.
147. $ 77880 by 300.

148. If the earth moves about the sun at the rate of 68000 miles an hour, how far does it move in 240 hours?

149. A square mile is 640 acres. How many acres has a State whose area is 7800 square miles?

MISCELLANEOUS EXERCISES.

150. Bought 20 barrels of flour at $ 8.50 a barrel, 8 tons of fine feed at $ 19.50 a ton, and oats for $ 63.25, and gave in payment a $ 500 bank-bill. How much should be paid back?

151. In a certain battle an army lost 416 killed and 3 times as many wounded. The enemy's loss was 5 times as many. What was the entire loss of killed and wounded in the battle?

152. A clerk had a salary of $ 1500 for a year of 52 weeks. His board cost him $ 4.25 a week; he wasted for cigars and liquor $ 1.30 a week; and his other expenses were for the year $ 150. How much did he save? How much could he have saved if he had avoided the cigars and liquor?

153. Two trains of cars leave Boston at the same time on the same railroad for the West. One goes at the rate of 31.50 miles an hour, and the other at the rate of 16.25 miles an hour. How far apart will the two trains be at the end of 48 hours?

154. A drover has bought 120 fat oxen, each weighing on an average 1360 pounds. What is the weight of the whole?

155. In a school of 56 pupils there are 29 boys whose average weight is 85 pounds. The girls average 79 pounds each. What is the weight of the school?

156. A merchant bought 130 yards of cloth at $ 3.75 a yard, and sold the whole for $ 573.95. How much did he make?

157. Multiply the sum of 842 and 796 by twice their difference.

158. When boards are $ 31.50 a thousand feet, and shingles $ 4.25 a thousand, what will 40 thousand feet of boards and 22 thousand of shingles cost?

159. James Hudson has a house worth $ 2500, another worth $ 1900, and 60 acres of land worth $ 75 an acre. How much more is the land worth than the two houses?

160. Bought 17 yards of silk at $ 2.75 a yard, 114 yards of carpeting at $ 1.80 a yard, and $ 17 worth of lining. What was the amount of the bill?

161. A drover bought 60 oxen at $ 50 a head, 120 sheep at $ 4.25 a head, and 28 cows at $ 45.50 a head. He returned 20 of the oxen at cost and bought 5 horses at $ 125 each. What was the cost of the whole to him?

162. Bought 24 car-loads of wheat, each car holding 325 bushels, at $ 1.48 a bushel. I sold the wheat at $ 1.65 a bushel. How much did I gain?

QUESTIONS.

50. What is multiplication? 51. What is the multiplicand? 52. The multiplier? 53. The product? The factors of the product?

54. What is the sign of multiplication? 55. What is a concrete number? 56. An abstract number?

58. What are principles of multiplication? 59. How are numbers written for multiplying? How do you multiply? 60. What is the proof?

61. How do you multiply by a number of tens, hundreds, etc?

DIVISION.

62. 1. James has 32 books. How many times 8 books has he?

2. How many quarts of nuts, at 9 cents a quart, can be bought for 54 cents?

3. How many times 9 cents are 54 cents? How many times 9 cents in 54 cents?

4. If 7 men share 42 pounds of tea, how many pounds will each man have?

5. Ella has 27 cents. How many pencils at 5 cents each can she buy, and how many cents left?

63. **Division** is finding how many times one number is contained in another; or, finding one of the equal parts of a number.

64. The **Dividend** is the number divided.

65. The **Divisor** is the number by which we divide.

66. The **Quotient** is the result of a division.

67. The **Remainder** is the part of the dividend left, when the latter does not contain the divisor an exact number of times.

68. The **Sign** of Division, ÷ , or : , means divided by. Thus,

16 ÷ 8, or 16 : 8, is read, sixteen divided by eight.

Division is also indicated by writing the divisor at the left of the dividend, with a curve,), between, or by writing the divisor under the dividend, with a horizontal line between. Thus,

2) 6, or $\frac{6}{2}$, may be read, six divided by two.

69. A **Parenthesis,** ( ), or **Vinculum,** ——, is used to include such numbers as are to be considered together. Thus, $6 + 8 \div 2 + 5 = 15$, but $(6 + 8) \div (2 + 5) = 2$.

NOTE. —The signs × and ÷ have no force in either direction beyond a + or a − unless a parenthesis is used. Operations indicated by them must be performed first.

70. The **Equal Parts** into which a number may be divided are named, according to their size. Thus, —

One of two equal parts, written $\frac{1}{2}$, is called *one half;*
One of three equal parts, written $\frac{1}{3}$, is called *one third;*
One of four equal parts, written $\frac{1}{4}$, is called *one fourth;*
Two of three equal parts, written $\frac{2}{3}$, is called *two thirds;*
Three of four equal parts, written $\frac{3}{4}$, is called *three fourths;*
and so on.

ORAL EXERCISES.

6. John has 84 cents. How many times 7 cents has he?

7. How many times 12 cents in 84 cents?

8. Ellen has 49 peaches. Should she divide them equally among 7 of her playmates, how many would each receive?

9. What number is one of the 7 equal parts of 49? What is $\frac{49}{7}$?

10. If $96 be equally divided among 12 men, what sum would each receive? How much is $\frac{1}{12}$ of $96?

11. In 84 days how many weeks? How many times 7 in 84? What is $\frac{1}{7}$ of 84?

What is

12.	**13.**	**14.**	**15.**	**16.**
$63 \div 7$?	8) 72?	72 : 9?	$\frac{72}{12}$?	$\frac{1}{3}$ of 36?
$64 \div 8$?	9) 81?	96 : 12?	$\frac{108}{9}$?	$\frac{1}{5}$ of 45?
$54 \div 9$?	5) 60?	99 : 11?	$\frac{56}{7}$?	$\frac{1}{8}$ of 96?
$49 \div 7$?	11) 66?	32 : 4?	$\frac{81}{9}$?	$\frac{1}{10}$ of 110?
$24 \div 8$?	10) 80?	63 : 9?	$\frac{120}{12}$?	$\frac{1}{11}$ of 132?

17. Give the quotients of the numbers in the following line divided by 2; by 4; by 6; by 8; by 10.

8, 12, 16, 20, 24, 32, 36, 48, 60, 64.

18. Give the quotients of the numbers in the following line divided by 3; by 5; by 7; by 9; by 8.

28, 33, 35, 42, 49, 63, 81, 96, 100.

Supply the necessary numbers in the following:

19.

$6 \times ? = 42$
$9 \times ? = 108$
$? \times 12 = 72$
$4 \times ? = 32$

20.

$11 \times ? = 121$
$? \times 8 = 64$
$7 \times ? = 42$
$? \times 9 = 72$

21.

$3 \times 4 \times ? = 84$
$2 \times ? \times 6 = 144$
$3 \times 3 \times ? = 81$
$3 \times 3 \times ? = 27$

22.

$\frac{84}{7} + 9 = ?$
$132 : ? = 12$
$144 : ? = 12$
$\frac{4 \times 9}{6} + 5 = ?$

23. In 24 how many times 2? 6? 8? 4? 12?

24. In 45 how many times 5? 3? 9? 15?

25. In 60 how many times 5? 10? 6? 4? 12? 15? 20?

26. At 8 cents each, how many cocoa-nuts can be bought for 88 cents?

27. If a dozen of eggs cost 48 cents, what does one cost?

28. How many pounds of meat can be bought for 72 cents at 8 cents a pound?

29. How many yards of cloth at $6 a yard will pay for 8 tons of coal at $9 a ton?

30. If 4 men do a piece of work in 12 days, how long will it take 6 men?

31. What number divided by 9 will give 6?

32. If 4 cords of wood cost $36, what will 7 cords cost?

33. How many twelve-quart cans will hold 84 gallons of milk?

34. If a train of cars runs 108 miles in 9 hours, what is the rate per hour?

71. Principles of Division.

1. *The dividend is the product of the divisor and the quotient.*

2. *When the divisor and the dividend are like numbers, the quotient will be an abstract number.*

3. *When the divisor is an abstract number, the dividend and the quotient will be like numbers.*

WRITTEN EXERCISES.

35. Divide 8574 by 6.

$$\begin{array}{lrl} \text{Divisor} & 6)\,8574 & \text{Dividend.} \\ & \overline{1429} & \text{Quotient.} \\ & \underline{6} & \\ \text{Proof} & 8574 & \end{array}$$

Solution. — We write the divisor at the left of the dividend, with a curve between them, and begin at the left to divide.

6 in 8 thousands, 1 thousand times, with 2 thousands, or 20 hundreds, remaining. We write the 1 thousand beneath in thousands' place.

Uniting the 20 hundreds with the 5 hundreds of the dividend, we have 25 hundreds. 6 in 25 hundreds, 4 hundred times, with 1 hundred, or 10 tens, remaining. We write the 4 hundreds beneath in hundreds' place.

Uniting the 10 tens with the 7 tens of the dividend, we have 17 tens. 6 in 17 tens, 2 tens times, with 5 tens, or 50 ones, remaining. We write the 2 tens beneath in tens' place.

Uniting the 50 ones with the 4 ones of the dividend, we have 54 ones. 6 in 54 ones, 9 times. We write the 9 ones beneath in ones' place.

The quotient is 1 thousand 4 hundreds 2 tens 9 ones, or 1429.

To prove the work, we multiply the quotient by the divisor, and have as the product the dividend (Art. 71).

36.	**37.**	**38.**	**39.**
4) 1532	7) 5747	5) 4855	8) 9136
40.	**41.**	**42.**	**43.**
6) 351.6	8) 97.84	7) 83.30	9) 40.203

44.

$$\begin{array}{rr} & 5)\,\underline{6373} \\ \text{Quotient} & 1274\frac{3}{5} \\ & \underline{5} \\ & 6370 \\ & \underline{3} \\ \text{Proof} & 6373 \end{array}$$

Solution. — Dividing, there is a final remainder of 3. The division of the 3 we indicate by $\frac{3}{5}$, and writing this as a part of the result, we have the quotient 1274$\frac{3}{5}$.

To prove the work, we multiply the integer of the quotient by the divisor, and adding the remainder, have the dividend.

45.	**46.**	**47.**	**48.**
5) 3576	12) 8143	8) 4899	7) 58965
49.	**50.**	**51.**	**52.**
4) 2693	3) 40.02	6) 364.27	9) 39.137

53. How many tons of coal at $5 a ton can be bought for $2025?

54. How far must a vessel sail each day to sail 1438 miles in 7 days?

55. Paid for 8 yards of broadcloth $43.60. What was the cloth a yard?

56. If the dividend is 20818, and the divisor 9, what is the quotient?

ORAL EXERCISES.

57. How many times is $\frac{1}{8}$ of 32 contained in $\frac{1}{3}$ of 60?

58. The product of two numbers is 56, and one of the numbers 8. What is the other number?

59. If the dividend is 72, and the divisor 9, what is the quotient?

60. Mr. Jones can earn in a month $81, and his son one ninth as much. How much can his son earn?

61. A farm of 120 acres has been divided into 12 lots. How many acres in each lot?

62. What is $\frac{1}{8}$ of 64? $\frac{1}{9}$ of 81? $\frac{1}{10}$ of 110? $\frac{1}{12}$ of 96?

63. One man can do a piece of work in 132 days. In what time can 11 men do it?

64. A man earned $144 in 12 weeks. What was that a week?

65. How many 10's in 100? How many 100's in 1000?

66. In 65 how many tens and what over? In 235 how many hundreds and what over?

67. How many times 20 in 100? How many times $(20 \div 4)$ in $(100 \div 4)$? How many times (20×2) in (100×2)?

72. A General Principle of Division.

Dividing or multiplying both dividend and divisor by the same number does not alter the quotient.

WRITTEN EXERCISES.

68. Divide 1185 by 12.

$$\begin{array}{rrl} & 98\frac{9}{12} & \text{Quotient.} \\ \text{Divisor } 12\,) & 1185 & \text{Dividend.} \\ & \underline{108} & \\ & 105 & \\ & \underline{96} & \\ & 9 & \end{array}$$

Solution. — 12 is in 118 tens, 9 tens times. We write the 9 tens over the tens of the dividend.

9 tens times 12 are 108 tens, which taken from the 118 tens of the dividend leaves 10 tens, and uniting with these 10 tens the 5 ones of the dividend, we have 105 ones. 12 is in 105, 8 times. We write the 8 ones above the ones of the dividend.

8 times 12 are 96, which taken from the 105 of the dividend leaves 9, and indicating its division by the divisor, we have $\frac{9}{12}$. Writing the $\frac{9}{12}$ as a part of the quotient, we have as the quotient $98\frac{9}{12}$.

69. Divide 9876 by 21.
70. Divide 3276 by 14.
71. Divide 31278 by 23.
72. Divide 106.97 by 19.
73. Divide 26.575 by 25.
74. Divide 1868.5 by 37.

73. Rule for Division.

Write the divisor at the left of the dividend with a curve between them.

Find how many times the divisor is contained in the few-

est left-hand figures of the dividend that will contain it, and write the result under or over the right-hand figure of the dividend used, as the first quotient figure.

Multiply the divisor by this quotient, subtract the product from the partial dividend used, and to the remainder annex the next dividend figure for a second partial dividend.

Divide and proceed as before, until all the dividend figures have been used.

If there is a final remainder, write it with the divisor under it, as a part of the quotient.

NOTE 1. — The division is called *Short Division* when only the divisor, dividend, and quotient are written; and *Long Division* when each process of the solution is written.

In short division the quotient is usually written below the dividend, and in long division over, or at the right of, the dividend.

NOTE 2. — When the divisor is an integer, and the quotient is written under or over the dividend, the decimal point in the quotient must be placed immediately under or over the point in the dividend.

74. PROOF. — Multiply the quotient by the divisor, and to the product add the remainder, if any. The result should be the dividend.

75. Divide 182.72 by 45, and prove the work.

	Solution.		*Proof.*	
	$4.06\frac{2}{45}$	Quotient.	$4.06\frac{2}{45}$	Quotient.
Divisor	45)182.72	Dividend.	45	Divisor.
	180		2030	
	2.72		1624	
	2.70		182.70	
	.02	Remainder.	.02	Remainder.
			182.72	Dividend.

76. Find the quotient of $ 114.48 divided by 36.

77. Find one twenty-eighth of $ 896.56.

78. Divide 347692 by 351.
79. Divide 8468.31 by 793.
80. Divide 947684 by 982.
81. Divide 3287.64 by 735.
82. Divide 6798.341 by 1234.
83. Divide $8392.476 by 987.
84. Divide 3004760 by 5942.
85. $1298763 \div 873 = ?$
86. $\frac{1}{841}$ of $63450 = ?$
87. $4127.098 \div 2007 = ?$
88. $8643.21 \div 987 = ?$
89. $(643 \times 857) \div 456 = ?$
90. $(984 \times 895) \div 359 = ?$
91. $\frac{8264 \times 13}{95} = ?$

92. What is the price of one sleigh if 85 cost $10625?

93. How many horses at $225 each will $5400 buy?

94. If $671178.90 is the valuation of a hamlet having 98 inhabitants, what is the average valuation to an inhabitant?

95. If the distance across the Atlantic Ocean is 3000 miles, in how many days will a ship, sailing 115 miles a day, make that distance?

96. What number multiplied by 512 gives a product of 1763.68?

97. What number multiplied by 2135 gives a product of 6419945?

98. Divide 49300 by 432.
99. Divide 700074 by 2047.
100. $5711.04 ÷ 108 = ?
101. $50000 ÷ 365 = ?

102. Divide 489 by 25 to hundredths.

```
     19.56
25) 489.00
    25
    ---
    239
    225
    ----
     14.0
     12.5
     ----
      1.50
      1.50
      ----
```

Solution. — 489 = 489.00. Dividing the 489.00 by 25, we obtain as the result required 19.56.

If the division had not terminated, we could have denoted this by placing the sign + after the hundredths in the quotient to indicate the incompleteness.

103. Divide 2722.5 by 44 to hundredths.
104. Divide 9164 by 86 to thousandths.
105. Divide $ 12625 by 404 to cents.
106. Divide 17552 by 128 to thousandths.

107. If as many persons die in thirty-three years as are equal to the entire population, how many persons die annually, to the nearest unit, on an average, out of every million?

The divisor a number of tens, hundreds, etc.

108. Divide 1467 by 10; by 100; by 1000.

$1467. \div 10 = 146.7$

$1467. \div 100 = 14.67$

$1467. \div 1000 = 1.467$

Solution. — The removal of a figure one place to the right decreases its value ten times (Art. 18). To divide a number by 10, or to find 1 tenth of a number, it is only necessary to move each figure of it one place to the right. This is done by moving the decimal point one place to the left, giving $146\frac{7}{10}$ as the quotient. In the same way two removals of the point to the left divides by 100, three removals by 1000, and so on. That is,

75. To divide by 10, 100, 1000, etc.,

Move the decimal point of the dividend as many places to the left as there are ciphers in the divisor.

109. Divide 5824 by 160.

$$\begin{array}{r} 36.4 \\ 16.0)\ 582.4 \\ \underline{48} \\ 102 \\ \underline{96} \\ 6.4 \\ \underline{6.4} \end{array} \qquad \text{or} \qquad \begin{array}{r} 36|\frac{64}{160} \\ 16|0)\ 582|4 \\ \underline{48} \\ 102 \\ \underline{96} \\ 64 \end{array}$$

Solution. — 160 is 16 × 10. Dividing both divisor and dividend by 10, by removing the decimal point one place to the left, the division becomes the same as 582.4 by 16, which gives as the quotient 36.4.

Or, if the exact remainder is required, we may indicate the division by 10, by marking off one place, and have as a quotient 582, and a remainder of 4 ones. Dividing 582 by 16, we have for a quotient 36, and a remainder 6 tens. Uniting the two remainders, we have as the true remainder 64.

110. There are 1000 mills in a dollar; how many dollars are there in 19650 mills?

111. Divide 3962 by 100; by 1000.

112. Divide 9108 by 200 to hundredths.

113. Divide 12386 by 90, and find the true remainder.

114. Divide 5886990 by 5400 to hundredths.

115. In the State of Alabama 1437272 acres have produced 51000 bales of cotton. What was that to an acre?

116. If light moves at the rate of 186000 miles in a second, how long is it in passing from the sun to the earth, a distance of 92000000 miles?

Divide:

117. 8496453 by 8291.
118. 2907654 by 3782.
119. 72659302 by 1234.
120. 67890123 by 5678.
121. 4567.890 by 2961.
122. 184.837 by 349.
123. 6239076 by 6384.
124. 1689.783 by 945.
125. 849 × 863 by 1252.
126. 84 × 96 × 25 by 1189.
127. $\overline{694 \times 87} + 956$ by 609.
128. 847 × 12 × 900 by 9.
129. 9008 × 7080 by 2090.
130. $945.65 by 850.
131. 843 – 159 by 29 × 7.
132. $849.625 by 975.

MISCELLANEOUS EXERCISES.

133. A farmer obtained $7665 for the apples from his orchard at $3 a barrel. How many barrels did he have?

134. Two men start from the same place and travel in opposite directions, one at the rate of 25 miles a day, and the other at the rate of 31 miles a day. When 5656 miles apart how many days had they traveled?

135. A cargo of 25 tons of coal was bought by the long ton of 2240 pounds, and sold by the short ton of 2000 pounds. How many short tons was the gain?

136. A farmer bought 75 acres of land at $34 an acre, and 85 acres at $20.40 an acre. What was the average cost of the whole an acre?

137. If a man can save $ 15 in each of the 12 months of a year, in how many years can he save enough to amount to $ 2520 ?

138. The product of two factors is 96000, and one of the factors is 150. What is the other factor ?

139. Bought a quantity of boards for $ 2650, and sold the same for $ 3286, and thereby gained $ 6 a thousand feet. How many thousand feet were there ?

140. The product of three factors is 33600. Two of the factors are 15 and 35. What is the third ?

141. Texas contains 274400 square miles and Massachusetts 7800. How many States of the size of Massachusetts might be made out of Texas, and how many square miles over ?

142. If the divisor is 350 and the dividend 262500, what is the quotient ?

143. A man bought 164 acres of land for $ 80 an acre, and sold a part for $ 4480, at the same rate. How many acres had he then left ?

144. If the dividend is 71142 and the quotient 1002, what is the divisor ?

145. A man exchanged 159 cords of wood at $ 5 a cord, for a horse valued at $ 144, and the balance in sheep at $ 3 apiece. How many sheep did he receive ?

QUESTIONS.

63. What is division ? 64. What is the dividend ? 65. The divisor ? 66. The quotient ? 67. The remainder ?

68. What is the sign of division ? 69. What is a parenthesis, or vinculum, used to include ?

71. What are principles of division ? 72. What is a general principle of division ?

73. How are the numbers written for dividing ? How do you divide ? How is a final remainder written ? 74. What is the proof of division ?

REVIEW.

ORAL EXERCISES.

76. **1.** What number added to 18 will make 27?

2. John had 37 cents. He gave 7 to his brother, and shared the remainder equally with his two sisters. What was his share?

3. A farmer had 18 chickens. The foxes killed 3 and he sold 7. How many has he left?

4. Thomas gave 17 cents for a whip, 13 cents for a slate, 12 cents for a copy-book, and 10 cents for pencils. How much change did he receive from a dollar bill?

5. $(5 \times 6 + 5) \div (14 - 7) = ?$

6. John had 30 marbles. He lost half of them and then gave away 5. How many had he left?

7. A man had $ 57; he lost at one time $ 5, and at another $ 10. How many had he left?

8. Bought 7 tons of coal at $ 8 per ton, and gave in payment 2 twenty-dollar bills and 2 ten-dollar bills. How much change should be received back?

9. I have 3 bags of nuts. There are 2 bushels in each bag. What is the whole worth at $ 3 a bushel?

10. George had in a basket 41 apples. He gave 5 to Susan, 7 to Lucy, and 6 to Henry. What is the value of those left at 2 cents each?

Find the result of:

11. $31 - 9 + (6 \times 3)$.

12. $(\overline{63 + 9} \div 12) \times (14 - 8)$.

13. $(\overline{11 \times 8} - 16) \div 9$.

14. $65 - (60 \div 5 + 30)$.

15. $(30 + 25 - 23) \times 3$.

16. $\overline{46 - 16 + 10} \div 10$.

17. $(81 \div 9) \times (75 \div 25)$.

18. $100 - (15 \times 5) \div 5$.

19. Two men start from the same place and go in the same direction; one travels at the rate of 5 miles an hour, and the other at the rate of 9 miles an hour. How far apart will they be in 12 hours?

20. A steamboat can run 10 miles an hour down stream, and 8 miles up stream. After running down stream 4 hours, how long will it be in returning?

21. If you should earn $ 60 in 10 weeks, and pay of your earnings $ 3 a week for board, how much will your net earnings be?

22. William had 98 peaches. He gave 18 to his brother, and shared the remainder equally with 9 others. How many did he give away in all?

23. If 6 men can do a piece of work in 8 days, in how many days can 4 men do it?

24. In what time can 6 men excavate a cellar, which 14 men can excavate in 3 days?

25. If 15 men can build a wall in 10 days, in what time can 25 men do it?

26. If 9 barrels of flour are worth $ 81, what are 7 barrels worth?

27. When 24 pounds of coffee can be bought for $ 6, how many pounds can be bought for $ 8?

28. If a man can earn $ 96 in 8 weeks, in what time can he earn $ 60?

29. A man having $ 95 bought 5 coats, and had $ 20 left. What did the coats cost apiece?

30. Two men start 132 miles apart and travel toward each other, one at the rate of 6 miles an hour, and the other at the rate of 5 miles an hour. How many miles must each travel before meeting?

31. A cistern can be emptied in 16 minutes by 5 pipes. In what time can it be emptied by only 2 pipes?

32. A and B start together and travel in the same direction, A traveling 21 miles a day and B 27 miles. When they have traveled 9 days, how much less than 60 miles are they apart?

WRITTEN EXERCISES.

33. The area of Illinois is 55405 square miles, and of Arkansas 52198 square miles. How many square miles is Illinois larger than Arkansas?

34. The minuend is 74760, and the subtrahend 34943. What is the difference?

35. The difference between two numbers is 4004, and the greater number is 5496. What is the smaller?

36. A man had $ 5000. He expended for a stable $ 2560.75, and for a horse and carriage $ 375.87. What had he left?

37. The multiplicand is 664 and the multiplier 19. What is the product?

38. Six men bought some property for $ 5670 and sold it for $ 7896.84. What was each man's share of the gain?

39. A merchant bought 3 casks of sugar, each weighing 255 pounds, at 9 cents a pound, and sold it at 11 cents a pound. How many dollars did he make?

40. A man sets out to travel 223 miles at the rate of 27 miles a day. When he has gone 61 miles, in how many days can he finish the distance?

41. What is the cost of 365 parlor organs at $ 97 each?

42. The divisor is 48, the quotient 596, and the remainder 10. What is the dividend?

43. Expended in goods $ 42.57, buying 17 yards of cloth at 15 cents a yard, 46 pounds of coffee at 28 cents a pound, 16 gallons of molasses at 76 cents a gallon, and 107 pounds of confectionery. How much was the confectionery a pound?

44. Bought 310 tons of coal for $ 1472.50 and sold it for $ 1549.60. How much did I make a ton?

45. Bought two lots of land; the first containing 144 acres at $ 12 an acre, and the second 108 acres at $ 15 an acre. I sold both lots at $ 18 an acre. What was the gain per acre?

46. The multiplicand is 407, and the product 10989. What is the multiplier?

47. Bought 288 barrels of flour for $ 1728, and sold it at a profit of $ 576. What did I get a barrel for it?

48. A gentleman bought a house for $ 5100, and farm stock to the amount of $ 715.80. He paid at one time $ 2013, and at another $ 1981.95. How much remained to be paid?

49. $(194 + 65) \times 7 + \frac{352 - 220}{11} - 952 = ?$

50. Bought 500 barrels of flour at $ 5.75 a barrel, 47 hundred-weight of cheese at $ 9.25 a hundred, and 15 barrels of pork at $ 21.50 a barrel. What was the amount of the whole?

51. Bought molasses for $ 9212, and sold it at $ 67 a hogshead, and gained $ 20. How many hogsheads were there?

52. A cargo of 125 tons of coal was bought by the long ton of 2240 pounds, and sold by the short ton of 2000 pounds. How many long tons was the gain?

53. I have $ 2973. I wish to invest it in as many horses as I can at $ 150 each, and the remainder in a carriage. How many horses can I buy, and what can I pay for the carriage?

54. $\frac{7781}{31} + 549 - \overline{(2128 \div 7) \times 2} = \text{what?}$

55. $(194 + 65) \times 7 + \overline{(352 - 220) \div 11} - \overline{952 \div (91 - 35)} = ?$

56. A man bought 360 acres of land at $ 45.50, and paid down $ 1368. He sold 125 acres at $ 63 an acre, and made another payment. How much did he then owe for the land?

57. The product of three numbers is 40800. One of the numbers is 150, and another 16. What is the third number?

58. The dividend is 18988 and the quotient $25\frac{13}{759}$. What is the divisor?

59. If I have a garden 320 feet long and half as wide, how many times must I walk around it to travel 100 miles of 5280 feet each?

60. A man has $ 1496, which he wishes to lay out in purchasing cows and oxen, an equal number of each. If he should pay $ 37 for each cow and $ 51 for each ox, how many of each can he buy?

61. If 216 pianos cost $ 112320, what will 519 cost?

62. How many pounds of coffee, at 38 cents a pound, will pay for 2 hogsheads of sugar containing 1160 pounds each, at 19 cents a pound?

63. A farmer having $ 3038, bought 15 tons of hay at $ 11 per ton, 3 yoke of oxen at $ 155 each, 375 sheep at $ 5 each, and spent the rest for cows at $ 41 apiece. How many cows did he buy?

64. A man in his will gave to each of his 2 sons $ 7600; to a third son $ 1500; to each of 3 daughters $ 3775, and the balance $ 6877 to his wife. His wife died, however, and the whole property was divided equally among his children; what did each receive?

65. The expenses of a picnic party of 9 gentlemen and 8 ladies were $ 2.40 each. The gentlemen paid all the expenses. What did each pay?

66. Sold 160 tons of coal at $ 5 per ton, and a number of tons at $ 3 per ton. The value of all the coal sold was $ 965. How many tons were there?

67. Bought 960 acres of land for $ 12000. Sold ¼ of it at $ 12 per acre, ⅓ of it at $ 15 per acre, and the remainder for $ 20 per acre. Did I gain or lose, and how much?

REVIEW QUESTIONS.

2. What is a number? 21. What is an integer? 55. What is a concrete number? 56. An abstract number?

4. What is notation? 5. Numeration? 18. What are principles of notation? 31. What is addition? 37. What are principles of addition? 41. What is subtraction? 46. What are principles of subtraction?

50. What is multiplication? 58. What are principles of multiplication? 63. What is division? 71. What are principles of division?

FACTORS.

77. 1. What two numbers multiplied together produce 2? 3? 5?

2. What numbers, other than itself and 1, will divide 6 without a remainder?

3. What number multiplied by 2 will produce 6?

4. Name all the numbers, other than itself and 1, which will divide 21 without a remainder.

5. What number multiplied by 3 will produce 21?

6. Name the numbers between 1 and 15 which are the product of two or more numbers greater than 1.

78. The **Factors** of a number are the integers which produce the number when multiplied together. Thus,

3 and 5 are factors of 15, and 2, 3, and 3 are factors of 18.

79. A **Composite Number** is a number having other factors than itself and one. Thus,

4, 6, 8, 9, and 10 are composite numbers.

NOTE. — One number is said to be *divisible* by another when there is no remainder in the division. Thus,

A composite number is divisible by any of its factors.

80. A **Prime Number** is a number having no other factors than itself and one. Thus,

1, 2, 3, 5, 7, 11 are prime numbers.

81. A **Prime Factor** is a factor that is a prime number.

NOTE. — As every integer is a factor of itself, and has one as the other factor, in speaking of the factors of a number we usually exclude the number itself and one.

82. Every prime number, except 2 and 5, has 1, 3, 7, or 9 for its unit figure.

NOTE. — 2 is a factor of any number ending in 0, or whose ones are divisible by 2. 3 or 9 is a factor of any number the sum of whose figures is divisible by 3 or 9. 5 is a factor of any number ending in 0 or 5.

WRITTEN EXERCISES.

7. Find the prime factors of 56.

$$\begin{array}{r} 2\underline{)\,56} \\ 2\underline{)\,28} \\ 2\underline{)\,14} \\ 7 \end{array}$$

Solution. — Dividing by the prime numbers 2, 2, and 2, the last quotient obtained is 7, which is also a prime number. The factors required are 2, 2, 2, 7 ; or 2^3, 7.

Ans. 2, 2, 2, 7 ; or 2^3, 7.

8. What are the prime factors of 84 ? Of 144 ? Of 160 ?

9. What are the prime factors of 462 ? Of 576 ? Of 1008 ?

83. Rule for finding the Prime Factors of a Number.

Divide the given number by any prime number above one that will exactly divide it. Divide the quotient, if composite, in like manner, and so continue until a prime quotient is found. The several divisors and the last quotient will be the prime factors.

Find the prime factors of

10. 210
11. 2772
12. 426
13. 6105
14. 1155
15. 2800
16. 3420
17. 7800
18. 16028
19. 17199
20. 10323
21. 12496

22. What is the largest prime factor of 1184 ?

23. Find the prime factors of 4389.

24. Find the prime factors of 6300.

25. What prime numbers multiplied together produce 40579 ?

CANCELLATION.

26. What is the quotient of 63 divided by 21?

27. What is one seventh of 63 divided by ⅐ of 21?

28. What is the quotient of ⅓ of 63, divided by ⅓ of 21?

29. What is the quotient of 5 × 3 × 2, divided by 2 × 3? Of 3 × 5, divided by 3? Of 2 × 5, divided by 2?

84. **Cancellation** is striking out the same factor from both dividend and divisor.

85. Principles of Cancellation.

1. *Striking out a factor of a number divides the number by the factor.*

2. *Striking out the same factors from dividend and divisor does not affect the quotient.*

WRITTEN EXERCISES.

30. Divide 3 times 80 by 48.

$$\frac{80 \times 3}{48} = \frac{\cancel{16} \times 5 \times \cancel{3}}{\cancel{16} \times \cancel{3}} = 5$$

Solution. — Indicating the division, and canceling, by striking out in both dividend and divisor the factors 16 and 3, common to both, there remains only the factor 5 in the dividend, which is the required quotient.

31. Divide 11 times 1476 by 6 times 132.

$$\frac{\overset{\overset{41}{\cancel{123}}}{\cancel{1476}} \times \cancel{11}}{\underset{\cancel{12}}{\cancel{132}} \times \underset{2}{\cancel{6}}} = 20\tfrac{1}{2}$$

Solution. — Striking out in dividend and divisor all the factors common to both, there remains in the dividend only the factor 41, and in the divisor only the factor 2. Completing, then, the division, we have for the quotient 20½.

32. Divide 45 × 20 × 7 by 49 × 4 × 9.

33. (54 × 3 × 4 × 15) ÷ (18 × 12 × 10) = ?

86. Rule for Cancellation.

Strike out all the factors common to both dividend and divisor, and then divide as may be required.

34. Divide 255 × 63 × 4 by 340 × 12 × 7.

35. Divide 18 × 16 × 10 × 5 by 12 × 8 × 6 × 2.

36. Divide 50 × 36 × 14 by 54 × 10 × 4 × 3.

37. Divide 545 × 105 × 11 by 35 × 33 × 5.

38. Multiply 64 by 63 and divide the product by 168.

39. What is the quotient of 36 × 21 × 14 divided by 27 × 7 × 6?

40. How many tons of hay at $24 a ton must be given for 4 cows at $42 each?

41. How many yards of cloth can be bought for $95, when 24 yards can be bought for $120?

42. If 100 men can perform a piece of work in 12 days, in how many days can 150 men perform it?

43. A fort has provisions for 225 men 12 months. How long will it last 675 men?

44. Exchanged 15 pieces of muslin, each containing 30 yards at 10 cents a yard, for 3 pieces of flannel, each containing 50 yards. What was the flannel a yard?

GREATEST COMMON DIVISOR.

45. What number will divide both 21 and 35, without a remainder?

46. What factor is common to 21 and 35? To 15 and 25?

47. What is the greatest number that will divide both 18 and 30, without a remainder?

48. What is the greatest factor common to 18 and 30? To 16 and 24?

87. A **Common Divisor** of two or more numbers is any factor found in each of them. Thus,

7 is a common divisor of 14 and 21.

88. The **Greatest Common Divisor** of two or more numbers is the greatest factor found in each of them. Thus,

6 is the greatest common divisor of 18 and 24.

89. Numbers are **Prime** *to each other* when they have no common factor or divisor. Thus,

9 and 14 are prime to each other.

90. Principle.

The greatest common divisor of two or more numbers is the product of all their common prime factors.

WRITTEN EXERCISES.

49. What is the greatest common divisor of 84 and 132?

$$\begin{array}{r} 2\underline{)\,84,\ 132} \\ 2\underline{)\,42,\ \ 66} \\ 3\underline{)\,21,\ \ 33} \\ 7,\ \ 11 \end{array}$$

Or, $84 = 2 \times 2 \times 3 \times 7$

$132 = 2 \times 2 \times 3 \times 11$

$2 \times 2 \times 3 = 12$, Ans.

Solution. — We find the prime factors common to the numbers are 2, 2, and 3. The product of these, 2 × 2 × 3 or 12, is the greatest common divisor required.

50. What is the greatest common divisor of 36, 81, 135?

51. What is the greatest common divisor of 24, 42, 54, and 60?

91. Rule for finding the Greatest Common Divisor.

Separate the numbers into their prime factors, and find the product of all such as are common to the numbers.

What is the greatest common divisor

52. Of 45 and 135?

53. Of 90 and 105?

54. Of 42 and 81?

55. Of 132 and 156?

56. Of 20, 26, and 38?

57. Of 32, 48, and 128?

58. Of 45, 72, and 81?

59. Of 24, 51, 105, and 729?

60. Having three rooms, the first 12 feet wide, the second 15 feet, and the third 18 feet, I wish to purchase a roll of the widest carpeting that will exactly fit each room without any cutting as to width. How wide must it be?

92. When the numbers are large, or cannot readily be separated into factors, —

Of two numbers divide the larger by the smaller, and the last divisor by the last remainder, until nothing remains. The final divisor is the greatest common divisor.

If more than two numbers are given, find the greatest common divisor of two of them, then of this divisor and a third number, and so on.

61. What is the greatest common divisor of 247 and 323?

```
                          247) 323 (1
                               247
                               ---
                                76) 247 (3
                                    228
                                    ---
Greatest common divisor = 19) 76 (4
                              76
                              --
```

Solution. — If 247 will exactly divide 323, it will be the greatest common divisor of 247 and 323. It will not exactly divide 323, for there is 76 remainder. If 76 will exactly divide 247, it will be the greatest common divisor of 76 and 247, and therefore of 247 and 323. It will not exactly divide 247, for there is 19 remainder. If 19 will exactly divide 76, it will be the greatest common divisor of 19 and 76, and of 76 and 247, and of 247 and 323. 19 will exactly divide 76, and therefore it is the greatest common divisor of 247 and 323.

Find the greatest common divisor of

62. 336 and 480.

63. 925 and 1475.

64. 308 and 506.

65. 172 and 1118.

66. 275 and 440.

67. 2145 and 3471.

68. 582 and 3724.

69. 10353 and 14877.

70. 3528 and 4424.

71. 1764 and 2660.

72. 744 and 906.
73. 728 and 808.
74. 756 and 1140.
75. 2883 and 3131.
76. 3178 and 3500.
77. 4872 and 9048.

LEAST COMMON MULTIPLE.

78. Name four numbers of which 4 is a factor.

79. 15 is an exact number of times what two numbers?

80. What is the least number that will contain both 3 and 7 an exact number of times?

81. What is the least number that both 4 and 9 will exactly divide?

93. A **Multiple** of a number is any number divisible by it (Art. 79, note). Thus,

5, 10, 15, etc., are multiples of 5.

94. A **Common Multiple** of two or more numbers is any number divisible by each of them. Thus,

48 is a common multiple of 4, 8, and 12.

95. The **Least Common Multiple** of two or more numbers is the least number divisible by each of them. Thus,

24 is the least common multiple of 4, 8, and 12.

96. Principle.

The least common multiple of two or more numbers contains each of the prime factors of those numbers, but no others.

WRITTEN EXERCISES.

82. What is the least common multiple of 21, 35, and 45?

$21 = 3 \times 7$
$35 = 5 \times 7$
$45 = 3 \times 3 \times 5$

Or,

$$\begin{array}{r|r} 3) & 21,\ 35,\ 45 \\ \hline 5) & 7,\ 35,\ 15 \\ \hline & 7,\ \ 3 \end{array}$$

$$3 \times 3 \times 5 \times 7 = 315$$

Solution. — Separating the numbers into their prime factors, we find the product of the different factors, using each the greatest number of times it occurs in any number, to be 3 × 3 × 5 × 7, or 315. This is the least common multiple, because it contains each prime factor of the numbers, but no other (Art. 96). Or, the different prime factors may be found by the second process annexed, the two methods being alike in principle.

83. What is the least common multiple of 7, 14, 15, and 21?

84. What is the least common multiple of 18, 28, 30, and 42?

97. Rule for finding the Least Common Multiple.

Separate the numbers into their prime factors. Take the product of all the different factors, using each factor the greatest number of times it occurs in any number.

NOTE 1. — The following rule is sometimes used.

Strike out any of the given numbers that are factors of any of the others, and divide the remaining numbers by any prime factor common to two or more of them.

Strike out from the resulting quotients and undivided numbers all that are factors of any of the rest, and divide as before.

Thus proceed until no two of the remaining numbers have a common factor. The product of the divisors and remaining numbers will be the least common multiple required.

NOTE 2. — The least common multiple of numbers prime to each other is their product.

Find the least common multiple

85. Of 21, 33, 56.

86. Of 63, 72, 84.

87. Of 66, 88, 110.

88. Of 81, 63, 135.

89. Of 8, 18, 24, 36.

90. Of 7, 25, 12, 41.

91. Of 28, 56, 100, 125.

92. Of 24, 42, 54, 180.

93. What is the least common multiple of 24, 96, 100, and 144?

94. What is the least common multiple of 4, 11, 18, 20, 36, and 48?

95. What is the least sum of money that can be exactly expended for sheep, cows, or oxen, at $ 5, $ 35, and $ 50 each, respectively?

MISCELLANEOUS EXERCISES.

96. What are the prime factors of 2520?

97. How many times are 2 and 3 respectively factors in 5760?

98. What is the greatest factor common to 689 and 1573?

99. Find the sum of all the prime numbers between 70 and 100.

100. Employ cancellation in dividing 14 × 15 × 16 × 24 × 48 × 60 by 7 × 30 × 8 × 8 × 6 × 12 × 3.

101. How many times is the greatest common divisor of 48, 36, 72, 24 contained in their least common multiple?

102. Find the sum of the composite numbers between 100 and 120, inclusive.

103. What is the difference between the greatest common divisor and the least common multiple of 160, 352, and 992?

104. Divide 1008 by 168, using prime factors and cancellation.

105. When hay is $ 24 a ton, how many barrels of flour, at $ 8, will exactly pay for 35 tons of hay?

QUESTIONS.

78. What are the factors of a number? 79. What is a composite number? 80. What is a prime number?

81. What is a prime factor? 83. How do you find the prime factors of a number?

84. What is cancellation? 85. What are principles of cancellation? 86. How is cancellation performed?

87. What is a common divisor of two or more numbers? 88. The greatest common divisor of two or more numbers? 90. What is a principle of common divisors? 91. How is the greatest common divisor found?

93. What is a multiple of a number? 94. A common multiple of two or more numbers? 95. The least common multiple of two or more numbers? 96. A principle of multiples? 97. How is the least common multiple found?

COMMON FRACTIONS.

98. 1. If a unit, as an apple, is divided into two equal pieces, what part of the whole will one piece be?

2. If an apple is divided into three equal pieces, what part of the whole will one piece be? Two pieces?

3. If a single thing is divided into four equal pieces, what part of the whole will one piece be? Two pieces? Three pieces?

4. How many halves in an apple? How many thirds? How many fourths?

5. What is meant by one half of a unit? By one third? By two thirds? By one fourth? By three fourths?

6. Which are the larger parts of an apple, halves or thirds? Thirds or fourths?

99. A **Fraction** is one or more of the equal parts of a unit.

The *Unit of the Fraction* is the unit divided, and a *Fractional Unit* is one of the equal parts into which it is divided.

100. The **Denominator** of a fraction is the number that shows into how many equal parts the unit is divided. Thus,

Three is the denominator of two thirds.

101. The **Numerator** of a fraction is the number that shows how many of the equal parts of the unit are taken. Thus,

Two is the numerator of two thirds.

102. The **Terms** of a fraction are its numerator and denominator. Thus,

2 and 3 are the terms of the fraction $\frac{2}{3}$.

103. A **Common Fraction** is a fraction expressed by writing the numerator above, and the denominator below, a dividing line. Thus,

Three fourths of a dollar is written $\$\frac{3}{4}$, 3 being the numerator, 4 the denominator, 1 dollar the *unit of the fraction*, and $\frac{1}{4}$ dollar the *fractional unit*.

104. An **Integer** may be expressed in a fractional form, by writing 1 under it for a denominator. Thus,

2 may be written $\frac{2}{1}$, and read 2 ones; 7 may be written $\frac{7}{1}$, and read 7 ones; etc.

105. A **Proper Fraction** is a fraction whose numerator is less than its denominator. Thus,

$\frac{5}{8}$ and $\frac{7}{11}$ are proper fractions.

106. An **Improper Fraction** is a fraction whose numerator is not less than its denominator. Thus,

$\frac{5}{5}$ and $\frac{13}{11}$ are improper fractions.

107. A **Mixed Number** is an integer and a fraction united. Thus,

$3\frac{1}{4}$, read three and one fourth, is a mixed number.

108. A **Fraction** may be regarded as an indicated division (Art. 63), the numerator being the dividend, and the denominator the divisor. Thus,

$\frac{3}{4}$ of 1 inch is the same as $\frac{1}{4}$ of 3 inches, or 3 inches divided by 4.

109. The **Value** of a fraction is the quotient of the numerator divided by the denominator.

EXERCISES.

Express in figures:

7. Three sevenths.
8. Seven elevenths.
9. Nine sixteenths.
10. Seventeen ones.

11. One twenty-first.

12. Eleven thirty-seconds.

13. Nineteen fortieths.

14. Three and three twenty-ninths.

15. Twenty-three and three fifths.

16. Eight and nine twelfths.

REDUCTION OF FRACTIONS.

ORAL EXERCISES.

17. In $\frac{1}{2}$ of an apple how many fourths of an apple? how many eighths?

18. In $\frac{2}{3}$ of an apple how many sixths? how many ninths?

19. Name a fraction equal to $\frac{1}{2}$. Name a fraction equal to $\frac{2}{3}$.

20. Express $\frac{3}{4}$ in terms 2 times as large. 3 times as large.

21. Change $\frac{3}{5}$ to $\frac{6}{10}$, $\frac{4}{5}$ to $\frac{8}{10}$, $\frac{1}{6}$ to $\frac{3}{18}$.

22. How is the fraction $\frac{5}{6}$ changed to twelfths?

23. How many halves are there in $\frac{4}{8}$? How many thirds are there in $\frac{8}{12}$?

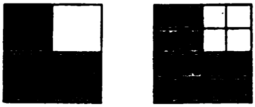
24. How many tenths of a melon in $\frac{10}{20}$? how many fifths?

25. Express $\frac{4}{5}$ in larger terms; $\frac{3}{7}$ in larger terms; $\frac{6}{18}$ in smaller terms; $\frac{12}{16}$ in smaller terms.

110. Reduction of Fractions is changing their form without changing their value.

To Larger Terms.

26. Change $\frac{3}{4}$ to sixteenths.



$$\frac{3\times 4}{4\times 4} = \frac{12}{16}$$

Solution.—The required denominator is 4 times the given denominator. Multiplying both terms of the fraction by 4 gives $\frac{12}{16}$.

It will be observed from the illustration, that $\frac{3}{4}$ of the square equals $\frac{12}{16}$ of it, the multiplication *increasing the number* of fractional units 4 times, and making each *one fourth as large*, so that the value of the fraction is not changed.

111. Principle.

Multiplying both terms of a fraction by the same number does not change its value.

27. Change to 12ths $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

28. Change to 18ths $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{5}{9}$, $\frac{7}{9}$, $\frac{8}{9}$, $\frac{4}{9}$.

29. Change to 20ths $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{4}$, $\frac{1}{10}$, $\frac{4}{5}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$.

30. Change to 24ths $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{7}{12}$, $\frac{11}{12}$.

31. Change to 36ths $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{9}$, $\frac{7}{9}$, $\frac{11}{12}$, $\frac{1}{4}$, $\frac{2}{9}$, $\frac{5}{12}$, $\frac{7}{12}$.

32. Change to 48ths $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{7}{12}$, $\frac{11}{12}$, $\frac{3}{16}$, $\frac{7}{24}$, $\frac{15}{16}$, $\frac{19}{24}$.

WRITTEN EXERCISES.

33. Change $\frac{3}{8}$ to twenty-fourths.

$$\frac{3}{8} = \frac{3\times3}{8\times3} = \frac{9}{24}$$

Or,

$$\frac{3}{8} \text{ of } 1 = \frac{3}{\cancel{8}} \text{ of } \frac{\overset{3}{\cancel{24}}}{24} = \frac{9}{24}$$

Solution. — To change 8ths to 24ths, we must multiply both terms of the fraction by 3. Doing this, we obtain $\frac{9}{24}$. Or,

Since 1 is 24 twenty-fourths, $\frac{1}{8}$ of 1 is $\frac{1}{8}$ of 24 twenty-fourths, or 3 twenty-fourths, and $\frac{3}{8}$ of 1 is 3 times 3 twenty-fourths, or $\frac{9}{24}$.

34. Change $\frac{2}{3}$ to seventy-fifths.

112. To change a fraction to larger terms:

Rule.

Divide the required denominator by the given denominator, and multiply both terms of the fraction by the quotient.

Change:

35. $\frac{3}{7}$ to 84ths.

36. $\frac{3}{4}$ to 32ds.

37. $\frac{4}{9}$ to 54ths.

38. $\frac{13}{14}$ to 196ths.

39. $\frac{5}{7}$ to 168ths.

40. $\frac{19}{21}$ to 189ths.

41. $\frac{3}{4}$ to 48ths.

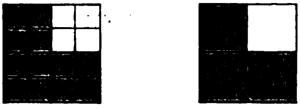
42. $\frac{5}{8}$ to 576ths.

To Smaller Terms.

113. A fraction is in its **Smallest Terms** when its terms have no common factor.

ORAL EXERCISES.

43. Change $\frac{12}{16}$ to smallest terms.



$\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

Solution.—Dividing both terms of $\frac{12}{16}$ by 4, we have $\frac{3}{4}$, whose terms have no common factor; hence, $\frac{12}{16}$ changed to smallest terms is $\frac{3}{4}$.

It will be observed from the illustration that $\frac{12}{16}$ of the square equals $\frac{3}{4}$ of it, the division *increasing the size* of the fractional units 4 times, while their *number is a fourth as large,* so that the value of the fraction is not changed.

114. Principle.

Dividing both terms of a fraction by the same number does not change its value.

Change to smallest terms:

44. $\frac{2}{4}$, $\frac{4}{6}$, $\frac{3}{6}$. $\frac{4}{8}$, $\frac{5}{10}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, $\frac{3}{12}$, $\frac{4}{12}$.

45. $\frac{3}{9}$, $\frac{6}{9}$, $\frac{6}{8}$, $\frac{6}{12}$, $\frac{10}{15}$, $\frac{8}{16}$, $\frac{12}{15}$, $\frac{14}{16}$, $\frac{9}{18}$, $\frac{15}{18}$.

46. $\frac{9}{20}$, $\frac{8}{10}$, $\frac{12}{18}$, $\frac{15}{20}$, $\frac{16}{20}$, $\frac{11}{22}$, $\frac{3}{21}$, $\frac{7}{21}$, $\frac{14}{21}$, $\frac{15}{21}$.

47. $\frac{8}{24}$, $\frac{18}{27}$, $\frac{21}{28}$, $\frac{18}{30}$, $\frac{13}{26}$, $\frac{24}{27}$, $\frac{21}{28}$, $\frac{20}{30}$, $\frac{16}{24}$, $\frac{18}{24}$.

48. $\frac{24}{30}$, $\frac{24}{28}$, $\frac{9}{30}$, $\frac{22}{32}$, $\frac{24}{32}$, $\frac{20}{32}$, $\frac{18}{32}$, $\frac{24}{36}$, $\frac{9}{36}$, $\frac{32}{36}$.

49. $\frac{8}{48}$, $\frac{12}{48}$, $\frac{18}{48}$, $\frac{30}{48}$, $\frac{36}{48}$, $\frac{45}{48}$, $\frac{42}{48}$, $\frac{30}{42}$, $\frac{28}{42}$, $\frac{26}{39}$.

50. $\frac{12}{60}$, $\frac{24}{60}$, $\frac{28}{60}$, $\frac{45}{60}$, $\frac{18}{60}$, $\frac{48}{64}$, $\frac{27}{63}$, $\frac{35}{63}$, $\frac{5}{65}$, $\frac{48}{72}$.

51. $\frac{50}{100}$, $\frac{75}{100}$, $\frac{40}{100}$, $\frac{80}{100}$, $\frac{70}{100}$, $\frac{35}{100}$, $\frac{60}{100}$, $\frac{20}{100}$, $\frac{25}{100}$, $\frac{90}{100}$.

WRITTEN EXERCISES.

52. Change $\frac{30}{75}$ to smallest terms.

$\frac{3)\,30}{3)\,75} = \frac{10}{25}$

$\frac{5)\,10}{5)\,25} = \frac{2}{5}$

Solution.—Dividing both terms of $\frac{30}{75}$ by 3, we have $\frac{10}{25}$, and dividing both terms of $\frac{10}{25}$ by 5, we have $\frac{2}{5}$, whose terms have no common factor; hence, $\frac{30}{75}$ changed to smallest terms is $\frac{2}{5}$.

53. Change $\frac{81}{108}$ to its smallest terms.

54. Change $\frac{120}{144}$ to its smallest terms.

115. To change a fraction to its smallest terms:

Rule.

Divide both terms of the fraction by any common divisor; treat the resulting fraction in the same way, and so continue until a fraction is found whose terms are prime to each other.

Note. — The greater the common divisor used, the shorter will be the process.

Change to their smallest terms:

55. $\frac{108}{120}$.
56. $\frac{84}{228}$.
57. $\frac{15}{175}$.
58. $\frac{425}{272}$.
59. $\frac{374}{361}$.
60. $\frac{319}{407}$.
61. $\frac{400}{450}$.
62. $\frac{648}{810}$.
63. $\frac{856}{936}$.
64. $\frac{121}{132}$.
65. $\frac{176}{1000}$.
66. $\frac{1519}{539}$.
67. $\frac{304}{1072}$.
68. $\frac{1261}{1649}$.
69. $\frac{182}{844}$.
70. $\frac{1995}{7035}$.

An Integer or Mixed Number to an Improper Fraction.

ORAL EXERCISES.

71. In 2 apples how many fourths of an apple? In 5 apples? In 8 apples?

72. Reduce 6 to fourths; 3 to fifths; 7 to sixths.

73. How many fourths of an apple in $3\frac{3}{4}$ apples? In $6\frac{1}{4}$ apples.

Change to improper fractions:

74. $1\frac{2}{3}$, $1\frac{3}{4}$, $1\frac{1}{2}$, $1\frac{5}{6}$, $1\frac{7}{8}$, $2\frac{4}{5}$, $3\frac{3}{7}$, $5\frac{1}{4}$, $4\frac{1}{2}$, $3\frac{2}{3}$.

75. $2\frac{5}{6}$, $2\frac{7}{8}$, $3\frac{1}{2}$, $3\frac{1}{3}$, $3\frac{3}{4}$, $3\frac{5}{6}$, $3\frac{7}{8}$, $4\frac{1}{4}$, $4\frac{2}{3}$, $4\frac{3}{4}$.

76. $4\frac{1}{5}$, $4\frac{2}{7}$, $4\frac{1}{3}$, $5\frac{3}{8}$, $5\frac{7}{12}$, $5\frac{6}{7}$, $5\frac{1}{2}$, $5\frac{3}{4}$, $5\frac{2}{3}$, $6\frac{1}{4}$.

77. $6\frac{2}{3}$, $6\frac{3}{4}$, $6\frac{5}{6}$, $6\frac{6}{7}$, $6\frac{7}{8}$, $6\frac{8}{9}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $8\frac{2}{3}$, $8\frac{3}{4}$.

78. $8\frac{4}{5}$, $9\frac{5}{6}$, $8\frac{7}{8}$, $4\frac{1}{12}$, $5\frac{1}{10}$, $4\frac{3}{7}$, $6\frac{5}{12}$, $8\frac{3}{11}$, $9\frac{4}{7}$, $5\frac{2}{7}$.

WRITTEN EXERCISES.

79. Change 19 to eighths.

19
8 eighths.
152
$\frac{}{8}$

Solution. — In 1 there are 8 eighths; in 19 there are 19 times 8 eighths, or $\frac{152}{8}$.

80. Change 197 to the form of a fraction.

81. Change $21\frac{5}{9}$ to ninths.

$21\frac{5}{9}$
9
194
$\frac{}{9}$

Solution. — In 1 there are 9 ninths, and in 21 there are 21 times 9 ninths, or 189 ninths; 189 ninths and 5 ninths are 194 ninths, or $\frac{194}{9}$.

82. Change $27\frac{5}{13}$ to thirteenths.

83. Change $37\frac{3}{11}$ to elevenths.

116. To change an integer or mixed number to an improper fraction:

Rule.

Multiply the integer by the denominator, and, if there is a fractional part, add its numerator to the product, and write the result over the denominator.

84. Reduce 93 to fifteenths; 107 to twenty-firsts.

85. Reduce 115 to a fraction whose denominator is 13.

Reduce to fractions in their smallest terms:

86. $9\frac{3}{16}$.
87. $56\frac{3}{19}$.
88. $104\frac{5}{7}$.
89. $4\frac{4}{153}$.

90. $14\frac{12}{13}$.
91. $98\frac{48}{97}$.
92. $3\frac{18}{19}$.
93. $142\frac{6}{7}$.

94. $81\frac{1}{60}$.
95. $25\frac{51}{274}$.
96. $153\frac{5}{14}$.
97. $173\frac{10}{1000}$.

An Improper Fraction to an Integer or Mixed Number.

ORAL EXERCISES.

98. How many dollars in 32 quarter-dollars? In 40 quarter-dollars? In \$$\frac{45}{4}$? In \$$\frac{42}{5}$? In \$$\frac{54}{8}$?

Change to integers or mixed numbers:

99. $\frac{8}{3}$, $\frac{11}{5}$, $\frac{15}{2}$, $\frac{16}{3}$, $\frac{7}{2}$, $\frac{9}{4}$, $\frac{11}{5}$, $\frac{12}{7}$, $\frac{13}{3}$, $\frac{14}{4}$.

100. $\frac{16}{5}$, $\frac{17}{3}$, $\frac{8}{5}$, $\frac{9}{2}$, $\frac{15}{7}$, $\frac{11}{6}$, $\frac{15}{6}$, $\frac{16}{7}$, $\frac{17}{5}$, $\frac{18}{8}$.

101. $\frac{19}{4}$, $\frac{20}{3}$, $\frac{21}{5}$, $\frac{22}{6}$, $\frac{23}{7}$, $\frac{24}{9}$, $\frac{25}{2}$, $\frac{26}{7}$, $\frac{27}{8}$, $\frac{28}{5}$.

102. $\frac{29}{6}$, $\frac{30}{7}$, $\frac{31}{8}$, $\frac{32}{5}$, $\frac{33}{4}$, $\frac{34}{3}$, $\frac{35}{9}$, $\frac{36}{11}$, $\frac{37}{12}$, $\frac{38}{9}$.

103. $\frac{39}{8}$, $\frac{40}{7}$, $\frac{41}{5}$, $\frac{42}{8}$, $\frac{43}{9}$, $\frac{44}{12}$, $\frac{45}{3}$, $\frac{46}{12}$, $\frac{47}{11}$, $\frac{48}{7}$.

WRITTEN EXERCISES.

104. Change $\frac{793}{19}$ to an integer or mixed number.

$$\begin{array}{r@{\,}l} & 41\frac{14}{19} \\ 19) & 793 \\ & \underline{76} \\ & 33 \\ & \underline{19} \\ & 14 \end{array}$$

Solution. — As 19 nineteenths are 1, 793 nineteenths will be as many ones as times 19 in 793, or $41\frac{14}{19}$. Ans. $41\frac{14}{19}$.

105. Change $\frac{516}{12}$ to an integer or mixed number.

106. Change $\frac{4521}{71}$ to an integer or mixed number.

117. To change an improper fraction to an integer or mixed number:

Rule.

Divide the numerator by the denominator.

Reduce to integers or mixed numbers:

107. $\frac{99}{11}$.

108. $\frac{245}{9}$.

109. $\frac{3216}{19}$.

110. $\frac{563}{563}$.

111. $\frac{966}{858}$.

112. $\frac{17}{12}$.

113. $\frac{131}{17}$.

114. $\frac{128}{1}$.

115. $\frac{2672}{81}$.

116. $\frac{1848}{459}$.

117. $\frac{265}{29}$.

118. $\frac{1000}{11}$.

119. $\frac{4123}{45}$.

120. $\frac{11110}{111}$.

121. $\frac{86400}{907}$.

Fractions to Fractions having a Common Denominator.

ORAL EXERCISES.

122. Express $\frac{1}{2}$ as tenths; $\frac{3}{5}$ as tenths.

123. Express $\frac{2}{3}$ and $\frac{1}{4}$ each as twelfths.

124. Change $\frac{3}{5}$ and $\frac{3}{4}$ to fractions having the same denominator.

125. Change $\frac{5}{6}$ and $\frac{4}{5}$ to fractions having the same denominator. What is a multiple of 6 and 5?

126. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{6}$ to fractions having the same denominator.

127. Express $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{7}{10}$ each as fortieths.

128. Express $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{5}{9}$ each as eighteenths.

129. What is the least common multiple of 3, 6, and 9?

118. Fractions have a **Common Denominator** when their denominators are alike.

119. The smallest denominator common to two or more fractions is their **Least Common Denominator.**

120. A **Common Denominator** of several fractions must be some *common multiple* of their denominators.

The **Least Common Denominator** of several fractions must be the *Least Common Multiple* of their denominators.

WRITTEN EXERCISES.

130. Change $\frac{2}{5}$ and $\frac{3}{7}$ to fractions having a common denominator.

$$\frac{2\times 7}{5\times 7}=\frac{14}{35}$$

$$\frac{3\times 5}{7\times 5}=\frac{15}{35}$$

Solution. — As the product of the denominators, 5 × 7, or 35, is their common multiple, 35 must be a common denominator of the fractions. Changing then the fractions to thirty-fifths, we have $\frac{14}{35}$ and $\frac{15}{35}$.

131. Change $\frac{6}{7}$ and $\frac{7}{8}$ to fractions having a common denominator.

132. Change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to fractions having the least common denominator.

$\frac{2}{\cancel{3}}$ of $\frac{\overset{4}{\cancel{12}}}{12} = \frac{8}{12}$

$\frac{3}{\cancel{4}}$ of $\frac{\overset{3}{\cancel{12}}}{12} = \frac{9}{12}$

$\frac{5}{\cancel{6}}$ of $\frac{\overset{2}{\cancel{12}}}{12} = \frac{10}{12}$

Solution. — The least common multiple of the denominators is 12, which must be the least common denominator of the fractions. Changing then the fractions to twelfths, we have $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$.

133. Change $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{5}{8}$ to fractions having the least common denominator.

134. Reduce $\frac{3}{4}$, $\frac{7}{9}$, and $\frac{11}{12}$ to fractions having the least common denominator.

121. To reduce to fractions having the least common denominator:

Rule.

Change each fraction to its smallest terms. Divide the least common multiple of the denominators by the denominator of each fraction, and multiply both terms of the fraction by the quotient.

NOTE. — When the denominators are mutually prime, take their product for the common denominator, and multiply each numerator by all the denominators except its own for the new numerator.

Reduce to fractions having the least common denominator:

135. $\frac{2}{9}$ and $\frac{7}{15}$.

136. $\frac{3}{7}$ and $\frac{4}{13}$.

137. $\frac{7}{12}$ and $\frac{31}{27}$.

138. $\frac{5}{48}$ and $\frac{9}{75}$.

139. $\frac{3}{14}$, $\frac{8}{21}$, and $\frac{9}{28}$.

140. $\frac{5}{6}$, $\frac{5}{9}$, and $\frac{12}{21}$.

141. $\frac{7}{15}$, $\frac{10}{21}$, and $\frac{17}{35}$.

142. $\frac{15}{8}$, $\frac{1}{2}$, and $\frac{12}{1}$.

143. $\frac{3}{25}$, $\frac{17}{45}$, $\frac{16}{55}$, and $\frac{3}{75}$.

144. $\frac{7}{10}$, $\frac{7}{100}$, $\frac{7}{1000}$, and 7.

145. $\frac{31}{4}$, $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$.

146. $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{3}{4}$.

147. Reduce $\frac{4}{9}$, $\frac{3}{5}$, and $\frac{8}{15}$ to fractions having the least common denominator, and show by the numerators which of the fractions has the greatest value.

ADDITION.

ORAL EXERCISES.

148. Mary paid $\frac{3}{8}$ of a dollar for a book, $\frac{7}{8}$ of a dollar for a hat, and $\frac{5}{8}$ of a dollar for a handkerchief. How many eighths of a dollar did she spend for all?

149. Smith owns $\frac{5}{16}$ of a vessel and Keen $\frac{3}{8}$. How many sixteenths does Keen own? What part of the vessel do both own?

Give the sum of the following fractions:

150.	**151.**	**152.**	**153.**	**154.**
$\frac{1}{2} + \frac{1}{3}$	$\frac{1}{2} + \frac{3}{4}$	$\frac{2}{3} + \frac{1}{2}$	$\frac{5}{12} + \frac{1}{15}$	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
$\frac{2}{3} + \frac{3}{4}$	$\frac{5}{6} + \frac{2}{3}$	$\frac{1}{9} + \frac{1}{6}$	$\frac{3}{8} + \frac{1}{9}$	$\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$
$\frac{1}{2} + \frac{5}{6}$	$\frac{3}{4} + \frac{5}{6}$	$\frac{2}{9} + \frac{5}{6}$	$\frac{1}{10} + \frac{2}{9}$	$\frac{1}{4} + \frac{1}{5} + \frac{1}{10}$
$\frac{3}{4} + \frac{1}{6}$	$\frac{2}{5} + \frac{1}{4}$	$\frac{4}{18} + \frac{5}{9}$	$\frac{5}{6} + \frac{4}{7}$	$\frac{2}{3} + \frac{3}{4} + \frac{1}{12}$
$\frac{3}{10} + \frac{1}{2}$	$\frac{3}{4} + \frac{1}{5}$	$\frac{3}{7} + \frac{1}{3}$	$\frac{3}{11} + \frac{2}{5}$	$\frac{3}{4} + \frac{2}{3} + \frac{5}{6}$

122. Like Fractions are like parts of the same unit. Thus,

$\frac{3}{8}$ of a dollar and $\frac{5}{8}$ of a dollar; also, $\frac{8}{11}$ and $\frac{9}{11}$ are like fractions.

123. Principle.

Only like fractions can be added.

WRITTEN EXERCISES.

155. What is the sum of $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{11}{12}$?

$$\frac{5}{6} + \frac{3}{8} + \frac{11}{12} =$$
$$\frac{20}{24} + \frac{9}{24} + \frac{22}{24} =$$
$$\frac{51}{24} = 2\frac{3}{24} = 2\frac{1}{8}$$

Solution. — Reducing the given fractions to fractions having the least common denominator, we have $\frac{20}{24}$, $\frac{9}{24}$, and $\frac{22}{24}$, which added give $\frac{51}{24} = 2\frac{3}{24}$, or $2\frac{1}{8}$.

156. What is the sum of $\frac{5}{8}$, $\frac{11}{12}$, and $\frac{13}{16}$?

157. What is the sum of $\frac{5}{14}$, $\frac{11}{18}$, and $\frac{9}{20}$?

124. Rule for Addition of Fractions.

Change the fractions, if necessary, to fractions having a common denominator, add the numerators, write the sum over the common denominator, and simplify the result, if needful.

158. Add $\frac{4}{9}$, $\frac{5}{14}$, and $\frac{5}{42}$.

159. Add $\frac{15}{21}$, $\frac{11}{21}$, and $\frac{17}{21}$.

160. Add $\frac{23}{24}$, and $\frac{61}{96}$.

161. Add $\frac{2}{7}$, $\frac{6}{11}$, and $\frac{8}{14}$.

162. Add $\frac{5}{17}$, $\frac{7}{51}$, and $\frac{9}{68}$.

163. Add $\frac{31}{6}$, $\frac{7}{9}$, and $\frac{61}{24}$.

164. Add $\frac{7}{8}$, $\frac{3}{10}$, $\frac{11}{16}$, and $\frac{7}{18}$.

165. Add $\frac{3}{25}$, $\frac{48}{50}$, $\frac{74}{75}$, and $\frac{81}{100}$.

166. What is the value of $\frac{2}{5} + \frac{23}{36} + \frac{17}{9} + \frac{7}{1}$?

125. When there are mixed numbers, *the fractions and the integers may be added separately, and the results united.*

167. What is the sum of $15\frac{3}{4}$ and $24\frac{5}{8}$?

168. What is the value of $37\frac{5}{6} + 109\frac{2}{7} + 341\frac{8}{21}$?

169. A man paid \$$4\frac{3}{4}$ for a hat, \$$16\frac{1}{2}$ for a coat, and \$$5\frac{7}{8}$ for a pair of boots. How much did he pay for the whole?

170. A has in his farm $160\frac{3}{4}$ acres, B has $67\frac{5}{16}$ acres, and C has $85\frac{7}{20}$ acres. What is the number of acres in the three farms?

SUBTRACTION.

ORAL EXERCISES.

171. A man owned $\frac{13}{16}$ of a ship and sold $\frac{7}{16}$. What part of the ship had he left?

172. $\frac{13}{16}$ less $\frac{7}{16}$ is what part of 1?

173. How much is $\frac{17}{19}$ less $\frac{11}{19}$? $\frac{37}{31}$ less $\frac{11}{31}$?

174. How much is $\frac{23}{28}$ less $\frac{3}{7}$?

Solution. — $\frac{3}{7}$ is $\frac{12}{28}$; and $\frac{23}{28}$ less $\frac{12}{28}$ is $\frac{11}{28}$.

175.	**176.**	**177.**
$\frac{2}{3} - \frac{1}{4} = ?$	$\frac{3}{5} - \frac{2}{10} = ?$	$\frac{2}{3} - \frac{1}{7} = ?$
$\frac{1}{2} - \frac{1}{5} = ?$	$\frac{7}{8} - \frac{3}{4} = ?$	$\frac{4}{7} - \frac{5}{21} = ?$
$\frac{8}{10} - \frac{1}{2} = ?$	$\frac{5}{8} - \frac{1}{4} = ?$	$\frac{1}{3} - \frac{1}{8} = ?$
$\frac{5}{6} - \frac{1}{3} = ?$	$\frac{2}{3} - \frac{1}{5} = ?$	$\frac{2}{3} - \frac{3}{8} = ?$
$\frac{7}{10} - \frac{1}{2} = ?$	$\frac{4}{5} - \frac{2}{3} = ?$	$\frac{5}{6} - \frac{1}{8} = ?$

178.

$\frac{2}{3} - \frac{1}{11} = ?$
$\frac{5}{6} - \frac{4}{18} = ?$
$\frac{5}{6} - \frac{3}{7} = ?$
$\frac{3}{4} - \frac{4}{9} = ?$
$\frac{3}{5} - \frac{5}{9} = ?$

179.

$\frac{5}{6} - \frac{5}{12} = ?$
$\frac{2}{7} - \frac{1}{11} = ?$
$\frac{4}{5} - \frac{3}{13} = ?$
$\frac{8}{9} - \frac{7}{12} = ?$
$\frac{11}{12} - \frac{5}{8} = ?$

180. Jane had \$$2\frac{1}{2}$, and spent \$$1\frac{3}{4}$. How much had she left?

126. Principle.

Only like fractions can be subtracted the one from the other.

WRITTEN EXERCISES.

181. From $\frac{13}{16}$ subtract $\frac{7}{12}$.

$\frac{13}{16} - \frac{7}{12} = \frac{39}{48} - \frac{28}{48}$, or $\frac{11}{48}$

Solution. — Reducing the given fractions to fractions having the least common denominator, we have $\frac{39}{48}$ and $\frac{28}{48}$. $\frac{28}{48}$ subtracted from $\frac{39}{48}$ leaves $\frac{11}{48}$.

182. Find the difference between $\frac{21}{24}$ and $\frac{31}{42}$.

183. From $\frac{29}{30}$ take $\frac{23}{30}$. **184.** From $\frac{17}{42}$ take $\frac{3}{14}$.

127. Rule for Subtraction of Fractions.

Change the fractions, if necessary, to fractions having a common denominator, find the difference of the numerators, and write it over the common denominator.

185. From $\frac{17}{24}$ take $\frac{7}{20}$.

186. From $\frac{11}{34}$ take $\frac{1}{10}$.

187. From $\frac{31}{36}$ take $\frac{9}{16}$.

188. From $\frac{167}{711}$ take $\frac{19}{711}$.

189. From $\frac{37}{51}$ take $\frac{48}{68}$.

190. From $\frac{97}{100}$ take $\frac{17}{100}$.

191. From $\frac{1}{10}$ take $\frac{19}{1000}$.

192. From $\frac{41}{34}$ take $\frac{17}{17}$.

193. What is the difference between $\frac{19}{96}$ and $\frac{43}{216}$?

194. I copied by mistake $\frac{15}{31}$ instead of $\frac{15}{30}$. What is the amount of error I made?

195. How much is $49\frac{2}{7}$ less $31\frac{3}{8}$?

$$
\begin{aligned}
49\tfrac{2}{7} &= 49\tfrac{10}{35} = 48\tfrac{45}{35}\\
31\tfrac{3}{5} &= \underline{31\tfrac{21}{35}}\\
&\qquad\;\; 17\tfrac{24}{35}
\end{aligned}
$$

Solution. — Changing the fractions to fractions having a common denominator, we have $49\frac{10}{35}$ and $31\frac{21}{35}$. As we cannot take $\frac{21}{35}$ from $\frac{10}{35}$, we take 1, or $\frac{35}{35}$, from 49, leaving 48; and adding the $\frac{35}{35}$ to $\frac{10}{35}$, we have $48\frac{45}{35}$. Subtracting $\frac{21}{35}$ from $\frac{45}{35}$, and 31 from 48, we have, as the result required, $17\frac{24}{35}$.

196. $31\frac{3}{4} - 10\frac{1}{6} = ?$

197. $63 - 54\frac{7}{12} = ?$

198. $73\frac{9}{10} - 67\frac{5}{100} = ?$

199. $29\frac{13}{18} - 16\frac{5}{6} = ?$

200. $311 - 30\frac{19}{31} = ?$

201. $103\frac{105}{108} - 99\frac{77}{96} = ?$

202. A boy paid for a sled \$ $7\frac{5}{8}$, and for skates \$ $1\frac{5}{6}$. How much more did he pay for the sled than for the skates?

203. The greater of two numbers is 150, and the smaller is $147\frac{17}{40}$. What is their difference?

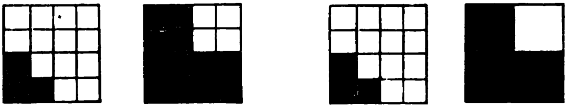
204. Two men start from the same place and travel in the same direction. When one has gone $17\frac{3}{20}$ miles and the other $19\frac{5}{8}$ miles, how far are they apart?

MULTIPLICATION.

A Fraction multiplied by an Integer.

ORAL EXERCISES.

205. Multiply $\frac{3}{16}$ by 4.



$\frac{3}{16} \times 4 = \frac{12}{16}$, or $\frac{3}{16 \div 4} = \frac{3}{4}$

Solution. — Multiplying the numerator of $\frac{3}{16}$ by 4 gives $\frac{12}{16}$, equal to $\frac{3}{4}$; or, dividing the denominator of $\frac{3}{16}$ by 4, gives $\frac{3}{4}$. In either case, $\frac{3}{16}$ is multiplied by 4. For it will be seen by the illustration that multiplying the numerator by 4 *increases* the *number* of fractional units 4 times, while their size remains unchanged; and that dividing the denominator by 4 *increases* the *size* of the fractional units 4 times, while their number is unchanged. The effect of either process, *therefore, is to* increase the value of the fraction 4 times.

206. At \$$\frac{3}{4}$ each, how many dollars will 12 arithmetics cost?

207. At the rate of $\frac{7}{8}$ of a bushel of grain a week, how many bushels of grain will a horse consume in 6 weeks?

208. If 1 hat costs \$$3\frac{1}{4}$, what will 8 hats cost?

209. Multiply the fractions in Exercises 27 to 32 by 3; by 4; by 5; by 6; by 7; by 8; by 9; by 10; by 12.

210. Multiply the mixed numbers in Exercises 74 to 78 by 4; by 5; by 6; by 8; by 9; by 10; by 12.

128. Principle.

A fraction is multiplied either by multiplying its numerator or by dividing its denominator.

WRITTEN EXERCISES.

211. Multiply $\frac{11}{28}$ by 7.

Solution.

$$7 \times \frac{11}{28} = \frac{11 \times 7}{28} = \frac{77}{28} = \frac{11}{4}, \text{ or } 2\frac{3}{4}$$

Or,

$$7 \times \frac{11}{28} = \frac{11}{28 \div 7} = \frac{11}{4}, \text{ or } 2\frac{3}{4}$$

212. Multiply $\frac{7}{27}$ by 6.

213. Multiply $\frac{19}{20}$ by 16.

214. Multiply $\frac{29}{31}$ by 40.

215. Multiply $\frac{49}{85}$ by 85.

216. Multiply $\frac{23}{75}$ by 35.

217. Multiply $\frac{16}{101}$ by 18.

218. Multiply $\frac{87}{96}$ by 48.

219. Multiply $\frac{108}{57}$ by 76.

220. What is the product of $63\frac{3}{4}$ by 5?

$$\begin{array}{lr} \frac{3}{4} \times 5 = \frac{15}{4} = & 3\frac{3}{4} \\ 63 \times 5 \quad = & 315 \\ \hline & 318\frac{3}{4} \end{array}$$

Or,

$$\begin{array}{r} 63\frac{3}{4} \\ 5 \\ 4)\overline{15} \\ \hline 3\frac{3}{4} \\ 315 \\ \hline 318\frac{3}{4} \end{array}$$

Solution. — $63\frac{3}{4}$ is 63 and $\frac{3}{4}$. $\frac{3}{4}$ multiplied by 5 gives $3\frac{3}{4}$, and 63 multiplied by 5 gives 315. These results added give, as the product required, $318\frac{3}{4}$.

221. Multiply $40\frac{7}{18}$ by 27.

222. Multiply $81\frac{4}{9}$ by 63.

223. Multiply $39\frac{3}{105}$ by 75.

224. Multiply $125\frac{5}{16}$ by 80.

225. What will 72 barrels of St. Louis flour cost at \$ $8\frac{5}{8}$ a barrel ?

An Integer multiplied by a Fraction.

ORAL EXERCISES.

226. If a bushel of corn costs 64 cents, what will $\frac{1}{2}$ of a bushel cost ? $\frac{1}{4}$ of a bushel ?

227. If a pound of tea costs 60 cents, what will $\frac{1}{4}$ of a pound cost ? $\frac{3}{4}$ of a pound ?

228. What is $\frac{5}{7}$ of 4 ?

Solution. — $\frac{1}{7}$ of 4 is $\frac{4}{7}$, and $\frac{5}{7}$ of 4 must be 5 times $\frac{4}{7}$, or $\frac{20}{7}$, equal to $2\frac{6}{7}$.

229. What is $\frac{3}{11}$ of 8 ? $\frac{4}{5}$ of 3 ? $\frac{6}{7}$ of 5 ? $\frac{5}{9}$ of 10 ?

230. What is the cost of $\frac{7}{8}$ of a cord of wood at \$ 6 a cord ? What is \$ 6 multiplied by $\frac{7}{8}$?

129. Principle.

Multiplying a number by a fraction is taking such a part of the number as is denoted by the fraction.

WRITTEN EXERCISES.

231. Multiply 180 by $\frac{4}{15}$.

$$\begin{array}{r} 15)\underline{180} \\ \overline{12} \\ \underline{4} \\ 48 \end{array} \qquad \text{Or,} \qquad \begin{array}{r} 180 \\ \underline{4} \\ 15)\,720 \\ \overline{48} \end{array}$$

Solution. — $180 \times \frac{4}{15}$ is the same as $\frac{4}{15}$ of 180. $\frac{1}{15}$ of $180 = 12$, and $\frac{4}{15}$ of $180 = 4$ times 12, or 48.

Or, as $\frac{4}{15}$ is the same as $\frac{1}{15}$ of 4, $180 \times \frac{4}{15}$ is the same as $\frac{1}{15}$ of 4 times 180. 4 times $180 = 720$, and $\frac{1}{15}$ of $720 = 48$.

232. Multiply 108 by $\frac{7}{9}$.

233. Multiply 89 by $\frac{5}{18}$.

234. Multiply 375 by $\frac{13}{15}$

235. Multiply 137 by $\frac{5}{7}$.

236. Multiply 144 by $\frac{11}{18}$.

237. Multiply 239 by $\frac{9}{13}$.

238. Multiply 404 by $\frac{3}{71}$.

239. Multiply 376 by $\frac{11}{17}$.

240. Multiply 33 by $3\frac{2}{5}$.

$$
\begin{array}{rl}
33 \times \frac{2}{5} = & 13\frac{1}{5} \\
33 \times 3 = & 99 \\
\hline
& 112\frac{1}{5}
\end{array}
\quad \text{Or,} \quad
\begin{array}{r}
33 \\
3\frac{2}{5} \\
\hline
5)\ 66 \\
\hline
13\frac{1}{5} \\
99 \\
\hline
112\frac{1}{5}
\end{array}
$$

Solution. — $3\frac{2}{5}$ is 3 and $\frac{2}{5}$; 33 multiplied by $\frac{2}{5}$ gives $13\frac{1}{5}$, and 33 multiplied by 3 gives 99. These results added give, as the product required, $112\frac{1}{5}$.

241. Multiply 110 by $9\frac{3}{4}$.

242. Multiply 85 by $13\frac{1}{6}$.

243. Multiply 84 by $9\frac{5}{12}$.

244. Multiply 145 by $11\frac{6}{7}$.

245. What cost $25\frac{5}{8}$ acres of land at $ 50 an acre?

A Fraction multiplied by a Fraction.

ORAL EXERCISES.

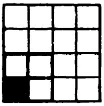
246. What part of a dollar is $\frac{1}{2}$ of $\frac{1}{2}$ of a dollar? $\frac{1}{3}$ of $\frac{1}{2}$ of a dollar?

247. What part of a dollar is $\frac{1}{2}$ of $\frac{2}{3}$ of a dollar? $\frac{1}{3}$ of $\frac{3}{4}$ of a dollar?

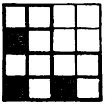
248. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{6}$?

249. What is $\frac{1}{3}$ of $\frac{3}{5}$? $\frac{1}{4}$ of $\frac{8}{9}$? $\frac{1}{5}$ of $\frac{5}{7}$? $\frac{1}{7}$ of $\frac{21}{23}$?

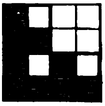
250. How much is $\frac{3}{4}$ of $\frac{3}{4}$?



$\frac{1}{4}$ of $\frac{1}{4} = \frac{1}{16}$



$\frac{1}{4}$ of $\frac{3}{4} = \frac{3}{16}$



$\frac{3}{4}$ of $\frac{3}{4} = \frac{9}{16}$

Solution. — $\frac{1}{4}$ of $\frac{3}{4}$ is $\frac{3}{16}$, and $\frac{3}{4}$ of $\frac{3}{4}$ is 3 times $\frac{3}{16}$, equal to $\frac{9}{16}$. This, also, appears by the illustration.

251.	252.	253.
$\frac{2}{3} \times \frac{3}{4} = ?$	$\frac{1}{2} \times \frac{1}{5} = ?$	$\frac{3}{10} \times \frac{1}{2} = ?$
$\frac{3}{5} \times \frac{3}{4} = ?$	$\frac{7}{8} \times \frac{3}{4} = ?$	$\frac{5}{8} \times \frac{1}{4} = ?$
$\frac{2}{3} \times \frac{1}{7} = ?$	$\frac{5}{21} \times \frac{2}{3} = ?$	$\frac{3}{8} \times \frac{2}{5} = ?$
$\frac{1}{11} \times \frac{2}{3} = ?$	$\frac{4}{18} \times \frac{3}{4} = ?$	$\frac{3}{7} \times \frac{5}{6} = ?$
$\frac{5}{12} \times \frac{5}{6} = ?$	$\frac{1}{11} \times \frac{2}{7} = ?$	$\frac{3}{13} \times \frac{4}{5} = ?$

254. Give the product of each pair of fractions in Exercises 150 to 153.

130. A *Compound Fraction* is a fraction of a fraction, as $\frac{3}{8}$ of $\frac{5}{8}$, and may be considered an expression of multiplication.

A *Simple Fraction* is a fraction not connected with another, and having both its terms integers.

WRITTEN EXERCISES.

255. Multiply $\frac{30}{31}$ by $\frac{5}{6}$.

$$\frac{30}{31}\times\frac{5}{6}=\frac{150}{186}=\frac{25}{31}, \text{ or, } \frac{\overset{5}{\cancel{30}}\times 5}{31\times\cancel{6}}=\frac{25}{31}$$

Solution. — $\frac{30}{31}$ multiplied by $\frac{5}{6}$ is the same as $\frac{5}{6}$ of $\frac{30}{31}$; $\frac{1}{6}$ of $\frac{30}{31}$ is $\frac{30}{186}$, and $\frac{5}{6}$ of $\frac{30}{31}$ is 5 times $\frac{30}{186}$, equal to $\frac{150}{186}$, or $\frac{25}{31}$.

NOTE.— By cancellation we may shorten the process, as shown in the second form of the work.

256. Multiply $\frac{18}{19}$ by $\frac{11}{12}$; $\frac{15}{36}$ by $\frac{24}{25}$.

131. Rule for Multiplication of Fractions.

Write the product of the numerators over the product of the denominators.

NOTE. — The rule is general, since a mixed number may be changed to an improper fraction, and an integer may be expressed in a fractional form.

257. Multiply $\frac{35}{72}$ by 27.

258. Multiply $\frac{36}{65}$ by $\frac{52}{99}$.

259. Multiply $\frac{267}{300}$ by $\frac{7}{5}$.

260. Multiply $\frac{7}{9}$ by $\frac{2}{3}$ of $\frac{6}{11}$.

261. Multiply $\frac{48}{5}$ by $\frac{4}{5}$ of $\frac{5}{16}$.

262. Multiply $\frac{35}{17}$ by $\frac{1}{7}$ of $\frac{51}{70}$.

263. What is the value of $\frac{3}{4}$ of $\frac{8}{9}$ of $3\frac{1}{3}$?

264. What is the value of $\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}$?

265. What cost $11\frac{3}{8}$ cords of wood at \$ $7\frac{5}{6}$ a cord?

266. What is the value of $\frac{11}{5}$ of $3\frac{1}{2}\times\frac{8}{5}\times\frac{2}{3}$ of 10?

267. What is the value of $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{9}{11}$ multiplied by $\frac{2}{3}$ of 18?

268. What is the product of $\frac{4}{7}$ of $8\frac{3}{10}$ multiplied by $\frac{4}{7}$ of $9\frac{1}{4}$?

269. What will $5\frac{7}{10}$ tons of hay cost at \$ $21\frac{9}{16}$ a ton?

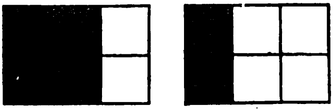
270. If a man walks $4\frac{9}{10}$ miles in an hour, how many miles can he walk in $\frac{3}{4}$ of $3\frac{7}{12}$ hours?

DIVISION.

A Fraction divided by an Integer.

ORAL EXERCISES.

271. What is $\frac{2}{3}$ divided by 2?



$$\frac{2 \div 2}{3} = \frac{1}{3}$$

Or,

$$\frac{2}{3 \times 2} = \frac{2}{6}$$

Solution. — $\frac{2}{3}$ divided by 2 is $\frac{2 \div 2}{3}$, equal to $\frac{1}{3}$. Or, as $\frac{2}{3}$ divided by 2 is the same as $\frac{1}{2}$ of $\frac{2}{3}$, we have $\frac{2}{3 \times 2}$, equal to $\frac{2}{6}$, or $\frac{1}{3}$.

It will be observed by the illustration that dividing the numerator by 2 *halves* the *number* of fractional units, while their size remains unchanged; and that multiplying the denominator by 2 *halves* the *size* of the fractional units, while the number remains unchanged. Either process divides the fraction by 2.

272. What is $\frac{9}{10}$ divided by 3? $\frac{15}{16}$ by 5?

273. Divide $\frac{21}{30}$ by 7; $\frac{45}{57}$ by 9; $\frac{63}{78}$ by 7; $\frac{99}{100}$ by 11.

274. Divide the fractions in Exercises 99 to 103 by 2; by 3; by 5; by 6; by 8; and simplify the result.

275. If 8 men can mow $\frac{3}{4}$ of a field in a day, what part of it can one man mow?

276. How is $\frac{3}{10}$ divided by 5? $\frac{3}{4}$ by 8? $\frac{4}{5}$ by 9?

277. Divide $\frac{7}{9}$ by 6; $\frac{11}{12}$ by 7; $\frac{9}{10}$ by 10; $\frac{11}{13}$ by 6.

278. If 7 pounds of coffee cost \$$2\frac{1}{10}$, how much will 1 pound cost?

Solution. — \$$2\frac{1}{10}$ = \$$\frac{21}{10}$; if 7 pounds cost \$$\frac{21}{10}$, 1 pound must cost $\frac{1}{7}$ of \$$\frac{21}{10}$, or \$$\frac{3}{10}$.

279. How much is $2\frac{1}{10}$ divided by 7? $11\frac{1}{3}$ divided by 17? $10\frac{5}{7}$ divided by 25?

280. If 9 bushels of wheat cost \$$19\frac{3}{4}$, what is the cost of 1 bushel?

132. Principle.

A fraction is divided either by dividing its numerator or by multiplying its denominator.

WRITTEN EXERCISES.

281. Divide $\frac{18}{23}$ by 9.

$$\frac{18}{23} \div 9 = \frac{18 \div 9}{23} = \frac{2}{23}$$

Or,

$$\frac{18}{23} \div 9 = \frac{18}{23 \times 9} = \frac{18}{207} = \frac{2}{23}$$

Solution. — As a fraction is divided by dividing its numerator, we divide the numerator by 9, and have $\frac{2}{23}$. Or, as a fraction is divided by multiplying the denominator, we multiply the denominator by 9, and have, as before, $\frac{2}{23}$.

282. Divide $\frac{7}{19}$ by 12.

283. Divide $\frac{16}{23}$ by 49.

284. Divide $\frac{3}{47}$ by 11.

285. Divide $\frac{105}{316}$ by 21.

286. Divide $\frac{67}{91}$ by 60.

287. Divide $\frac{64}{97}$ by 16.

288. Divide $\frac{1}{131}$ by 5.

289. Divide $\frac{450}{533}$ by 75.

290. Divide $17\frac{3}{8}$ by 6.

$$\begin{array}{r} 6)\,17\frac{3}{8} \\ \hline 2\frac{43}{48} \end{array}$$

Or,

$$17\frac{3}{8} = \frac{139}{8}$$

$$\frac{139}{8} \div 6 = \frac{139}{8 \times 6} = \frac{139}{48} = 2\frac{43}{48}$$

Solution. — 6 in $17\frac{3}{8}$, 2 times, and $5\frac{3}{8}$ remainder. $5\frac{3}{8} = \frac{43}{8}$, and $\frac{43}{8}$ divided by 6 gives $\frac{43}{48}$, which, united with the partial quotient 2, gives $2\frac{43}{48}$.

Or, $17\frac{3}{8} = \frac{139}{8}$, and $\frac{139}{8}$ divided by 6 gives, as before, $2\frac{43}{48}$.

291. When $28 is paid for $2\frac{1}{10}$ tons of hay, what part of a ton will $1 purchase?

292. When $\$107\frac{7}{10}$ is shared equally by 7 men, how many dollars will each man receive?

293. The product of two numbers is $30\frac{3}{8}$, and one of the numbers is 9. What is the other number?

294. When 12 cords of wood cost $\$76\frac{1}{2}$, what is the cost a cord?

An Integer divided by a Fraction.

ORAL EXERCISES.

295. How many fourths of a dollar in $ 1? In $ 3?

296. How many times is $\frac{1}{4}$ in 1? In 2? In 3?

297. In 4 how many times $\frac{1}{2}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{4}$? $\frac{2}{4}$?

298. At $ $\frac{3}{4}$ a pound, how many pounds of tea can be bought for $ 6?

Solution. — As many pounds as times $\frac{3}{4}$ in 6. In 6 there are $\frac{24}{4}$, and $\frac{3}{4}$ are contained in $\frac{24}{4}$ 8 times. Ans. 8 pounds.

299. In how many days can a horse consume 4 bushels of oats, if he consume $\frac{4}{15}$ of a bushel a day?

300. Divide any given integer by the fractions in Exercises 27 to 33.

301. At $ $1\frac{1}{4}$ a yard, how many yards of cloth can be bought for $ 10?

302. In how many weeks will a boy earn $ 8, if he is paid $ $2\frac{2}{5}$ a week?

WRITTEN EXERCISES.

303. Divide 65 by $\frac{5}{7}$.

$$65 = \frac{455}{7}; \quad \frac{455}{7} \div \frac{5}{7} = 91$$

Or,

$$65 \div \frac{5}{7} = \frac{\overset{13}{\cancel{65}} \times 7}{\cancel{5}} = 91$$

Solution. — $65 = \frac{455}{7}$; and $\frac{5}{7}$ is contained in $\frac{455}{7}$ as many times as 5 in 455, or 91 times. Or, $\frac{1}{7}$ is contained in 65, 7 times 65 times, and $\frac{5}{7}$, $\frac{1}{5}$ of 7 times 65 times, or 91 times. This process is the same as multiplying the dividend by the divisor inverted.

304. Divide 49 by $\frac{16}{29}$.

305. Divide 27 by $\frac{11}{12}$.

306. Divide 84 by $\frac{24}{25}$.

307. Divide 49 by $\frac{14}{15}$.

308. Divide 60 by $\frac{6}{17}$.

309. Divide 98 by $\frac{11}{17}$.

310. Divide 128 by $3\frac{7}{35}$.

311. Divide 432 by $2\frac{2}{3}$.

312. Divide 100 by $\frac{17}{19}$.

313. Divide 118 by $5\frac{3}{8}$.

314. How many yards of cloth at $ $\frac{7}{8}$ a yard can be bought for $ 56?

315. If a family consumes $2\frac{3}{4}$ pounds of sugar a week, in what time will it consume 44 pounds?

316. There is a board 17 feet long, which I wish to saw into pieces $2\frac{3}{7}$ feet long. What will be the number of pieces?

A Fraction divided by a Fraction.

ORAL EXERCISES.

317. How many times $\frac{2}{5}$ of a yard in $\frac{4}{5}$ of a yard?

318. At $\frac{3}{8}$ of a dollar a pound, how many pounds of coffee can be bought for $\frac{9}{8}$ of a dollar?

319. How many times $\frac{2}{5}$ in $\frac{4}{5}$? $\frac{3}{8}$ in $\frac{9}{8}$? $\frac{2}{9}$ in $\frac{14}{9}$?

320. How many pounds of raisins at $\frac{3}{16}$ of a dollar a pound can be bought for $\frac{3}{4}$ of a dollar?

Solution. — As many pounds as times $\frac{3}{16}$ in $\frac{3}{4}$. In $\frac{3}{4}$ there are $\frac{12}{16}$, and $\frac{3}{16}$ are contained in $\frac{12}{16}$, 4 times. Ans. 4.

321. How many times $\frac{2}{10}$ in $\frac{4}{5}$? $\frac{1}{2}$ in $\frac{4}{8}$? $\frac{3}{8}$ in $\frac{9}{12}$?

322. If it takes $\frac{3}{4}$ of a yard of cloth for a vest, how many vests can be made from $2\frac{1}{4}$ yards?

323.	324.	325.
$\frac{3}{16}\div\frac{1}{8}=?$	$\frac{7}{8}\div\frac{3}{4}=?$	$\frac{9}{2}\div\frac{1}{3}=?$
$\frac{7}{8}\div\frac{2}{5}=?$	$\frac{5}{6}\div\frac{1}{2}=?$	$2\frac{1}{3}\div\frac{4}{5}=?$
$\frac{12}{14}\div\frac{4}{7}=?$	$\frac{9}{10}\div\frac{3}{4}=?$	$2\frac{3}{8}\div\frac{5}{8}=?$
$\frac{5}{7}\div\frac{2}{3}=?$	$\frac{5}{6}\div\frac{2}{3}=?$	$6\frac{5}{6}\div\frac{2}{3}=?$
$\frac{3}{4}\div\frac{1}{2}=?$	$\frac{9}{12}\div\frac{3}{8}=?$	$3\frac{1}{2}\div\frac{1}{6}=?$

WRITTEN EXERCISES.

326. Divide $\frac{4}{5}$ by $\frac{2}{3}$.

$$\frac{4}{5}\div\frac{2}{3}=\frac{12}{15}\div\frac{10}{15}=\frac{12}{10}=\frac{6}{5}=1\frac{1}{5}$$

Or,

$$\frac{4}{5}\div\frac{2}{3}=\frac{\overset{2}{\cancel{4}}}{5}\times\frac{3}{\cancel{2}}=\frac{6}{5}=1\frac{1}{5}$$

Solution. — $\frac{4}{5}\div\frac{2}{3}$ is equivalent to $\frac{12}{15}$ divided by $\frac{10}{15}$, or to $\frac{12}{10}$, equal to $\frac{6}{5}$, or $1\frac{1}{5}$.

Or, as $\frac{4}{5}\div\frac{1}{3}$ is $\frac{4\times 3}{5}$, $\frac{4}{5}\div\frac{2}{3}$ is $\frac{4\times 3}{5\times 2}$, equal to $\frac{6}{5}$, or $1\frac{1}{5}$.

This is the same as multiplying the dividend by the divisor inverted.

327. What is the quotient of $\frac{7}{8}$ divided by $\frac{4}{7}$?

328. What is the quotient of $\frac{12}{25}$ divided by $\frac{9}{10}$?

133. Rule for Division of Fractions.

Change the fractions, if necessary, to fractions having a common denominator, and divide the numerator of the dividend by the numerator of the divisor. Or,

Invert the divisor, and proceed as in multiplication of fractions.

NOTE 1. — The rule is general, since a mixed number may be changed to an improper fraction, and an integer may be expressed in a fractional form.

NOTE 2. — A fraction having a fraction in one or both of its terms, as in $\frac{\frac{1}{2}}{8}$, $\frac{2\frac{1}{2}}{\frac{3}{4}}$, $\frac{\frac{3}{5}}{\frac{7}{8}}$ is called a *Complex Fraction*, and may be considered an expression of division.

329. Divide $\frac{25}{39}$ by $\frac{10}{13}$.

330. Divide $\frac{12}{49}$ by $\frac{3}{7}$.

331. Divide $\frac{11}{12}$ by $\frac{47}{56}$.

332. Divide $\frac{17}{20}$ by $\frac{3}{5}$.

333. Divide $\frac{38}{275}$ by $\frac{133}{385}$.

334. Divide $\frac{41}{33}$ by $\frac{63}{11}$.

335. Divide $9\frac{3}{8}$ by $4\frac{1}{2}$.

336. Divide $\frac{208}{135}$ by $\frac{130}{441}$.

337. Divide $17\frac{8}{21}$ by $5\frac{5}{6}$.

338. Divide $100\frac{5}{9}$ by $8\frac{2}{3}$.

339. How much is tea a pound when $\frac{14}{15}$ of a pound cost $\frac{14}{25}$ of a dollar?

340. How much is hay a ton when $\frac{5}{8}$ of a ton cost $11\frac{1}{4}$?

341. If a man walks $3\frac{3}{5}$ miles in an hour, in what time will he walk $15\frac{3}{10}$ miles?

342. Simplify $\frac{3\frac{3}{4}}{2\frac{2}{3}}$.

Solution. — Considering the fraction as an expression of division, we reduce its terms to improper fractions, and, dividing, have $\frac{45}{32}$, or $1\frac{13}{32}$.

$$\frac{3\frac{3}{4}}{2\frac{2}{3}} = \frac{15}{4} \div \frac{8}{3} = \frac{15}{4} \times \frac{3}{8} = \frac{45}{32} = 1\frac{13}{32}$$

Or,

$$\frac{3\frac{3}{4}}{2\frac{2}{3}} = \frac{45}{32} = 1\frac{13}{32}$$

Or, multiplying both terms of the complex fraction by 12, the least common multiple of the denominators of the fractional parts, we have, as before, $\frac{45}{32}$, or $1\frac{13}{32}$.

343. Simplify $\frac{\frac{9}{13}}{\frac{4}{5}}$. **345.** Simplify $\frac{6\frac{2}{7}}{\frac{3}{8}}$. **347.** Simplify $\frac{19}{2\frac{4}{11}}$.

344. Simplify $\frac{12}{\frac{5}{8}}$. **346.** Simplify $\frac{5\frac{1}{2}}{11}$. **348.** Simplify $\frac{6\frac{2}{3}}{8\frac{5}{6}}$.

349. Reduce $\frac{10\frac{5}{7}}{83\frac{1}{4}}$ to its simplest form.

350. If a man can do $\frac{18}{36}$ of a piece of work in $23\frac{3}{4}$ days, in what time can he do the whole?

351. At the rate of $31\frac{6}{14}$ miles an hour, in what time will a train of cars move $125\frac{5}{7}$ miles?

To find the Fraction one Number is of another.

352. One is what part of 2? Of 3? Of 4? Of 5?

353. What part of 5 dollars is 1 dollar? 2 dollars? 3 dollars? 4 dollars?

354. What part of 7 miles is 3 miles? 4 miles? 5 miles? 6 miles?

355. What part of 5 is $\frac{2}{3}$?

Solution. — 1 is $\frac{1}{5}$ of 5, and $\frac{2}{3}$ of 1 is $\frac{2}{3}$ of $\frac{1}{5}$ of 5, or $\frac{2}{15}$ of 5.

356. What part of 4 dollars is $\frac{5}{6}$ of a dollar?

357. What part of $\frac{9}{13}$ is $\frac{3}{13}$? What part of $\frac{5}{6}$ is $\frac{2}{3}$, or $\frac{4}{6}$?

358. What part of 8 dollars is $2\frac{1}{2}$ dollars?

134. The **Fraction** one number is of another is found by *dividing the number denoting the part by the number denoting the whole.*

WRITTEN EXERCISES.

359. What part of 125 is 75? **363.** What part of 20 is $3\frac{1}{3}$?

360. What part of 7 is 6? **364.** What part of 90 is $6\frac{1}{4}$?

361. What part of 12 is $\frac{15}{18}$? **365.** $\frac{13}{21}$ is what part of $\frac{13}{18}$?

362. What part of 186 is 93? **366.** $\frac{27}{52}$ is what part of $\frac{36}{65}$?

367. What fraction of 84 is 91?

368. What fraction of 3⅔ is 2½?

369. A tree, whose height was 85 feet, was broken off in a gale, 55 feet from the top. What part of the tree was left standing?

370. A sportsman started up a flock of 144 birds, and shot ⅔ of them. What number escaped?

371. If 25 yards of carpeting cost $37.50, what should 35 yards cost at the same rate?

372. John has 156 cents, and Peter 106 cents. How does Peter's money compare with John's?

373. If a man can do a piece of work in 7¾ days, what part of it can he do in 5⅙ days?

374. What will be the cost of ⅞ of a cord of wood, when 12 cords cost $$68\frac{4}{25}$?

To find the Whole when a Fractional Part is given.

375. 4 is ½ of what number? 6 is ½ of what number? 8 is ⅓ of what number? 3 is ⅛ of what number?

376. I spent $6, which was ⅕ of all my money; how much had I?

377. I lost ⅔ of my sheep, but had 8 left; how large a flock had I at first?

378. 18 years is ¾ of my age; how old am I?

Solution. — Since 18 years is 3 fourths of my age, 1 fourth of my age is ⅓ of 18 years, or 6 years; 4 fourths of my age is 4 times 6 years, or 24 years. Ans. 24 years.

379. $\frac{6}{7}$ of what number is 24?

380. 32 is ⅘ of what number? 48 is $\frac{8}{9}$ of what number?

381. 36 is $\frac{9}{10}$ of what number? 54 is $\frac{6}{10}$ of what number?

382. 42 is ⅞ of what number? 84 is $\frac{7}{12}$ of what number?

383. 63 is $\frac{9}{11}$ of what number? 75 is $\frac{25}{31}$ of what number?

384. How far is it to Boston if ⅞ the distance is 21 miles?

385. My house is insured for $\frac{2}{3}$ its value, or for $ 4000; what is it worth ?

386. The cost of my buggy is $\frac{3}{4}$ the cost of my horse. The horse cost $ 360 ; what did the buggy cost ?

WRITTEN EXERCISES.

387. 1728 is $\frac{19}{16}$ of what number ?

$$\frac{\overset{108}{\cancel{1728}}}{\cancel{16}} \times 19 = 2052$$

Solution. — Since 1728 is $\frac{19}{16}$ of an unknown number, $\frac{1}{16}$ of the unknown number is $\frac{1}{16}$ of 1728, or 108; $\frac{19}{16}$ of the unknown number are 19 times 108, or 2052. Ans. 2052.

388. 351 is $\frac{9}{25}$ of what number ?

135. To find the whole when a fractional part is given, *divide the part by the numerator of the fraction, and multiply the quotient by the denominator.*

389. 1368 is $\frac{9}{11}$ of what number ?

390. 891 is $\frac{11}{23}$ of what number ?

391. 8000 is $\frac{125}{100}$ of what number ?

392. What is $\frac{4}{5}$ of a number if 273 is $\frac{21}{50}$ of it ?

393. My income in 1881 was $ 1600, or $\frac{8}{15}$ of my income in 1880 ; what was the falling off ?

394. $\frac{12}{17}$ of 340 is $\frac{8}{11}$ of what number ?

395. $\frac{3}{5}$ is $\frac{9}{10}$ of what number ?

396. $7\frac{1}{2}$ is $\frac{5}{17}$ of what number ?

397. Sold goods for $ 462, and thereby lost $\frac{3}{7}$ of the cost; what was the cost ?

398. A man failing in business could pay me only $ 3930, which was $\frac{3}{5}$ of what he owed me. How much did he owe me ?

399. A has $ 2000, and B has $1\frac{5}{16}$ times as much. How much has B ?

400. My house and furniture cost $ 10901, and the furniture cost $\frac{2}{9}$ as much as the house. How much did the house cost ?

MISCELLANEOUS EXERCISES.

401. At \$ $\frac{5}{8}$ a yard, how many yards of cloth can be bought for \$ $43\frac{1}{2}$?

402. If $2\frac{1}{2}$ pounds of butter cost \$ $\frac{15}{16}$, what is that a pound?

403. How many yards of cloth $\frac{4}{5}$ of a yard wide will equal 12 yards $\frac{3}{4}$ of a yard wide?

404. If $\frac{7}{8}$ of a barrel of oatmeal is worth \$ 5.60, what is a barrel worth?

405. What is the value of $\dfrac{\frac{7}{8}}{\frac{3}{11}}$ of $12\frac{1}{2}$ divided by $\dfrac{\frac{1}{3}}{7\frac{1}{2}}$ of $8\frac{3}{4}$?

406. What number must be multiplied by $7\frac{3}{8}$ that the product may be 20?

407. A can walk $3\frac{7}{11}$ miles in 60 minutes, and B can walk $\frac{9}{11}$ as fast as A. How long will it take B to walk the same distance?

408. If \$ $106\frac{7}{8}$ is shared equally among 8 men, how much will each man's share be?

409. If of \$ 350 you should spend \$ 125, what part of the money would remain?

410. There is a board 19 feet in length, which I wish to saw into pieces $2\frac{3}{7}$ feet long. What will be the number of pieces, and how many feet will remain?

411. How much is coal a ton when $10\frac{1}{8}$ tons can be bought for \$ $67\frac{1}{2}$?

412. What is the relative value of $\frac{2}{21}$ and $\frac{1}{18}$?

413. Reduce $\dfrac{4\frac{2}{3} \text{ of } 5\frac{1}{7} \times 8}{28\frac{1}{2} \div 7\frac{1}{8}}$ to its simplest form.

414. What number must $2\frac{1}{3}$ be multiplied by to give $4\frac{1}{5}$?

415. Find the value of $\frac{1}{9}$ of $1\frac{1}{4} \times 4\frac{1}{2}$ divided by $\frac{5}{56}$ of $1\frac{1}{3} \times 3\frac{1}{2}$.

416. Jones and Smith plowed for a certain sum of money; Jones plowed $6\frac{1}{4}$ acres, and Smith $9\frac{3}{8}$ acres. What should be Smith's money, if Jones's share is \$ 20?

417. What number must be taken from $12\frac{3}{4}$, and the remainder multiplied by $10\frac{4}{5}$, that the product shall be 50?

418. When $236 are paid for $11\frac{4}{5}$ acres, what will $20\frac{7}{10}$ acres cost?

419. In counting my money I found I had $969, which was just $2\frac{3}{8}$ times as much as my brother had. How many dollars had I more than my brother?

QUESTIONS.

99. What is a fraction? 100. What is the denominator of a fraction? 101. What is the numerator? 102. What are the terms of a fraction?

105. What is a proper fraction? 106. An improper fraction? 107. A mixed number?

108. How may a fraction be regarded? 109. What is the value of a fraction?

110. What is reduction of fractions? 112. How is a fraction reduced to larger terms? 113. When is a fraction reduced to its smallest terms? 114. What is a principle of fractions? 115. How is a fraction reduced to smallest terms?

116. How is an integer or mixed number reduced to an improper fraction? 117. How is an improper fraction reduced to an integer or mixed number?

118. When have fractions a common denominator? 120. What must be a common denominator of two or more fractions? 121. How are fractions reduced to their least common denominator?

122. What are like fractions? 123. What fractions only can be added? 124. How do you add fractions? 126. What fractions only can be subtracted? 127. How do you subtract one fraction from another?

128. How is a fraction multiplied? 129. How is a number multiplied by a fraction? 131. How is one fraction multiplied by another?

132. How is a fraction divided? 133. How is one fraction divided by another?

REVIEW.

ORAL EXERCISES.

136. **1.** Name the only even prime number.

2. Show that 56 is a composite number.

3. What is the greatest factor common to 16 and 24?

4. What is the least common multiple of 3, 4, and 6?

5. Reduce $9\frac{4}{5}$ to fifths; $7\frac{5}{8}$ to eighths; $10\frac{3}{11}$ to elevenths.

6. Reduce to smallest terms $\frac{27}{35}$; $\frac{8}{56}$; $\frac{18}{81}$; $\frac{72}{108}$; $\frac{50}{125}$.

7. A has worked $\frac{2}{5}$ of a day, and B $\frac{1}{2}$ of a day. What part of a day have both worked?

8. $\frac{1}{4}$ of a pole is in the water, $\frac{2}{5}$ in the mud, and the rest in the air. What part is in the air?

9. James is 18 years old, and his age is $\frac{3}{7}$ of the age of his father. How old is his father?

10. Damon sold a house lot for $\$30$, which was $\frac{3}{5}$ of what it cost him. What was the cost of the lot?

11. Multiply $\frac{7}{8}$ by 6; $\frac{11}{12}$ by 7; 5 by $\frac{3}{7}$; $\frac{4}{5}$ by $\frac{3}{4}$.

12. If a man can mow $1\frac{2}{5}$ acres of grass in a day, how much can he mow in 6 days?

13. If cloth is worth $\frac{9}{10}$ of a dollar a yard, what is $\frac{5}{8}$ of a yard worth?

14. Divide $\frac{9}{15}$ by 3; $\frac{11}{12}$ by 4; 8 by $\frac{2}{3}$; $\frac{3}{5}$ by $\frac{1}{4}$; $2\frac{1}{2}$ by $\frac{3}{4}$.

15. Bought 5 bushels of wheat for $\$7\frac{3}{4}$. What was it a bushel?

16. How much is $\frac{1}{3}$ of $8\frac{4}{9}$; $\frac{1}{8}$ of $5\frac{3}{7}$; $\frac{1}{9}$ of $7\frac{2}{5}$?

17. There is a pole standing so that $\frac{3}{7}$ of it is in the water, and $\frac{2}{3}$ as much in the mud. How much is in the mud?

18. If $\frac{3}{4}$ of a bushel of grain will serve a horse a week, how many weeks will $6\frac{3}{4}$ bushels serve him?

19. If $1\frac{2}{3}$ tons of hay will keep a cow 8 weeks, how many cows will 10 tons keep for the same time?

20. What number is that, $\frac{1}{3}$ of which exceeds $\frac{1}{4}$ of it by 3?

21. The difference between $\frac{5}{6}$ and $\frac{7}{12}$ of a number is 2. What is the number?

22. If to William's age there is added $\frac{2}{3}$ of his age he will be 25 years old. What is his age?

23. A man bought a coat and a hat for $30. The hat cost $\frac{3}{7}$ as much as the coat. What was the cost of each?

24. A and B made an even exchange of horses. By the trade A lost $40, which was $\frac{2}{7}$ of the value of the horse he had at first. What was the value of each horse?

25. Joseph spent $\frac{7}{15}$ of his money for clothes. He then paid away 6\frac{2}{5}$, which was just $\frac{1}{5}$ of all he had left. How many dollars had he at first?

26. If 25 is $\frac{5}{8}$ of some number, what is $\frac{3}{4}$ of the same number?

27. A boy spent $\frac{5}{8}$ of his money, and then had given him $\frac{1}{4}$ as much as he had left. What part of his money did he then have?

28. What cost 12 apples at the rate of 7 for 5$\frac{1}{4}$ cents?

29. If 6 men can do a piece of work in 4$\frac{2}{5}$ days, in what time will 1 man do it? In what time will 5 men do it?

30. If you can travel 13$\frac{5}{8}$ miles in 3 hours, how many miles can you travel in 8 hours?

31. If 5 men can cut 6$\frac{7}{8}$ cords of wood in a day, how many cords can 7 men cut in the same time?

32. If A can do a piece of work in 7 days, and B in 5 days, what part of it can both do in a day by working together?

33. If A can build a wall in 3 days, which B can build in 4 days, in what time can they build it by working together?

WRITTEN EXERCISES.

34. Change $\frac{2157}{3741}$ to smallest terms.

35. Find the product of the common prime factors of 1728 and 2880.

36. What number contains each of the prime factors of 243, 972, 576, but no others?

37. Show that $\frac{5+6}{8+7}$ is greater than $\frac{5}{8}$ and less than $\frac{6}{7}$.

38. Find the sum of $\frac{38}{60}$, $\frac{3}{4}$, and $\frac{17}{100}$.

39. Find the sum of $\frac{3}{4}$, $1\frac{1}{8}$, $\frac{17}{5}$, and $13\frac{7}{10}$.

40. What number added to $\frac{117}{237}$ will make 16 ?

41. If $9\frac{7}{8}$ yards cut from a roll of cloth leaves $24\frac{1}{8}$ yards, what is the length of the roll ?

42. What is the value of $2\frac{5}{6}$ times $\frac{7}{8}$ of $\frac{9}{10}$?

43. Find the number that must be added to $\frac{3}{7}$ of $\frac{5}{9}$ to make $\frac{1}{2}$ of $\frac{6}{9}$.

44. A man has $229\frac{1}{2}$ pounds of honey, which he wishes to pack in boxes containing $8\frac{1}{2}$ pounds each. How many boxes will he require?

45. What fraction must $\frac{31}{37}$ be divided by to make the quotient $3\frac{10}{37}$?

46. Find the least number which, divided by 6, by 8, and by 9, gives in every case a remainder 4.

47. The tide rose $\frac{5}{6}$ of a foot in one hour, $\frac{13}{15}$ the next, and $\frac{5}{8}$ the third hour. How much did it rise in the 3 hours ?

48. Change $\frac{4}{7}$ to an equivalent fraction, having 91 for its denominator.

49. How many cakes would be required for a school of 53 children, of whom 27 are boys, if each girl has $\frac{1}{3}$ of a cake, and each boy $\frac{1}{2}$ as much again as each girl?

50. How many times does $\frac{1}{12} + \frac{1}{4} + \frac{1}{8}$ contain $\frac{1}{3} + \frac{1}{6} + \frac{1}{9}$?

51. What is the product of $\frac{2}{3}$ of $\frac{12}{7}$ multiplied by $\frac{3\frac{1}{2}}{16}$?

52. What fraction of 3 bushels is $\frac{7}{12}$ of $2\frac{3}{4}$ bushels?

53. The difference between $\frac{6}{7}$ and $\frac{7}{8}$ of a number is 10. What is the number?

54. Reduce $\frac{5}{7}$ of $\frac{3}{15}$ of $\frac{4}{16}$ of $8\frac{3}{4} \times \frac{11}{5}$ to a simple fraction in its smallest terms.

55. Sold a horse for \$$105\frac{3}{4}$, which was $\frac{3}{4}$ of its cost. What was its cost?

56. What cost $13\frac{7}{10}$ tons of coal at \$$7\frac{7}{8}$ a ton ?

57. Bought $1\frac{7}{8}$ bushels of corn for \$$3\frac{33}{64}$; what was it a bushel?

58. How many acres of land at $\$17\frac{3}{4}$ an acre can be bought for $\$350\frac{9}{16}$?

59. When $\$2.13$ are paid for $\frac{3}{8}$ of a cord of wood, what cost 1 cord? What cost $10\frac{1}{2}$ cords?

60. A boy, after spending $\frac{2}{5}$ of his money in oranges, finds that $\frac{4}{7}$ of what he has left is 24 cents. How much money had he at first?

61. If 28 men can build an embankment in 42 days, in how many days can $\frac{3}{4}$ as many men build it?

62. How much cambric that is $\frac{3}{4}$ of a yard wide will line $6\frac{3}{4}$ yards of cloth that is $1\frac{1}{4}$ yards wide?

63. What is the value of $(\frac{5}{7} \text{ of } 1\frac{6}{13}) \div \dfrac{2\frac{5}{7}}{3\frac{1}{4}}$?

64. A owns $\frac{5}{16}$ of a store, B $\frac{7}{12}$, and C the remainder. The profits were $\$960$. What was the share of each?

65. When $\$1\frac{1}{8}$ will buy $\frac{7}{8}$ of a gallon of oil, what part of a gallon will $\$\frac{9}{11}$ buy?

66. If $\frac{2}{3}$ of a store is worth $\$3300$, what is the value of $\frac{3}{10}$ of the same?

67. How many pounds of tea can be bought for $\$23\frac{5}{8}$, if $7\frac{1}{4}$ pounds cost $\$5\frac{7}{16}$?

68. If 4 bushels of corn cost $\$3\frac{1}{16}$, how much will 7 bushels cost?

69. A will do $\frac{2}{3}$ as much as B. The board of each is worth $\$\frac{3}{4}$ a day. If B is paid $\$1\frac{1}{2}$ a day and board, what should be paid to A in addition to his board?

70. A saves $\frac{1}{4}$ of his income, and B, having the same income, spends $1\frac{1}{2}$ times as much as A, and finds himself $\$62\frac{1}{2}$ in debt at the end of the year. What was the income of each?

71. A cistern of 960 gallons is emptied by two pipes, A and B, in 5 and 7 minutes, respectively. How much water will pass through each if both are opened together?

72. What is the value of $\dfrac{1}{6 + 7\frac{1}{2}} + \frac{3}{5} \text{ of } 1\frac{2}{3} + (1\frac{2}{7} \div 1\frac{1}{4}) + \frac{9}{35}$?

DECIMAL FRACTIONS.

137. A **Decimal Fraction** is a fraction whose unit is divided into tenths, hundredths, thousandths, etc.

138. A **Decimal** is a decimal fraction expressed without its denominator by means of the decimal point (Art. 20). Thus,

0.7, 0.05, and 0.168 are decimals.

139. A **Mixed Decimal** is an integer and a decimal. Thus,

17.06 and 5.305 are mixed decimals.

140. The method of writing decimals, in continuation of the notation of integers (Art. 25), is shown in the following

TABLE.

	Integer.						*Decimal.*									
PLACE-NAMES.	Hundred-thousands	Ten-thousands	*Thousands*	Hundreds	Tens	*Ones*	Decimal Point	Tenths	Hundredths	*Thousandths*	Ten-thousandths	Hundred-thousandths	*Millionths*	Ten-millionths	Hundred-millionths	*Billionths*
FIGURES.	7	9	3,	1	5	4	.	6	3	8,	4	7	8,	1	3	9
GROUP-NAMES.	Thousands.			Ones.				Thousandths.			Millionths.			Billionths.		

The number in the table is, seven hundred ninety-three thousand one hundred fifty-four, and six hundred thirty-eight thousand four hundred seventy-eight million one hundred thirty-nine billionths.

NOTE 1. — The place-names above and below ones correspond to each other, except that the decimals have the termination *ths*.

NOTE 2. — A decimal with a common fraction is called a *complex decimal;* as, $0.16\frac{4}{5}$, read sixteen and four-fifths hundredths.

141. The **Denominator** of a decimal is 1, with as many ciphers annexed as there are places in the decimal. Thus,

0.6 is $\frac{6}{10}$, 0.15 is $\frac{15}{100}$, 0.011 is $\frac{11}{1000}$, and 0.06$\frac{3}{7}$ is $\frac{6\frac{3}{7}}{100}$.

WRITTEN EXERCISES.

Read the following:

1. 0.35
2. 0.035
3. 0.7077
4. 0.0095
5. 0.0007
6. 0.4003
7. 5.55555
8. 180.06$\frac{1}{2}$
9. 4.7307$\frac{2}{3}$
10. 473.7$\frac{3}{4}$
11. 0.47378
12. 0.000931
13. 667.6671
14. 6.676671
15. 66766.71
16. 6676.671
17. 14.000014
18. 1.0000001
19. 0.0000077
20. 7000000.7
21. 96.00300315

Write decimally:

22. Ten, and eleven hundredths.

23. Sixty-seven, and seven tenths.

24. Six hundred seven, and nine hundred six thousandths.

25. One thousand five hundred forty-one, and one hundred seventy-eight ten-thousandths.

26. Sixteen, and one hundred twenty-two hundred-thousandths.

27. Thirty-seven thousand four hundred eighty-eight hundred-thousandths.

28. Nine hundred fifty-eight thousand four hundred thirty-three millionths.

29. Four thousand four hundred four, and fifty-nine and three-fourths hundredths.

30. Sixty-eight, and three hundred three thousand eight hundred seven and two-sevenths millionths.

31. Forty million three hundred three, and forty-three thousandths.

32. Forty-seven, and nine million nine hundred ninety-nine thousand nine hundred ninety-nine ten-millionths.

Express in decimal form :

33. $\frac{93}{100}$.	**38.** $7\frac{7\frac{1}{4}}{100}$.	**43.** $1000\frac{101}{10000}$.
34. $\frac{8}{1000}$.	**39.** $\frac{10345}{10000000}$.	**44.** $879\frac{19}{10000000}$.
35. $\frac{13141}{100000}$.	**40.** $9060501\frac{3\frac{1}{4}}{10}$.	**45.** $6958\frac{103}{10000}$.
36. $68\frac{87}{100000}$.	**41.** $\frac{56}{10000000}$.	**46.** $\frac{9}{10000000}$.
37. $945\frac{125}{1000}$.	**42.** $4167\frac{36}{100}$.	**47.** $\frac{287}{1000000000}$.

REDUCTION.

A Decimal to Smaller or Larger Denominator.

ORAL EXERCISES.

48. How many tenths in 1? How many hundredths? How many thousandths?

49. How many hundredths in 2 tenths? In 3 tenths? In 9 tenths?

50. In 0.01 how many thousandths? In 0.1 how many hundredths? How many thousandths?

51. How many tenths in 0.50? In 0.70? In 0.900? In 2.50?

52. How many hundredths in 0.600? In 0.850? In 6.500?

142. Principle.

Annexing a cipher to a decimal, or removing a cipher from the right of a decimal, does not change its value. (See Arts. 111, 114.)

WRITTEN EXERCISES.

53. Write 0.16, 3, and 1.014 as decimals having the least common denominator.

0.16 = 0.160
3. = 3.000
1.014

Solution. — The smallest order of decimals in the given numbers is thousandths.

0.16 expressed as thousandths is 0.160, and 3. expressed as thousandths is 3.000.

The required decimals are 0.160, 3.000, and 1.014.

54. Reduce 71.500 to tenths.

55. Change 19, 43.6, 0.64, and 53 to thousandths.

56. Express 15.600, 4.7, and 13 as hundredths.

57. Reduce 18.0156, 401.6, and 176.4700 to decimals having the least common denominator.

A Decimal to a Common Fraction.

58. How many halves in .5? In .50?

59. How many fourths in .25? In .75? In .750?

60. How many fifths in 0.4? In 0.6? In 0.8?

61. How many twentieths in 0.05? In 0.15? In 0.45?

WRITTEN EXERCISES.

62. Change .355 to a common fraction in its smallest terms.

$.355 = \frac{355}{1000} = \frac{71}{200}$

Solution.—As .355 is 355 thousandths, it may be written $\frac{355}{1000}$, which, changed to its smallest terms, is $\frac{71}{200}$.

63. Change .225 to a common fraction in its smallest terms.

64. Change .875 to a common fraction in its smallest terms.

143. To change a decimal to a common fraction:

Rule.

Omit the decimal point, write the denominator, and change the fraction to its smallest terms.

Reduce to common fractions in smallest terms:

65. .025
66. .561
67. .054
68. $.66\frac{2}{3}$
69. .625
70. .375
71. .368
72. 11.75
73. $4.43\frac{3}{4}$
74. .09375
75. $9.37\frac{1}{2}$
76. 115.875
77. .01375
78. 200.96
79. .015625

A Common Fraction to a Decimal.

80. How many tenths in 1? In $\frac{1}{2}$? In $\frac{1}{5}$? In $\frac{2}{5}$?

81. How many hundredths in 1? In $\frac{1}{4}$? In $\frac{3}{4}$? In $\frac{2}{5}$?

82. How many thousandths in 1? In $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{3}{4}$?

WRITTEN EXERCISES.

83. Change $\frac{5}{8}$ to a decimal.

$$8\overline{)\underline{5.000}}$$
$$.625$$

Solution. — $\frac{5}{8}$ is equal to $\frac{1}{8}$ of 5. 5 equals 50 tenths, or 5.0 ; $\frac{1}{8}$ of 50 tenths is 6 tenths, with 2 tenths, equal to 20 hundredths, remaining. $\frac{1}{8}$ of 20 hundredths is 2 hundredths, with 4 hundredths, equal to 40 thousandths, remaining. $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Ans. .625.

84. Change $\frac{3}{5}$ to a decimal.

85. Change $\frac{8}{11}$ to a decimal of four places.

$$11\overline{)\underline{8.0000}}$$
$$.7272\tfrac{8}{11}$$

Solution. — $\frac{8}{11}$ is equal to $\frac{1}{11}$ of 8. As there are to be four places in the decimal, we annex four decimal places of ciphers to the numerator, reducing it to 8.0000. $\frac{1}{11}$ of 8.0000 is $.7272\frac{8}{11}$.

86. Change $\frac{5}{12}$ to a complex decimal of three places.

87. Change $\frac{4}{11}$ to a complex decimal of four places.

144. To reduce a common fraction to a decimal:

Rule.

Annex decimal ciphers to the numerator, divide by the denominator, and point off as many decimal orders in the quotient as there are ciphers annexed.

Note 1. — When the division does not terminate, or has been carried as far as is desirable, the remainder may be expressed as a common fraction and made a part of the result.

Note 2. — When an approximate result is sufficient, a fraction of $\frac{1}{2}$ or more than $\frac{1}{2}$ in a result may be rejected, and the last figure of the decimal be made to express 1 more. Thus,

$.7272\frac{8}{11}$ approximately expressed to the *nearest ten-thousandth* is .7273.

Note 3. — When the fraction as a part of a decimal is unimportant, it may be omitted, and the incompleteness of the result simply marked by +. Thus,

.7272+ may be written instead of $.7272\frac{8}{11}$.

Reduce to decimals:

88. $\frac{11}{40}$.	**91.** $\frac{14}{20}$.	**94.** $4\frac{1}{64}$.
89. $\frac{7}{16}$.	**92.** $\frac{8}{25}$.	**95.** $2\frac{3}{32}$.
90. $\frac{1}{32}$.	**93.** $\frac{612}{125}$.	**96.** $5\frac{1}{128}$.

97. Reduce $\frac{1}{13}$ to a complex decimal of four places.

98. Reduce $\frac{7}{12}$ to the nearest ten-thousandth.

99. Reduce $\frac{18}{19}$ to an approximate decimal of four places.

100. Reduce $\frac{7}{75}$ to a complex decimal of four places.

101. Reduce $\frac{5}{37}$ to the nearest millionth.

102. Change to the nearest thousandth $\frac{2}{3}$, $\frac{21}{38}$, and $\frac{3}{250}$, and find the sum of the results.

103. Reduce to the nearest ten-thousandth, and add, $15\frac{5}{6}$, $20\frac{5}{8}$, $12\frac{4}{5}$, and 2.68.

104. Reduce $3\frac{41}{333}$ to the nearest millionth, and subtract the result from 15.057.

145. For rules for Addition and Subtraction of Decimals, see Arts. 38 and 47.

MULTIPLICATION.

ORAL EXERCISES.

105. How much is 3 times 2 tenths? 3 times 0.3?

106. How much is 7 times 1 hundredth? 7 times $\frac{5}{100}$? 9 times 0.06?

107. How much is $\frac{1}{10}$ of 1? Of 3? $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of .3?

108. $\frac{1}{10} \times \frac{1}{10}$? $\frac{3}{10} \times \frac{2}{100}$? $3 \times .3$? $4 \times .3$? $.04 \times .3$?

109. How many places of decimals in the product when tenths are multiplied by tenths? When hundredths are multiplied by tenths?

WRITTEN EXERCISES.

110. Multiply 31.5 by .07.

$$\begin{array}{r} 31.5 \\ .07 \\ \hline 2.205 \end{array}$$

Solution. — As .07 is the same as $\frac{1}{100}$ of 7, 31.5 multiplied by .07 is the same as $\frac{1}{100}$ of 7 times 31.5. 7 times 31.5 = 220.5, and $\frac{1}{100}$ of 220.5, which is found by removing the decimal point two places to the left (Art. 75), is 2.205.

111. What is the product of 671 by .305?

112. What is the product of 18.72 by 7.1?

146. Rule for Multiplication of Decimals.

Multiply as integers, and point off as many figures for decimals in the product as there are decimal places in both factors.

NOTE. — If there are not figures enough in the product, supply the deficiency by prefixing ciphers.

	113.	114.
Multiply	.126	3.18
By	.016	.00029
	756	2862
	126	636
Product	.002016	.0009222

115. Multiply 5.64 by 45.

116. Multiply 96.5 by 100.

117. Multiply 6.34 by .0023.

118. Multiply 42.2 by 2.004.

119. Multiply 1671 by .013.

120. Multiply .563 by 47.

121. Multiply 19634 by .0073.

122. Multiply .0703 by .0055.

123. Multiply .0505 by .001.

124. Multiply .0076 by .017.

125. What is the product of one million by one millionth ?

126. What is the cost of 35.75 yards of cloth, $ 4.50 a yard ?

127. How much must be paid for 13.375 cords of wood, at $ 4.62 a cord ?

128. What is the product of one hundred one thousandths by ten thousand one hundred one hundred-thousandths ?

DIVISION.

ORAL EXERCISES.

129. How many times 2 tenths in 8 tenths ? 3 tenths in 9 tenths ?

130. How much is $\frac{1}{2}$ of 8 tenths ? $\frac{1}{3}$ of 9 tenths ?

131. How much is $\frac{1}{3}$ of 15 hundredths ? $\frac{1}{4}$ of .24 ? $\frac{1}{5}$ of .45 ?

132. Divide .8 by .2 ; .63 by 7 ; .63 by .07.

133. The product of two factors is .72, and one of the factors is 9. What is the other factor ?

134. The product of two factors is .72, and one of the factors is .9. What is the other factor?

135. In .72 divided by 9, how many places of decimals are there in the quotient? In .72 divided by .9?

WRITTEN EXERCISES.

136. Divide 46.48 by .4.

‸4.) 46‸4.8
116.2

Solution. — Both dividend and divisor may be multiplied by 10 without changing the quotient (Art. 72). This is done by moving the point one place to the right (Art. 75). Dividing as in integers, and placing the quotient point under the new dividend point, we have the quotient 116.2.

137. Divide .7935 by .23.

3.45
‸23.) ‸79.35
69
103
92
115
115

Solution. — To make the divisor an integer, we multiply both divisor and dividend by 100, by moving the point in each two places to the right. Dividing as in integers, and placing the quotient point over the new dividend point, we have 3.45 as the quotient required.

138. Divide 1.264 by 4.

139. Divide .00115 by .05.

147. Rule for Division of Decimals.

If the divisor is an integer, divide as in integers, and point off as many decimal places in the quotient as there are such places in the dividend.

If the divisor is a decimal, make it an integer by moving the decimal point a sufficient number of places to the right; move the decimal point in the dividend as many places to the right, annexing ciphers, if necessary, and then divide.

NOTE 1. — When the division does not terminate, or has been carried as far as is desirable, the remainder may be expressed as a common fraction and made a part of the result; or, the fraction may be rejected and an approximation expressed to the nearest decimal; or, the sign + may be used to mark the incompleteness of the division (Notes, Art. 144).

NOTE 2. — When the divisor is an integer, the decimal point in the quotient must invariably be placed directly under or over that of the dividend. If it is desirable to divide without changing decimal points, it may be done, care being taken to give the quotient as many decimal places as those in the dividend exceed those in the divisor.

140. Divide 37.4 by 4.5 to three decimal places.

```
       8.311+
4‸5.) 37‸4.000
      360
      ---
       140
       135
       ---
         50
         45
         --
          50
          45
          --
           5
```

141. Divide 6.0512 by 3.7 to the nearest thousandth.

```
      1.635
3‸7.) 6‸0.512
      37
      --
      235
      222
      ---
       131
       111
       ---
        202
        185
        ---
         17
```

142. Divide 783.5 by 6.25.

143. Divide .0189 by .025.

144. Divide .01001 by .001.

145. Divide 1.0665 by .00135.

146. Divide .08748 by 1.08.

147. Divide .817 by .9147.

148. Divide 1.365 by 1000.

149. Divide 72 by .018.

150. Divide 555 by .0037.

151. Divide .0016016 by .00143.

152. Divide ninety-five and three tenths by two hundred sixty-four thousandths, to the nearest thousandth.

153. Divide one hundred eight and twenty-nine thousandths by seven and two tenths, to three decimal places.

154. If a man can walk three and seventy-five hundredths miles in an hour, in how many hours can he walk seven hundred eighty-seven and five-tenths miles?

MISCELLANEOUS EXERCISES.

155. Write four hundred three ten-thousandths; four hundred, and three ten-thousandths.

156. Write and read: 0.00567; 2.13007; 1.00157.

157. Express as decimals: $\frac{87}{10000}$; $\frac{162}{100000}$; $\frac{9}{1000000}$.

158. Find the mixed decimal equal to $4\frac{3}{8} + \frac{19}{20}$.

159. From 45 subtract 36.00073.

160. Find the least fraction which is to be added to the sum of 25.7, 8.389, and 23.056 to make the result an integer.

161. Find the difference between 6346 and .6346; 4.2 and .0042; .0000005 and .00005.

162. Simplify, and express the result in a decimal form, $3\frac{5}{8} + 17\frac{1}{4} + 476 + 3.125$.

163. Change .03125 to a common fraction in its smallest terms.

164. Change 4, $2\frac{3}{4}$, 17, .136, and .0408 to equivalent decimals having a common denominator, and find their sum.

165. If the year is considered 365.25 days, instead of 365.242264, how great will be the error in 1880 years?

166. The dividend is 7423.973, the quotient 12.130, and the remainder .413. What is the divisor?

167. Divide $7498.70 among A, B, and C, so that A shall have just $749.83 more than each of the others.

168. What is the value of $20004 + (20.104 \times 5.07) - (6.44 \div .005)$?

169. Add $2\frac{2}{3}$, $4\frac{21}{38}$, and 51.652, expressing the sum to the nearest thousandth.

170. A man willed his property to his three sons, — to the youngest he gave $968.49; to the second, 3.4 times as much as to the youngest; and to the eldest, 3.7 times as much as to the second. Required the value of his property.

171. If a train of cars moves at the rate of 30.25 miles an hour, find to the nearest ten-thousandth how many hours it will take to go 150.75 miles.

172. Reduce to its simplest form $\frac{1}{.01} - \frac{.0009}{.4 \times .0005 + .002 \times .0125}$.

173. Dividing a certain sum by .027, the quotient is 6116, and the remainder .003. What is the dividend?

174. Bought wheat at $0.94 per bushel, to the amount of $59.22, and sold it for $70.56. What was made on a bushel?

175. What is the cost of 60.5 tons of coal when 0.9 of a ton costs $6.66?

176. Find .3 of $.064\frac{2}{3}$ of $1728.

177. Divide (12.8 ÷ .16) by (.128 ÷ .0016).

178. What part of .84 is .012?

179. If .6875 of a gallon of wine costs $3.75, what will 40.25 gallons cost?

180. If $\frac{1}{8}$ of an acre costs $$15\frac{7}{8}$, what will 46.78 acres cost?

181. If one pound costs $$0.18\frac{5}{8}$, how many pounds can be bought for $14.40?

182. If 35.84 cubic feet of water weigh a ton, what will be the weight of 2458.6 cubic feet?

183. If 48 is .08 of some number, what is .7 of it?

184. The product of three factors is 5.76; one of them is .024, another is .06; find the third.

185. How many times is the difference of $(\frac{5}{6} \div 0.33\frac{1}{3})$ and $(0.87\frac{1}{2} \div \frac{1}{8})$ contained in their product?

QUESTIONS.

137. What is a decimal fraction? 138. What is a decimal? 139. A mixed decimal? 141. What is the denominator of a decimal?

142. What is the principle in relation to annexing or cutting off a cipher from the right of a decimal?

143. How is a decimal reduced to a common fraction? 144. How is a common fraction reduced to a decimal?

146. How do you multiply in decimals? 147. How do you divide in decimals?

UNITED STATES MONEY.

STATES OF AMERICA In God we trust UNITED
ONE DOLLAR
1 DOLLAR 1864
UNITED STATES OF AMERICA IN GOD WE TRUST TWENTY D.
UNITED STATES OF AMERICA FIVE D.
UNITED STATES OF AMERICA HALF DOL.
UNITED STATES OF AMERICA 2½ D.
3 DOLLARS 1870
UNITED STATES OF AMERICA IN GOD WE TRUST TEN D.
UNITED STATES IN GOD WE TRUST QUAR. DOL.
UNITED STATES OF AMERICA 5 CENTS
ONE DIME
ONE CENT

148. TABLE.

10 mills (m.) are 1 cent, c. or ¢.
10 cents " 1 dime, d.
10 dimes " 1 dollar, $.
10 dollars " 1 eagle, e.

149. **Coins** are pieces of metal stamped by authority of the government to circulate as money.

150. The principal **Coins** of the United States are:— BRONZE,—the cent; NICKEL,—the five-cent; SILVER,—the dime, quarter-dollar, half-dollar, and dollar; GOLD,—the dollar, quarter-eagle, three-dollar, half-eagle, eagle, and double-eagle.

151. In ordinary business transactions, eagles, dimes, and mills are rarely mentioned, eagles being expressed as dollars, dimes as cents, and mills as a fraction of a cent. Thus,

3 eagles, 2 dollars, 5 dimes, 8 cents, 5 mills are written, $ 32.58½.

152. To change *cents* to *mills, multiply* by 10.
To change *dollars* to *cents, multiply* by 100.
To change *dollars* to *mills, multiply* by 1000.
Thus,

45 cents = 450 mills; $ 5 = 500 cents, or 5000 mills.

153. To change *mills* to *cents, divide* by 10.
To change *cents* to *dollars, divide* by 100.
To change *mills* to *dollars, divide* by 1000.
Thus,

850 mills = $ 0.85; 365 cents = $ 3.65; 47000 mills = $ 47.

154. The dollar being the unit, of which cents are hundredths and mills are thousandths, it follows that

All the rules for the processes in decimals are applicable to processes in United States money.

ORAL EXERCISES.

1. How many mills in 7 cents? In 7 cents 4 mills?
2. How many cents in 70 mills? In 75 mills?
3. Change $ 5 to cents.
4. Change $ 5 to mills.
5. Change $ 3.50 to cents.
6. Change $ 6.37 to mills.
7. Change 500 cents to dollars.
8. Change 5000 mills to dollars.
9. Change 350 cents to dollars.
10. Change 6370 mills to dollars.
11. In changing cents to dollars, how many places to the left is the decimal point moved?

12. In changing mills to dollars, how many places to the left is the decimal point moved?

13. Paid for a bag of flour $ 1.25, for coffee 40 cents, and for soap 20 cents. How much was paid for the whole?

14. A boy earned one week $ 3.50, and the next 75 cents. How much did he earn in all?

15. Jane has 60 cents, Alice 15 cents, and Henry $ 1.20. How much have they all?

16. Bought an arithmetic for $ 1.25, and gave a two-dollar bill in payment. What change should I receive?

17. Bought a plow for $ 12.50, and sold it for $ 15. How much did I make?

18. How much is the profit on flour bought for $ 7.50, and sold for $ 9.25?

19. If a man can earn $ 1.50 in one day, how much can he earn in 3 days?

20. How much must be paid for 5 pairs of shoes at $ 2.25 a pair?

21. At 25 cents a yard, how many yards of cloth can be bought for $ 1.75?

22. At 50 cents a bushel, how many bushels of apples can be bought for $ 4.75?

WRITTEN EXERCISES.

23. Bought a coat for $ 13.50, a vest for $ 2.63, a hat for $ 5, and a pair of boots for $ 6.13. What was the amount?

24. A man paid for tea 63 cents, for butter 80 cents, for flour $ 7.50, and for other articles $ 32. What did he pay for the whole?

25. Mr. Avery bought a farm for $ 6500, and paid down $ 1356.85. How much more had he to pay?

26. How much will be received for 56 pounds of crackers at 14 cents a pound, and 128 loaves of bread at 9 cents a loaf?

27. How many yards of cloth at 19 cents a yard can be bought for $ 47.50?

28. A man bought 491 bushels of corn at 81 cents a bushel. He used 29 bushels, and sold the rest at 95 cents a bushel. How much did he make?

29. Bought a house and farm of 36 acres for $ 7975. The house is worth $ 4560. What is the value of the land an acre, to the nearest cent?

30. A farmer sold 187 acres of land at $ 37.50, and 131 acres at $ 63 an acre. At what rate an acre, to the nearest cent, did he sell the whole?

31. At 67 cents a yard, how many yards of cloth can be bought for $ 29.70?

32. At 9 cents a pound, how many pounds of beef can be bought for $ 68.40?

33. When 7 barrels of onions are worth $ 26.25, what are 43.50 barrels worth?

34. Bought land for $ 1040.98, and sold 35.45 acres at $ 20.40 an acre; the rest was worth to me $ 20.50 an acre. How many acres did I buy?

ALIQUOT PARTS.

155. Aliquot Parts of a number are such parts of the number as will exactly divide it. Thus,

2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10.

156. Computations are often abridged by use of the following

Aliquot Parts of a Dollar.

50 cents = $\frac{1}{2}$ of $ 1.
$33\frac{1}{3}$ cents = $\frac{1}{3}$ of $ 1.
25 cents = $\frac{1}{4}$ of $ 1.
20 cents = $\frac{1}{5}$ of $ 1.
$16\frac{2}{3}$ cents = $\frac{1}{6}$ of $ 1.

$12\frac{1}{2}$ cents = $\frac{1}{8}$ of $ 1.
10 cents = $\frac{1}{10}$ of $ 1.
$8\frac{1}{3}$ cents = $\frac{1}{12}$ of $ 1.
$6\frac{1}{4}$ cents = $\frac{1}{16}$ of $ 1.
5 cents = $\frac{1}{20}$ of $ 1.

ORAL EXERCISES.

35. What will 42 yards of cloth cost at $16\frac{2}{3}$ cents a yard?

Solution. — $16\frac{2}{3}$ cents is $\frac{1}{6}$ of $ 1. Since one yard costs $ $\frac{1}{6}$, 42 yards will cost 42 times $ $\frac{1}{6}$, or $ $\frac{42}{6}$, or $ 7.

36. At $12\frac{1}{2}$ cents a pound, what will 96 pounds of sugar cost?

37. At 25 cents a box, how much will 60 boxes of strawberries cost?

38. At $12\frac{1}{2}$ cents a pound, how many pounds of sugar can be bought for $ 12?

Solution. — At $12\frac{1}{2}$ cents a pound, $1 will buy 8 pounds, and $12 will buy 12 times 8 pounds, or 96 pounds.

39. At 25 cents a box, how many boxes of strawberries can be bought for $15?

40. At $33\frac{1}{3}$ cents a yard, how many yards of cloth can be bought for $ $9.33\frac{1}{3}$?

41. At 50 cents a gallon, how many gallons of molasses can be bought for $ 11.50?

42. At 20 cents a dozen, how many dozen eggs can be bought for $10.80?

WRITTEN EXERCISES.

43. What will 1832 pairs of shoes cost at $1.75 a pair?

$ 1.75 = $ 1 + $ $\frac{1}{2}$ + $ $\frac{1}{4}$.
$ 1832 = cost at $ 1.
916 = " .50
458 = " .25
$ 3206 = " $ 1.75

Solution. — 1832 pairs at $1 will cost $1832; at $ $\frac{1}{2}$, will cost $\frac{1}{2}$ of $ 1832, or $ 916; and at $ $\frac{1}{4}$, will cost $\frac{1}{2}$ of $ 916, or $ 458. $ 1832 + $ 916 + $ 458 = $ 3206.

44. What will 144 bushels of wheat cost at $ $1.37\frac{1}{2}$ a bushel?

45. What will 84 yards of cloth cost at $66\frac{2}{3}$ cents a yard?

46. What will 60 pairs of boots cost at $ 3.87½ a pair?

47. At $ 5.12½ a dozen, how many dozen pocket-knives can be bought for $ 502.25?

48. At $ 1.62½ a pair, how many pairs of shoes can be bought for $ 29.25?

49. At $ 0.87½ each, how much must be paid for 56 caps?

50. At $ 2.25 a day, how much will a man earn in 302 days?

51. At 37½ cents a yard, how many yards of cloth can be bought for $ 793 87½?

52. What will 575 melons cost at $ 16 a hundred?

Solution. — 575 is equal to 5.75, or 5¾ hundreds. 5¾ hundreds, at $ 16 a hundred, will cost 5¾ times $ 16, or $ 92.

53. What will 1360 herrings cost at 37½ cents a hundred?

54. What will 650 melons cost at $ 12.50 a hundred?

55. What will 2250 feet of boards cost at $ 20.50 a thousand feet?

Solution. — 2250 is equal to 2.250, or 2¼ thousands; and 2¼ times $ 20.50 = $ 46.12½.

56. How much will 5650 feet of plank cost at $ 44 a thousand feet?

57. What is the cost of 4565 feet of joist at $ 23 a thousand feet; 13640 feet of boards at $ 53.55 per thousand feet; and 15250 shingles at $ 4.50 per thousand?

58. If 18500 bricks are sold for $ 155.40, what is the cost per thousand?

Solution. — 18500 = 18.5 thousands. Since 18.5 thousands cost $ 155.40, 1 thousand will cost $ 155.40 ÷ 18.5, or $ 8.40.

59. If 8375 feet of spruce scantling cost $ 100.50, what is the cost of a thousand feet?

60. If 650 melons cost $ 81.25, what is the cost a hundred?

61. Bought a farm, containing 75.8 acres, at $ 31.50 an acre, and sold it for $ 2274. What was the loss per acre?

ACCOUNTS AND BILLS.

157. An **Account** is a record of articles bought or sold, cash paid or received, or services rendered.

158. A **Debtor** is one who owes a debt, and a **Creditor** is one to whom a debt is owed.

159. A **Bill** is a written statement of an account made out by the creditor for the debtor.

A bill is **Receipted** when its payment is acknowledged in writing by the creditor, or by some one authorized to sign for him.

NOTE. — Dr. is for "debtor," Cr. for "creditor," M. for "thousand," and @ for "at."

WRITTEN EXERCISES.

Copy and find the amounts and balances of the following accounts and bills: —

62.

Henry Otis,

In Account with JAMES DELANY, Dr.

1881.

Jan.	*3*	*To 1 bbl. Molasses 44 gal. @ 55¢*	*$*	
"	"	" *100 lb. Carolina Rice* " *7¢*		
Feb.	*8*	" *50 lb. Mocha Coffee* " *28¢*		
Mar.	*17*	" *1 bbl. Sugar 328 lb.* " *9¢*		
Apr.	*22*	" *1 box Tea 35 lb.* " *45¢*		
			$	

63.

New York, May 16, 1881.

Mr. James Cooper,

Bought of ARTHUR GILMAN & CO.

TERMS CASH.

5 bbl. Clear Pork .... @	*$ 20.50* ...	*$*	
3 bbl. New Mess Beef "	*12.50* ...		
2 bbl. Lard, 421 lb. .... "	*0.11* ...		
364 lb. Smoked Hams "	*0.10* ...		
		$	

Received Payment,

Arthur Gilman & Co.

64.

Providence, June 11, 1881.

Mr. William Baker,

To JOHN O'BRIEN, Dr.

To 6 days' Labor .... @	*$ 2.50*	*$*	
" *6 days' Labor of Boy* "	*1 25*		
" *2 bbl. Rockland Lime* "	*0.95*		
" *3450 Bricks per M.* "	*12.00*		
" *Carting* .... "	*2.75*		
Cr.			
By 1 Month's Rent	*10.00*		
		$	

Received Payment,

John O'Brien,

By Peter O'Brien.

65.

Burlington, July 19, 1881.

Mr. George Allen,

1881. **Bought of ALBERT LANE & SONS.**

May	9	61 yd. American Prints	@	7¢	$
"	11	84 yd. Avon Sheeting	"	8¢	
"	13	94 yd. Samoset Ticking	"	19¢	
June	7	10 doz. Merino Hose	"	$2.24	
"	11	83 yd. Prints	"	6¢	
					$

Paid, July 21, 1881,

Albert Lane & Sons.

66.

New Orleans, July 1, 1882.

Harlan, Jones, & Co.,

1882. **In Account with FELIX REMOND, Dr.**

June	3	15 bbl. N. P. Flour	@	$8.25	$
"	5	10 bbl. St. Louis Flour	"	7.40	
"	9	3 tubs Butter, 121 lb.	"	.30	
"	15	125 lb. Smoked Ham	"	.10	
"	25	43 lb. Cheese...............	"	.11	
		Cr.			
"	9	Merchandise.......		96.50	
"	15	Cash.................		50.00	
		Balance due F. R.			$

Received Payment, July 3, 1882,

Felix Remond.

67. Trenton, Oct. 3, 1881. Andrew Shaw bought of Ingram, Smith, & Co., 25 pairs of Kip Boots at $ 2.50 a pair; 20 pairs Calf Boots at $ 2.75 a pair; 30 pairs of New Brunswick Rubber Boots at $ 3 a pair; 15 pairs of Kip Brogan Shoes at $ 1.25 a pair; and there was charged for carting 25 cents. Make out Mr. Shaw's bill and find its amount.

68. St. Paul, Sept. 15, 1881. Hooper, Blake, & Dudley sold Jacob Van Husen 100 bbl. Best Test Patent Flour at $ 8.25 a barrel; 50 bbl. Wilbur's Extra Flour at $ 6.50 a barrel; and charged for prepaid freight $ 6.25. Make out the bill and receipt it for Hooper, Blake, & Dudley.

69. Make out a bill, June 1, 1881, for Joseph McIntire, for 3 months' services rendered him at $ 28 a month.

70. Chicago, Nov. 1, 1882. William Asbury owed George W. Ogden & Bro. for 50 tons of Franklin Coal at $ 5.25 a ton, bought Oct. 2; 75 tons of Cumberland Coal at $ 4.75 a ton, bought Oct. 10; 25 cords of Pine Wood at $ 4.75 a cord, bought Oct. 16: and George W. Ogden & Brother owed him for cash paid Oct. 10, $ 262.50; and for bill of merchandise rendered Oct. 20, $ 31.65. Make out the account and receipt for the balance in your name for George W. Ogden & Brother.

71. Nov. 10, 1881. Arthur Roberts bought of Jordan, Marsh, & Co 64½ yards of tapestry carpet at 87½ ¢; 27 yards Brussels carpet at $ 1.85; 18¾ yards oilcloth at 37½ ¢; the making and laying cost $ 13.22. What was the amount of the bill?

QUESTIONS.

148. Recite the table of United States money. 150. What are the principal coins of the United States? 154. Why are the rules for the processes in decimals applicable to processes in United States money?

155. What are aliquot parts of a number? 156. What are some of the aliquot parts of a dollar?

157. What is an account? 158. A debtor? A creditor? 159. What is a bill? When is a bill receipted?

WEIGHTS AND MEASURES.

160. 1. How can the distance between two places be found and expressed?

2. How can you determine the space occupied by a floor? The space enclosed by the walls of a room?

3. How can you determine the amount of water in a pitcher? The quantity of grain in a bin?

4. How can you tell how much sugar you have bought?

LENGTH MEASURES.

161. A **Line** is that which has length only, as the distance between two corners of this book.

162. **Linear,** or **Length Measures** are those used in measuring lines or distances.

TABLE.

12 inches (in.) are 1 foot, ft.
3 feet " 1 yard, yd.
5½ yd., or 16½ ft. " 1 rod, rd.
320 rd., or 5280 ft. " 1 mile, mi.

5. How many inches in 4 ft.? In 5½ ft.? In ½ yd.? In ¾ yd.?

6. How many yards in 72 ft.? In 90 in.? In 16½ ft.? In 38 ft.? In ½ rd.?

7. How many rods in ½ mi.? In ¼ mi.? In ⅛ mi.? In 33 yd.?

8. At $2 a foot, what will it cost to build 24 rods of stone wall?

9. Required the distance in yards around a room 13 feet square.

10. How many rods of fence are required to enclose a lot 25 rods wide and twice as long?

11. In 2 yd. 2 ft. 2 in., how many inches?

12. What cost 27 feet of picture-cord at 10 cents a yard?

13. What cost 27 in. of ribbon at 60 cents a yard?

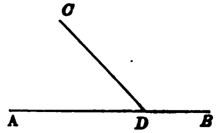
14. How many inches in 0.33⅓ of 2 yards?

SURFACE MEASURES.

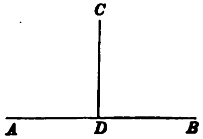
163. A **Surface** is that which has length and breadth only, as this page, or the outside of a block.

A *plane surface* is one that does not change its direction.

164. An **Angle** is the difference in direction of two straight lines which meet at a point, called the *vertex* of the angle; as *A D C* or *C D B*.

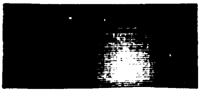


165. Two lines are *perpendicular* to each other when they meet so as to form equal adjacent angles. The angles thus formed are called *Right Angles.*



Thus the lines *A B, C D* are perpendicular to each other, and form the right angles, *A D C, C D B.*

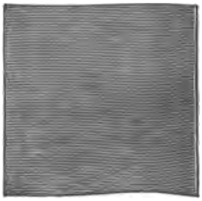
166. A **Rectangle** is a plane surface with four straight sides and four right angles.



A Rectangle.

167. A **Square** is a rectangle with equal sides.

168. **Surface,** or **Square Measures** are those used in measuring surfaces.



An Inch *Square*, or Square Inch.

TABLE.

144	square inches (sq. in.)	are	1 square foot, sq. ft.
9	square feet	"	1 square yard, sq. yd.
30¼	square yards, or 272¼ sq. ft.	"	1 square rod, sq. rd.
160	square rods	"	1 acre, A.
640	acres	"	1 square mile, or section, sq. mi.

An *acre* contains 43,560 square feet. A *rood* (R.) is 40 square rods.

ORAL EXERCISES.

15. How many square inches in ½ sq. ft.? In ¾ sq. ft.? In 0.5 sq. ft.? In $\frac{11}{12}$ sq. ft.?

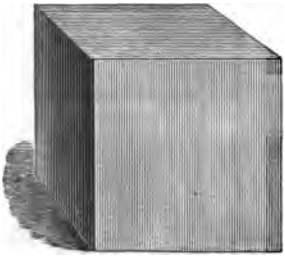
16. How many square feet in 3 sq. yd.? In 5½ sq. yd.? In 200 sq. in.? In 7 sq. yd. 8 sq. ft.?

17. In 47 surface feet how many surface yards? In 80 sq. ft.? In 100 sq. ft.?

18. How many square rods in ¼ A.? In $\frac{7}{10}$ A.? In 0.75 A.?

19. What part of an acre is 120 sq. rd.? 50 sq. rd.?

VOLUME MEASURES.


An Inch *Cube*, or Cubic Inch.

169. A **Solid,** or **Volume,** is that which has length, breadth, and thickness; as a brick or a box.

170. A **Cube** is a solid bounded by six equal squares.

171. **Cubic Measures** are those used in measuring solids or volumes.

TABLE.

1728 cubic inches (cu. in.) are 1 cubic foot, cu. ft.
27 cubic feet " 1 cubic yard, cu. yd.

Also,

16 cubic feet " 1 cord foot, cd. ft.
8 cord feet, or 128 cu. ft. " 1 cord, cd.

ORAL EXERCISES.

20. In 3 cu. yd. how many cubic feet? In 10⅓ cu. yd.? In 81 cu. ft. how many cubic yards? In 108 cu. ft.?

21. In 4 cd. how many cord feet? In 5½ cd.?

22. In 64 cu. ft. how many cord feet?

23. How many cord feet equal a cubic yard?

CAPACITY MEASURES.

172. Liquid Measures are used in measuring liquids.

173. Dry Measures are used in measuring grain, roots, fruit, etc.

TABLES.

Liquid.

4 gills (gi.) are 1 pint, pt.
2 pints " 1 quart, qt.
4 quarts " 1 gallon, gal.

Dry.

2 pints (pt.) are 1 quart, qt.
8 quarts " 1 peck, pk.
4 pecks " 1 bushel, bu.

The *gallon* contains 231 cubic inches; 4 *quarts* dry measure, 268⅘ cubic inches; and the *bushel*, 2150.42 cubic inches.

A *hogshead* as a measure is 63 gallons; and a *barrel*, 31½ gallons.

ORAL EXERCISES.

24. How many gills in 5 pt.? In 9¼ pt.? In 2 qt.? In 1 gal.? In ⅜ qt.?

25. How many quarts in 7 gal.? In 8½ gal.? In 48 pt.? In 100 gi.? In 3 gal. 3 qt.?

26. How many gallons in 41 qt.? In 53 qt.? In 60 pt.? In 120 gi.? In 89 qt.?

27. How many pecks in 48 quarts? How many bushels?

28. Change to larger measures, 100 pt. of milk; 64 qt. of nuts; 84 gi. of oil.

29. What is a six-gallon can of milk worth at 5 cents a quart?

30. I take a pint of milk at night and a quart in the morning. What is my weekly milk-bill at 7 cents a quart?

31. What cost 6¾ bu. of potatoes at $ ¼ a bushel?

32. I burn a pint of kerosene every night. What will a 2 weeks' supply cost me at 20 cents a gallon?

33. Bought nuts for $2 a bushel, and sold them for 10 cents a quart. How much did I gain on 5 bushels?

WEIGHTS.

174. Troy Weights are used in weighing gold, silver, and jewels.

175. Avoirdupois Weights are used in weighing all common articles.

TABLES.

Troy.			Avoirdupois.		
24 grains (gr.)	are	1 pennyweight.	16 ounces (oz.)	are	1 pound, lb.
20 pwt.	"	1 ounce, oz.	100 pounds	"	1 cental, ctl.
12 ounces	"	1 pound, lb.	2000 pounds	"	1 ton, T.

1. The *long ton* of 2240 avoirdupois pounds is used at custom-houses, and in weighing coal at the mines.

2. A pound avoirdupois is equal to 7000 grains, so that 192 ounces avoirdupois are equal to 170 ounces troy, and 144 pounds avoirdupois are equal to 175 pounds troy. A *hundred-weight* (cwt.) is the same as the cental.

ORAL EXERCISES.

34. In avoirdupois weight, 3 lb. are how many ounces? 5½ lb.? 2¼ lb.? 3 lb. 7 oz.? 2 lb. 9 oz.? 3⅝ lb.? .75 lb.?

35. In avoirdupois weight, 56 oz. are how many pounds? 65 oz.? 27 oz.? 2500 lb. are how many tons? 4000 lb.? 7000 lb.?

36. Change to the next larger troy weight, 35 oz.; 75 oz.; 60 pwt.; 84 pwt.; 48 gr.; 30 gr.; 100 oz.

37. Change to the next smaller troy weight, 3 lb.; 2½ lb.; 4 lb. 7 oz.; 6 oz.; 5½ oz.; 2 pwt.; 2¼ pwt.; ¾ lb.; ⅘ oz.; ⅞ pwt.; 0.6 lb.; 0.8 oz.

38. What cost 2 lb. 4 oz. of butter at 32 cents a pound?

39. What can I get for 7 pwt. of old silver at $1.10 an oz.?

40. How much hay at $25 a ton can be bought for $55?

41. If 200 lb. can be packed in a barrel, how many barrels will be needed to pack 2 T. 8 ctl.?

TIME.

176. Time is a measured portion of duration.

TABLE.

60 seconds (sec.)	are	1 minute, min.
60 minutes	"	1 hour, h.
24 hours	"	1 day, d.
365 days	"	1 common year, y.
366 days	"	1 leap year, l. y.

Also,

7 days	are	1 week, wk.
12 calendar months (mo.)	"	1 year, y.
100 years	"	1 century.

1. The *Civil Day* begins and ends at 12 o'clock, midnight.

2. The *Calendar Months*, and the number of days in each, are, —

1. January,	31 days.	7. July,	31 days.
2. February, 28 or 29	"	8. August,	31 "
3. March,	31 "	9. September,	30 "
4. April,	30 "	10. October,	31 "
5. May,	31 "	11. November,	30 "
6. June,	30 "	12. December,	31 "

3. The *Solar Year* is 365 d. 5 h. 48 min. 49.7 sec., or very nearly 365¼ days. The fraction in 4 years amounts to nearly a day. Hence,

When the number of any year is divisible by 4 *and not by* 100, *and when it is divisible by* 400, *the month of February has* 29 *days, and the year is called a leap year.*

ORAL EXERCISES.

42. How many minutes in 2 h.? In 3½ h.? In ¾ h.? In 3 h. 20 min.?

43. How many hours in 210 min.? In 3 d.? In ½ wk.?

44. How many days in 96 h.? In 60 h.?

45. How many minutes from 10.25 A. M. to 11.45 A. M.? From 11.50 A. M to 1.35 P. M.?

46. What part of a minute is 42 sec.? What part of an hour is 48 min.?

47. How many days from Mar. 31 to May 31? From June 7 to July 15? From Oct. 19 to Nov. 12?

48. How old is a child on July 4, 1883, who was born May 31, 1880?

49. If a three weeks' vacation begins June 21, when does it end? If it ends Oct. 9, when does it begin?

50. What day was 25 days before Mar. 4, 1880? Sixty days after?

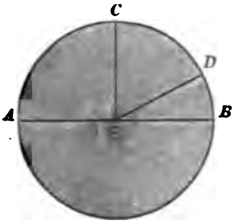
51. How long from Thursday noon to 4.30 P. M. of Friday?

52. How long is the night when the sun sets at 7.35 and rises at 4.30?

53. Name the next six leap years.

54. What century is this? In what century did 1620 come?

ARC AND ANGLE MEASURES.



177. A **Circle** is a plane surface bounded by a curve, all points of which are equally distant from a point within called the *center.*

The *circumference* of a circle is its boundary line.

An *arc* of a circle is any part of the circumference; as *A C* or *D B.*

The *diameter* of a circle is a straight line drawn through its center, and terminated by the circumference; as *A B.*

The *radius* of a circle is half its diameter; as *A E* or *E D.*

178. A **Degree** is $\frac{1}{360}$ of any circumference.

179. An **Angle** whose vertex is the center of a circle is measured by the arc between its sides.

Thus, the arc *A C* measures the angle *A E C.*

TABLE.

60 seconds (″)	are	1 minute, ′.
60 minutes	"	1 degree, °.
360 degrees	"	1 circumference, C.

1. A *degree of the earth's equator* contains 69.16 miles, or about 69⅙ miles.

2. A *sign* is an arc of 30°; as *D B.* A *sextant* is an arc of 60°; as *D C.* A *quadrant* is an arc of 90°; as *A C.* A *semi-circumference* is an arc of 180°; as *A C B.* A *right angle* is an angle of 90°; as *A E C.*

NOTE. — The earth, by turning on its axis once in 24 hours, causes $\frac{1}{24}$ of 360°, or 15°, of longitude to pass under the sun, from east to west, in 1 hour's time.

ORAL EXERCISES.

55. How many degrees in 2 right angles? In 4 right angles?

56. How many minutes in 3½°? In ¾°?

57. In 195′ how many degrees? In 360′?

58. How many signs in a quadrant? Draw an angle of 45°; another of 75°.

59. How many degrees does the minute-hand of a clock move in 3 hours? The hour-hand?

60. How many degrees does the minute-hand of a watch move in 3½ hours?

61. How many degrees in half a meridian circle? How many miles in one such degree?

MISCELLANEOUS MEASURES.

180. Counting.

12 ones	are	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 ones	"	1 score.

181. Paper.

24 sheets	are	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

ORAL EXERCISES.

62. How many pencils in a gross? How many score?

63. What cost 15 doz. pens at $ 1 a gross?

64. How many sheets in a ream? In ½ ream?

65. What will 72 sheets cost at 15 cts. a quire?

66. If I buy paper at 20 cents a quire, and sell for a cent a sheet, how much do I gain on a ream?

67. Name some articles that are sold by the gross? By the dozen?

68. How much will a great gross of tacks cost at 10 cents a paper?

69. Bought 3 gross of pens at $ 1.50 a gross, and sold them for 2 cents each; how much did I gain?

70. How many dozen pint bottles will be needed to hold 5 gal. 3 qt. of currant wine?

71. If I use 1 ream of paper in 10 weeks, how many sheets do I use per day?

72. If you can count four score in a minute, how many can you count between 8.50 and 9.10, A. M.?

73. How many gross of boxes at 2¢ each can be bought for $5.76?

74. If the sun requires 24 hours to make his apparent journey around the earth, how many degrees does he travel in an hour?

75. A druggist buys a cask of liquor containing 30 gallons for $30, and sells it at $0.75 a pint. What does he gain?

QUESTIONS.

161. What is a line? 162. What are linear measures? Recite the table of linear measures.

163. What is a surface? A plane surface? 164. What is an angle? 165. When are two lines perpendicular to each other? 166. What is a rectangle? 167. A square? 168. For what are surface, or square measures used? Give the table.

169. What is a solid, or volume? 170. What is a cube? 171. What are cubic measures? Give the table.

172. For what are liquid measures used? Give the tables. 173. What articles are measured by the bushel?

174. For what are troy weights used? Give the table. 175. For what are avoirdupois weights used? Give the table. What is the long ton? Which is the heavier, a pound of gold or a pound of sugar?

176. What is time? Recite the table. When does the civil day begin and end? Name the calendar months. When has February 29 days?

177. What is a circle? The circumference of a circle? The diameter? 178. What is a degree? 179. How is an angle measured? Give the table. What is a degree at the equator? What is a sign? A right angle?

180. Give the counting table. 181. Give the table for paper.

COMPOUND NUMBERS.

182. 1. How many inches in 3 feet 6 inches?

2. How many feet and inches in 42 inches?

3. How many quarts in 6 gallons 3 quarts?

4. How many gallons and quarts in 27 quarts?

183. A **Denomination** is the name of a unit of measure or weight.

184. A **Denominate Number** is a number composed of units of one or more denominations. Thus,

42 inches, 6 pounds 4 ounces, are denominate numbers.

185. A **Simple Number** is a number of a single kind or denomination. Thus,

2, $3, 5 months, 7 yards, are simple numbers.

186. A **Compound Number** is a number composed of two or more denominations of the same general kind. Thus,

3 ft. 6 in., 6 gal. 3 qt., are compound numbers.

REDUCTION.

To change Denominate Numbers to Smaller Denominations.

ORAL EXERCISES.

5. How many inches in 7 feet? In 5 feet 6 inches?

6. How many gills in 5 gallons? In 2 gal. 3 qt.?

7. How many ounces in 3 lb. troy? In 8 lb. 7 oz.?

8. How many minutes in 4 h. 20 min.? In 5 h. 15 min.?

9. How many quires of paper in 10 reams? How many sheets in 3 quires 16 sheets?

10. How many days in 12 weeks 6 days? In 11 weeks 4 days?

11. How many quarts in 3 pecks? In 2 bu. 1 pk.?

187. **Reduction Descending** is changing denominate numbers to smaller denominations.

WRITTEN EXERCISES.

12. How many inches in 124 rd. 4 yd. 2 ft.?

124 (rd.) 4 yd. 2 ft.
$5\frac{1}{2}$ yd.
62
620
686 (yd.)
3 ft.
2060 (ft.)
12 in.
24720 in.

Solution. — 1 rd. = $5\frac{1}{2}$ yd.; 124 rd. = 124 × $5\frac{1}{2}$ yd., or 682 yd., which, with 4 yd. added, are 686 yd.

1 yd. = 3 ft.; 686 yd. = 686 × 3 ft., or 2058 ft., which, with 2 ft. added, are 2060 ft.

1 ft. = 12 in.; 2060 ft. = 2060 × 12 in., or 24720 in. 124 rd. 4 yd. 2 ft. are 24720 inches.

13. How many pints in 19 bu. 3 pk. 7 qt. 1 pt.?

14. How many cubic feet in 48 cu. yd. 15 cu. ft.?

15. How many pounds in 17 T. 17 cwt. 90 lb.?

188. Rule for Reduction Descending.

Multiply the largest denomination by the number of units it takes of the next smaller denomination to equal one of that larger, and to the product add the given number, if any, of the smaller denomination.

Multiply the sum in like manner, and so proceed until the given number is changed to units of the required denomination.

16. Reduce 12 A. 144 sq. rd. 144 sq. ft. to feet.

17. Reduce 5 cu. yd. 23 cu. ft. 725 cu. in. to cubic inches.

18. Reduce 60 gal. 3 qt. 1 pt. to pints.

19. Reduce 13 bu. 2 pk. 7 qt. 1 pt. to pints.

20. Reduce 47 miles to feet.

21. Reduce 72 lb. 10 oz. 15 pwt. 7 gr. to grains.

22. Reduce 43 T. 13 cwt. 20 lb. to pounds.

23. Reduce 365 d. 5 h. 48 min. 50 sec. to seconds.

24. How many cubic inches in 8 cu. yd. 10 cu. ft. 728 cu. in.?

25. How many seconds in 45° 28′ 54″ ?

26. How many cubic feet in 25 cd.?

27. How many gills in 40 gal. 3 qt. 0 pt. 2 gi.?

28. How many minutes in 67 wk. 6 d. 9 h. 52 min.?

29. How many sheets of paper in 4 bundles 1 ream of paper?

30. What is the value of 1 A. 80 sq. rd. of land at 5 cents a square foot?

31. What will be the cost of grading 3 m. 195 rd. of road at $5.50 a rod?

32. Change $\frac{7}{8}$ of a mile to feet.

Solution. — 1 mile = 5280 ft.; $\frac{7}{8}$ mi. = $\frac{7}{8}$ of 5280 ft., or $\frac{7}{\cancel{8}} \times \overset{660}{\cancel{5280}}$ ft. = 4620 ft.

33. Reduce $\frac{1}{20}$ of a mile to yards.

34. What part of a grain is $\frac{1}{8640}$ of a pound troy?

35. What part of a second is $\frac{1}{89600}$ of a day?

36. What part of a gill is $\frac{7}{288}$ of a gallon?

37. Reduce $\frac{2}{15}$ of a bushel to quarts.

38. Reduce 1.375 gallons to pints.

$$\begin{array}{r l} 1.375 & \text{(gal.)} \\ 4 & \text{qt.} \\ \hline 5.500 & \text{(qt.)} \\ 2 & \text{pt.} \\ \hline 11.000 & \text{pt.} \end{array}$$

Solution. — 1 gal. = 4 qt.; 1.375 gal. = 1.375 × 4 qt., or 5.5 qt.

1 qt. = 2 pt.; 5.5 qt. = 5.5 × 2 pt., or 11 pints.

39. Reduce .024 of a ton to pounds.
40. Reduce .0075 of an acre to square feet.
41. Reduce .3945 of a day to minutes.
42. How many square yards in 1.364 acres?

To change Denominate Numbers to Larger Denominations.

ORAL EXERCISES.

43. How many feet in 84 inches? In 66 inches?
44. How many gallons in 160 gills? In 88 gills?
45. How many pounds troy in 36 ounces? In 103 ounces?
46. How many hours in 260 minutes? In 315 minutes?
47. How many reams in 200 quires of paper? In 76 sheets of paper how many quires?
48. How many weeks in 90 days? In 81 days?
49. How many pecks in 24 quarts? In 72 quarts?

189. Reduction Ascending is changing denominate numbers to larger denominations.

WRITTEN EXERCISES.

50. Change 410 feet to larger denominations.

3 ft.	410 ft.
5½ yd.	136 yd. + 2 ft.
2	2
11 hf.-yd.	272 hf.-yd.
	24 rd. + 8 hf.-yd., or 4 yd.

Solution. — As in 3 feet there is 1 yard, there are as many yards in 410 feet as times 3 feet in 410 feet, which are 136, and 2 feet remainder. As in 5½ yards there is 1 rod, there are as many rods in 136 yards as times 5½ yards in 136 yards, or times 11 half-yards in 272 half-yards, or 12, and 8 half-yards, or 4 yards, remainder.

In 410 feet there are 24 rd. 4 yd. 2 ft.

51. Change 1279 dry pints to larger denominations.
52. Change 1311 cubic feet to larger denominations.
53. Change 35790 pounds to larger denominations.

190. Rule for Reduction Ascending.

Divide the given number by the number of units it takes of that denomination to equal one of the next larger, and reserve the remainder, if any. Divide the quotient in like manner, and so proceed until the required denomination is reached.

The last quotient, and the several remainders, if any, will be the number sought.

NOTE. — Reduction Ascending and Reduction Descending, being reverse processes, are proofs of each other.

Change to larger denominations:

54. 562068 square feet.
55. 273749 cubic inches.
56. 487 liquid pints.
57. 879 dry pints.
58. 248160 feet.
59. 419887 grains.
60. 87320 pounds.
61. 31556930 seconds.
62. 391256 cubic inches.
63. 163734″.
64. 3200 cubic feet.
65. 1306 gills.
66. 684592 minutes.
67. 4320 sheets of paper.

68. At 5 cents a square foot, how many acres and square rods of land can be bought for $ 3267 ?

69. At $ 5.50 a rod, how many miles and rods of road can be graded for $ 6352.50 ?

70. Change 3080 ft. to the fraction of a mile.

$\frac{3080}{5280}$ mi. $= \frac{7}{12}$ mi.

Solution. — 1 mi. = 5280 ft. ; 3080 ft. are $\frac{3080}{5280}$ mi., which, changed to smallest terms, is $\frac{7}{12}$ mi.

71. Reduce 88 yards to a fraction of a mile.

72. What part of a pound troy are 3000 grains ?

73. What part of a day are 12600 seconds ?

74. What part of a gallon are 24 gills ?

75. Reduce $4\frac{4}{15}$ quarts to a fraction of a bushel.

76. Reduce 11 pints to a decimal of a gallon.

$$\begin{array}{r@{\ }l} 2)\ \underline{11.000} & \text{pt.} \\ 4)\ \underline{5.500} & \text{qt.} \\ 1.375 & \text{gal.} \end{array}$$

Solution. — As in 1 quart there are 2 pints, there are in 11 pints $\frac{1}{2}$ as many quarts, or $11 \div 2 = 5.5$ quarts.

As in 1 gallon there are 4 quarts, there are in 5.5 quarts $\frac{1}{4}$ as many gallons, or $5.5 \div 4 = 1.375$ gallons.

77 Reduce 48 pounds to a decimal of a ton.

78. Reduce 326.7 square feet to a decimal of an acre.

79. What decimal of a day is 568.08 minutes ?

80. How many acres in $6601.7\dot{6}$ square yards ?

To change Denominate Fractions to Integers of Smaller Denominations.

ORAL EXERCISES.

81. How many quarts in $\frac{3}{4}$ of a peck ?

82. How many hours in $\frac{7}{8}$ of a day ?

83. How many feet and inches in $\frac{5}{6}$ of a yard ?

Solution. — $\frac{5}{6}$ of a yard $= \frac{5}{6}$ of 3 feet $= 2\frac{1}{2}$ feet ; $\frac{1}{2}$ of a foot $= \frac{1}{2}$ of 12 inches $=$ 6 inches ; $\frac{5}{6}$ of a yard $=$ 2 feet 6 inches.

84. How many days and hours in $\frac{5}{8}$ of a week ?

85. How many ounces and pennyweights in .8 of a pound ?

86. How many quarts and pints in .6 of a gallon ?

191. A **Denominate Fraction** is a fraction of a denominate number.

WRITTEN EXERCISES.

87. Reduce $\frac{7}{9}$ of a bushel to units of smaller denominations.

$$\textit{Solution.}\left\{\begin{array}{l} \frac{7}{9}\text{ bu.} = \frac{7}{9}\text{ of 4 pk.} = 3\frac{1}{9}\text{ pk.} \\ \frac{1}{9}\text{ pk.} = \frac{1}{9}\text{ of 8 qt.} = 0\frac{8}{9}\text{ qt.} \\ \frac{8}{9}\text{ qt.} = \frac{8}{9}\text{ of 2 pt.} = 1\frac{7}{9}\text{ pt.} \end{array}\right.$$

Ans. 3 pk. 0 qt. $1\frac{7}{9}$ pt.

88. Reduce $\frac{2}{9}$ of a mile to units of smaller denominations.

89. What is the value of .83 of a bushel in smaller denominations?

First Solution.

.83 bu. = .83 of 4 pk. = 3.32 pk.
.32 pk. = .32 of 8 qt. = 2.56 qt.
.56 qt. = .56 of 2 pt. = 1.12 pt.

Ans. 3 pk. 2 qt. 1.12 pt.

Second Solution.

$$\begin{array}{r} .83 \text{ bu.} \\ \underline{4} \\ 3.32 \text{ pk.} \\ \underline{8} \\ 2.56 \text{ qt.} \\ \underline{2} \\ 1.12 \text{ pt.} \end{array}$$

90. Reduce .53 rods to units of smaller denominations.

192. Rule for Reduction of Denominate Fractions to Integers.

Change the fraction, as far as possible, to an integer of the denomination next smaller. If there is a fraction in the result, proceed with it in like manner, and so continue as far as required.

Reduce to integers of smaller denominations:

91. $\frac{3}{7}$ of an acre.
92. $\frac{8}{9}$ of a pound troy.
93. $\frac{7}{11}$ of a common year.
94. $\frac{7}{9}$ of a mile.
95. .6725 of a cental.
96. .282 of a ton.
97. .875 of a rod.
98. .761 of a day.

To change Denominate Integers to Fractions of Larger Denominations.

ORAL EXERCISES.

99. What part of a peck is 6 quarts?
100. What part of a day is 21 hours?
101. What fraction of a pound is 14 ounces?
102. What fraction of a yard is 2 feet 6 inches?
103. What fraction of a gallon is 2 quarts 1 pint?

104. What decimal of a bushel is 3 pecks?

105. What decimal of an ounce is 16 pennyweights?

WRITTEN EXERCISES.

106. What fraction of a bushel is 3 pk. 0 qt. $1\frac{7}{9}$ pt.?

$$\textit{Solution.}\left\{\begin{array}{l} 1\frac{7}{9}\text{ pt.} = \frac{16}{9}\text{ pt.} = \frac{16}{9} \div 2\text{, or }\frac{8}{9}\text{ qt.} \\ 0\frac{8}{9}\text{ qt.} = \frac{8}{9} \div 8\text{, or }\frac{1}{9}\text{ pk.} \\ 3\frac{1}{9}\text{ pk.} = \frac{28}{9}\text{ pk.} = \frac{28}{9} \div 4\text{, or }\frac{7}{9}\text{ bu., }\textbf{Ans.} \end{array}\right.$$

107. What part of a mile is 71 rd. 1 ft. 10 in.?

108. What decimal of a bushel is 3 pk. 2 qt. 1.12 pt.?

$$\textit{Solution.}\left\{\begin{array}{l} 1.12\text{ pt.} = 1.12 \div 2 = .56\text{ qt.} \\ 2.56\text{ qt.} = 2.56 \div 8 = .32\text{ pk.} \\ 3.32\text{ pk.} = 3.32 \div 4 = .83\text{ bu.} \end{array}\right\}\text{ Or, }\begin{array}{r} 2)\underline{\ 1.12}\text{ pt.} \\ 8)\underline{\ 2.56}\text{ qt.} \\ 4)\underline{\ 3.32}\text{ pk.} \\ .83\text{ bu.} \end{array}$$

Ans. .83 bu.

109. Reduce 2 yd. 2 ft. 8.94 in. to a decimal of a rod.

193. Rule for Reduction of Integers to a Denominate Fraction.

Change the given number of the smallest denomination to a fraction of the next larger. Write this fraction as a part of the number of that larger denomination. Change, in like manner, the number thus formed, and so proceed as far as required.

110. Reduce 68 sq. rd. 155 sq. ft. to a fraction of an acre.

111. Reduce 10 oz. 13 pwt. 8 gr. to a fraction of a pound.

112. Reduce 232 d. 10 h. 21 min. to a fraction of a year.

113. Reduce 248 rd. 4 yd. 2 ft. 8 in. to a fraction of a mile.

114. Reduce 67 lb. 4 oz. to a decimal of a cental.

115. Reduce 5 cwt. 64 lb. to a decimal of a ton.

116. Reduce 4 yd. 2 ft. 5.25 in. to a decimal of a rod.

117. Reduce 18 h. 15 min. 50.4 sec. to a decimal of a day.

194. When it is required to find the part that one denominate number is of another,

Reduce the numbers to the same denomination, and divide the result denoting the part by that denoting the whole.

118. What part of 2 A. 112 sq. rd. is 144 sq. rd?

119. What part of 3 mi. 120 rd. 4 yd. is 2 mi. 120 rd. 3 yd.?

120. What part of 1 lb. 4 oz. 12 pwt. 12 gr. is 5 oz. 10 pwt.?

121. What decimal of 74 mi. 80 rd. is 9 mi. 90 rd.?

122. What decimal of 7 bu. 1 pk. 5 qt. is 82 bu. 3 pk. 1 qt.?

ADDITION.

195. The operations with compound numbers differ from those with simple numbers only in the fact that compound numbers have an *irregular* instead of a decimal scale.

The principles, however, being the same, no special rules are required for adding, subtracting, multiplying, or dividing compound numbers.

WRITTEN EXERCISES.

123. Add 15 lb. 11 oz. 19 pwt. 22 gr., 71 lb. 10 oz. 13 pwt. 17 gr., and 65 lb. 9 oz. 17 pwt. 14 gr.

15 lb.	11 oz.	19 pwt.	22 gr.
71	10	13	17
65	9	17	14
153 lb.	8 oz.	11 pwt.	5 gr.

Solution. — The sum of the grains is 53 gr. = 2 pwt. 5 gr. We write the 5 gr. beneath, and add the 2 pwt. with the pennyweights.

The sum of the pennyweights is 51 pwt. = 2 oz. 11 pwt. We write the 11 pwt. beneath, and add the 2 oz. with the ounces.

The sum of the ounces is 32 oz. = 2 lb. 8 oz. We write the 8 oz. beneath, and add the 2 lb. with the pounds.

The sum of the pounds is 153 lb., which we write beneath, and have, as the entire sum, 153 lb. 8 oz. 11 pwt. 5 gr.

124.

mi.	rd.	ft.	in.
21	295	11	1
45	279	10	11
35	214	9	9
58	276	16	10
54	70	16	1
216	177	14½	8
Or,			
216	177	15	2

125.

A.	sq. rd.	sq. ft.	sq. in.
65	159	272	143
80	134	260	116
14	110	166	135
66	58	177	131
50	151	69	117
278	135	131¼	66
Or,			
278	135	131	102

In Ex. 124, the ½ ft. in the sum, reduced to units of a lower denomination, gives 6 in., which added with the 8 in. of the sum, and reduced, gives the second form of answer. In Ex. 125, the ¼ sq. ft., in like manner, gives 36 sq. in., which added to the 66 sq. in. of the sum, and reduced, gives the second form of answer.

126. What is the sum of 14 cu. yd. 20 cu. ft. 1463 cu. in., 9 cu. yd. 20 cu. ft. 1463 cu. in., 11 cu. yd. 23 cu. ft. 67 cu. in., and 27 cu. yd. 1305 cu. in.?

127. Find the sum of 18 gal. 3 qt., 60 gal. 3 qt. 1 pt., 61 gal. 3 qt., and 57 gal. 3 qt. 1 pt.

128. Find the sum of 15 d. 23 h. 55 min. 17 sec., 13 d. 15 h. 17 min. 38 sec., 10 d. 23 h. 42 min. 17 sec., 16 d. 16 h. 38 min. 47 sec., and 20 d. 52 min. 57 sec.

129. What is the sum of 28° 56′ 58″, 21° 51′ 37″, and 13° 39′ 57″?

130. Add .761 d. 4¼ h. and 1 h. 8 min. 31 sec.

.761 d. =	18 h.	15 min.	50.4 sec.
4¼ h. =	4	15	0
	1	8	31
	23 h.	39 min.	21.4 sec.

Solution. — .761 d. and 4¼ h. are each reduced to units of smaller denominations, and added with the 1 h. 18 min. 31 sec., give, as the answer, 23 h. 39 min. 21.4 sec.

131. Find the sum of $\frac{7}{11}$ of a ton, $\frac{17}{25}$ of a cental, and 1 T. 2 cwt. 3 lb.

132. A man traveled one day $60\frac{3}{4}$ miles, the second day 50 mi. 120 rd., and the third day $56\frac{5}{8}$ miles. How far did he travel in the three days?

133. I have three lots, — the first contains $\frac{5}{7}$ of an acre, the second $\frac{2}{3}$ of an acre, and the third $\frac{13}{14}$ of an acre. How many acres have I?

SUBTRACTION.

134. Find the difference between 15 lb. 3 oz. 12 pwt., and 9 lb. 1 oz. 17 pwt.

lb.	oz.	pwt.
15 lb.	3 oz.	12 pwt.
9	1	17
6 lb.	1 oz.	15 pwt.

Solution. — As 17 pwt. cannot be taken from 12 pwt., we take 1 oz., = 20 pwt., from the 3 oz., leaving 2 oz., and add to the 12 pwt., which gives 32 pwt.; 17 pwt. taken from 32 pwt. leaves 15 pwt. We write the 15 pwt. beneath.

1 oz. from 2 oz. leaves 1 oz., which we write beneath; 9 lb. from 15 lb. leaves 6 lb. Ans. 6 lb. 1 oz. 15 pwt.

	135.	136.
From	73 bu. 2 p. 5 qt.	17 mi. 311 rd. 1 yd. 1 ft. 3 in.
Take	59 3 7	3 79 1 2 7

137. From 116 A. 53 sq. rd. 100 ft. 113 in. take 87 A. 137 sq. rd. 100 sq. ft. 113 sq. in.

138. The longitude of Boston is 71° 4′ 9″ W. and that of Chicago 87° 35′ W. What is the difference in the longitude of the two places?

139. From $\frac{11}{18}$ of a pound troy take 2 oz. 19.2 gr.

140. From a hogshead of molasses $\frac{4}{5}$ had leaked out. How much remained?

141. From .367 of a year take .761 of a day.

142. The distance between two places is .7895 of a mile. How much is that more than $\frac{3}{8}$ of a mile?

To find the Time between two Dates.

143. What is the time from May 16, 1819, to April 9, 1881?

Solution.

May 16, 1819, to May 16, 1880, = 61 y.
May 16, 1880, to March 16, 1881, = 10 mo.
April 16, 1881, to April 9, 1881, = 24 d.
May 16, 1819, to April 9, 1881, = 61 y. 11 mo. 24d.

196. Rule.

Find first the number of entire years, then the number of entire calendar months remaining, and then the remaining days.

144. What is the time between Oct. 16, 1876, and Aug. 9, 1882?

145. A note was given Nov. 15, 1879, and paid July 5, 1881. How long did it run?

146. How long from the surrender of Cornwallis, Oct. 19, 1781, to the battle of New Orleans, Jan. 8, 1815?

147. What time elapsed from the declaration of American independence, July 4, 1776, to the emancipation proclamation, Jan. 1, 1863?

148. How many days are there from Dec. 19, 1879, to Mar. 16, 1880?

$$12 + 31 + 29 + 16 = 88$$

Solution. — There are 12 days remaining in December, 31 in January, 29 in February, and 16 in March, or, in all, 88 days.

149. A note dated April 9 is to be paid June 8, 1881. How long has it to run?

150. Find the exact time, in days, between May 25, 1880, 10 P. M., and March 4, 1881, 9 A. M.

MULTIPLICATION.

151. Multiply 11 bu. 3 pk. 2 qt. by 7.

$$\begin{array}{r} 11\text{ bu. }3\text{ pk. }2\text{ qt.} \\ 7 \\ \hline 82\text{ bu. }2\text{ pk. }6\text{ qt.} \end{array}$$

Solution. — 7 times 2 qt. are 14 qt. = 1 pk. and 6 qt. We write the 6 qt. under the quarts, and reserve the 1 pk. to add to the 7 times 3 pk.

7 times 3 pk. = 21 pk., which plus the 1 pk. reserved = 22 pk. = 5 bu. and 2 pk. We write the 2 pk. under the pecks, and reserve the 5 bu. to add to the 7 times 11 bu.

7 times 11 bu. = 77 bu., which plus the 5 bu. reserved = 82 bu. Ans. 82 bu. 2 pk. 6 qt.

152. Multiply 17 wk. 4 d. 23 h. 47 min. by 8.

153. Multiply 3 mi. 40 rd. 4 yd. 2 ft. by 12.

154. If 1 load of hay weighs 1 T. 3 cwt. 17 lb., what will 9 loads weigh?

155. How much land in 14 farms of 25 A. 60 sq. rd. 21 sq. yd. each?

156. If the moon moves in her orbit 13° 11′ 35″ in 1 day, how far will she move in 20 days?

DIVISION.

157. Find $\frac{1}{7}$ of 82 bu. 2 pk. 6 qt.

$$\begin{array}{r|l} 7) & 82\text{ bu. }2\text{ pk. }6\text{ qt.} \\ \hline & 11\text{ bu. }3\text{ pk. }2\text{ qt.} \end{array}$$

Solution. — $\frac{1}{7}$ of 82 bu. = 11 bu., with a remainder of 5 bu. The 5 bu. = 20 pk., which added to the 2 pk. in the dividend = 22 pk. $\frac{1}{7}$ of 22 pk. = 3 pk., with a remainder of 1 pk. The 1 pk. = 8 qt., which added to the 6 qt. = 14 qt. $\frac{1}{7}$ of 14 qt. = 2 qt. Ans. 11 bu. 3 pk. 2 qt.

158. Divide 139 wk. 6 d. 10 h. 16 min. by 8.

159. Divide 40 mi. 210 rd. 5 yd. 2 ft. by 12.

160. If 12 spoons weigh 3 lb. 10 oz. 11 pwt., what is the weight of each spoon?

161. What is the daily motion of the moon, if it moves 197° 38′ 45″ in 15 days?

162. A planter has 1534 gal. 1 qt. 1 pt. of molasses, which he wishes to put into 25 equal casks. What must be the least capacity of each cask to exactly receive the molasses?

MISCELLANEOUS EXERCISES.

163. How many inches in 18 rd. 5 yd. 2 ft. 11 in.?

164. What will 5 T. 17 cwt. 25 lb. of iron cost at 3 cents a pound?

165. At 3 cents a pound, how many tons of iron can be bought for $ 396.18?

166. What fraction of a common year is $\frac{8}{27}$ of a day?

167. In one barge there are 50 T. 5 ctl. 75 lb., and in another 47 T. 17 ctl. 35 lb. How many tons in both barges?

168. Two boats start in a race, and one of them gains 5 feet upon the other in every 55 yards. How many rods will it have gained at the end of 2 miles?

169. If 2 A. 65 sq. rd. can be plowed in a day, how much can be plowed in $8\frac{1}{4}$ days?

170. Find the exact number of days from June 11, 1879, to Aug. 5, 1881.

171. If 9 acres produce 21 T. 537 lb. of hay, what does one acre produce?

172. If a pendulum vibrates 47 times in a minute, in what time will it vibrate 13267583 times?

173. What decimal of 20 acres is 7 A. 148 sq. rd.?

174. If a man can cut 24 cords 102 cubic feet of wood in 12 days, how many cord feet can he cut in one day?

175. How many silver spoons, each weighing 2 oz. 10 pwt., can be made from a bar of silver weighing 11 lb. 5 oz. 10 pwt.?

176. A stationer buys 25 reams of commercial note paper at $1.75 a ream, and retails it at 12 cents a quire, with the exception of one outside quire of each ream, which he sells at 8 cents. How much does he make?

177. How many years, months, and days did a man live who was born March 15, 1767, and died June 8, 1845?

178. Bought a cask of oil, containing 68½ gallons, at 72 cents a gallon; ⅜ having leaked out, the remainder was sold at 90 cents a gallon. Did I make or lose, and how much?

179. Show that any article is worth as many five-cent pieces a cental as dollars a ton.

180. A carpenter sent two of his apprentices to ascertain the length of a certain fence. The first made it 17 rd. 16 ft. 11 in., and the second made it 18 rd. 5 in. The carpenter, fearing they might both be wrong, measured for himself, and found it to be 17 rd. 5 yd. 1 ft. 11 in. What was the difference in their measurements?

181. If A and B should commence, March 5, 1882, to go to bed at the same hour, and A should rise at ¼ before 6 o'clock and B at ¾ past 7, how much more time for labor would A have had than B, by March 5, 1900, paying attention to the leap years?

QUESTIONS.

183. What is a denomination? 184. A denominate number? 185. A simple number? 186. A compound number?

187. What is reduction descending? 188. How is a denominate number reduced to smaller denominations?

189. What is reduction ascending? 190. How is a denominate number reduced to larger denominations?

191. What is a denominate fraction? 192. How is a denominate fraction reduced to integers of smaller denominations? 193. How are denominate integers reduced to fractions of larger denominations?

196. How is the difference between dates, in years, months, and days, found?

THE METRIC SYSTEM.

197. The **Metric System** of weights and measures, now coming into use in the United States, has for its base a unit called the *meter*.

NOTE. — This system, in extensive use in the arts and sciences, adopted for the United States Coast Survey, and partially employed in the Mint and General Post Office, was legalized for use in the United States by Congress in 1866.

198. The **Meter**, which was intended to be, and is very nearly, *the ten-millionth part of the distance on a meridian from the equator to the pole*, is the principal unit of lengths, and the standard unit from which all metric measures are derived.

199. The **Are**,* the principal unit of the measures of land, is a square whose side is ten meters.

200. The **Stere**, the principal unit of the measures of wood and stone, is a cube whose edge is a meter.

201. The **Liter**, the principal unit of the measures of capacity, is a cube whose edge is the tenth of a meter.

202. The **Gram**, the principal unit of weight, is the weight of a cube of pure water at its greatest density, whose edge is a hundredth part of a meter.

203. The **Names** of the *divisions* of the unit are formed by prefixing to the name of the unit the Latin words, *milli* for 1000th, *centi* for 100th, and *deci* for 10th; and the *names of multiples*, by prefixing the Greek, *deka* for 10, *hekto* for 100, *kilo* for 1000, and *myria* for 10000.

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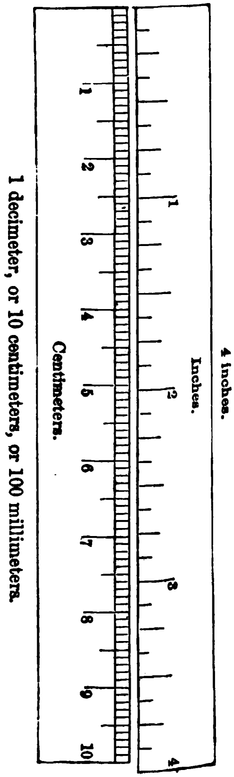
* Are is pronounced *air;* stere, *stair;* and liter, *lee'ter*. All metric names have the accent on the first syllable.

204. In the metric system, as in United States money, only a few of the denominations are much used. These will be distinguished in the tables by the difference in type.

The unit corresponds to the dollar, and *deci*, *centi*, *milli* to *dimes*, *cents*, *mills*.

205. LENGTH MEASURES.

10 millimeters (^{mm})	are	1 **centimeter**, ^{cm}.
10 centimeters	"	1 decimeter, ^{dm}.
10 decimeters	"	1 **METER**, ^{m}.
10 meters	"	1 dekameter, ^{Dm}.
10 dekameters	"	1 hektometer, ^{Hm}.
10 hektometers	"	1 **kilometer**, ^{Km}.
10 kilometers	"	1 myriameter, ^{Mm}.



Equivalents.

1 centimeter = 0.3937 inch.
1 decimeter = 3.937 inches.
1 meter = 39.37 inches.
1 kilometer = 0.6214 mile.

1. The *meter* is used in measuring woven fabrics and short lengths and distances.
2. The *kilometer* is the unit in measuring roads and long distances.
3. The *decimeter* is nearly 4 inches; the *meter*, about 3 feet $3\frac{3}{8}$ inches; and the *kilometer*, about 200 rods, or $\frac{5}{8}$ of a mile.

206. A **Metric Number** is written with the decimal point separating the unit from its decimal parts. Thus,

$1^{Km}\ 5^{Hm}\ 7^{Dm}\ 3^{m}\ 4^{dm}\ 2^{cm}$, written as meters, is 1573.42^{m}, and, written as kilometers, is 1.57342^{Km}.

207. In reading metric numbers, the name of the unit may be applied to all on the left of the decimal point, and the name of the smallest denomination denoted to all on the right of the point. Thus,

42.73 ^{m} may be read forty-two, and seventy-three hundredths meters; or, forty-two meters, and seventy-three centimeters.

8.675 ^{Km} may be read eight, and six hundred seventy-five thousandths kilometers; or, eight kilometers, and six hundred seventy-five meters.

1. How many meters and hundredths of a meter are expressed by 7.25 ^{m}?

2. How many kilometers and thousandths of a kilometer are expressed by 8.407 ^{Km}?

3. How many meters and centimeters does 7.25 ^{m} express?

4. How many kilometers and meters does 8.407 ^{Km} express?

5. How many centimeters in 4.15 ^{m}? How many meters in 7.384 ^{Km}?

6. Reduce 784 centimeters to meters; 6453 meters to kilometers.

208. SURFACE MEASURES.

100 square millimeters ($^{sq\ mm}$) are 1 **square centimeter**, $^{sq\ cm}$.
100 square centimeters " 1 square decimeter, $^{sq\ dm}$.
100 square decimeters " 1 **square meter**, $^{sq\ m}$, or centare.
100 square meters centares " 1 square dekameter, $^{sq\ Dm}$.
100 square dekameters Ares " 1 square hektometer, $^{sq\ Hm}$.
100 square hektometers " 1 **square kilometer**, $^{sq\ Km}$.

Also,

100 centares (^{ca}), or sq. meters, are 1 ARE, ^{a}.
100 ares " 1 **hektare**, ^{Ha}.

Equivalents.

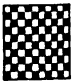
1 square centimeter	= 0.1550 sq. inch.	1 are	= 3.954 sq. rods.
1 square decimeter	= 0.1076 sq. foot.	1 hektare	= 2.471 acres.
1 square meter	= 1.196 sq. yards.	1 sq. kilometer	= 0.3861 sq. mile.

1. The *square meter* is used in measuring ordinary surfaces; the *square kilometer*, in measuring the area of countries; and the *are* and *hektare*, in measuring land.

2. The *square meter* is about 10¾ square feet, or 1⅕ square yards; and the *hektare*, about 2½ acres.

3. As 100 units of a smaller denomination make a unit of a denomination next larger, the scale is 100, and two places of figures must be allowed for each denomination. Thus,

31 ^{Ha} 14 ^{a} 17 ^{ca}, written as ares, is 3114.17 ^{a}, which may be read 3114 ares, and 17 centares; and, written as hektares, is 31.1417 ^{Ha}, which may be read 31 hektares, and 1417 centares.


1 Square Centimeter.

7. In 4 square meters how many square decimeters? In 4 square centimeters how many square millimeters?

8. Express 65.41 ^{a} as centares; as hektares.

9. How many ares in 5734 ^{ca}? How many hektares in 6893 ^{a}?

209. VOLUME MEASURES.

1000 cubic millimeters ($^{cu\,mm}$) are 1 cubic centimeter, $^{cu\,cm}$.
1000 cubic centimeters " 1 cubic decimeter, $^{cu\,dm}$.
1000 cubic decimeters " 1 **cubic meter**, $^{cu\,m}$, or STERE.

Also,

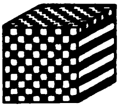
10 decisteres (^{dst}) are 1 **stere**, st.

Equivalents.

1 cubic centimeter	= 0.061 cu. inch.	1 cubic meter	= 1.308 cu. yards.
1 cubic decimeter	= 61.022 cu. inches.	1 stere	= 0.2759 cord.

1. The *cubic meter*, the unit of ordinary solids, takes the name of *stere* when applied to the measuring of wood and lumber.

2. The *cubic decimeter* is about 61 cubic inches; the *cubic meter*, or *stere*, about $35\frac{1}{3}$ cubic feet.



1 Cubic Centimeter.

3. Where, as in the table, 1000 units of a smaller denomination make a unit of a denomination next larger, the scale is 1000, and three places of figures must be allowed for each denomination. Thus,

$13^{\text{cu m}}\ 406^{\text{cu dm}}\ 578^{\text{cu cm}}$, written as cubic meters, is $13.406578^{\text{cu m}}$, which may be read 13 cubic meters, and 406578 cubic centimeters; and, written as cubic decimeters, is $13406.578^{\text{cu dm}}$.

10. In $6^{\text{cu m}}$ how many cubic decimeters? In $8^{\text{cu dm}}$ how many cubic centimeters?

11. In $7000^{\text{cu mm}}$ how many cubic centimeters? In $9000^{\text{cu dm}}$ how many cubic meters?

12. Express $76.006^{\text{cu dm}}$ as cubic centimeters; as cubic meters.

13. Express $0.3125^{\text{cu m}}$ as cubic decimeters; as cubic centimeters.

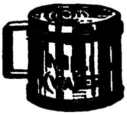
210. CAPACITY MEASURES.

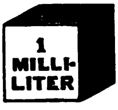
10 milliliters ($^{\text{ml}}$)	are	1 **centiliter**, $^{\text{cl}}$.
10 centiliters	"	1 deciliter, $^{\text{dl}}$.
10 deciliters	"	1 LITER, $^{\text{l}}$.
10 liters	"	1 dekaliter, $^{\text{Dl}}$.
10 dekaliters	"	1 **hektoliter**, $^{\text{Hl}}$.
10 hektoliters	"	1 kiloliter, $^{\text{Kl}}$.

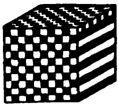
Equivalents.

1 liter = 61.022 cu. inches.
1 liter = 1.0567 liquid quarts.
1 liter = 0.908 dry quart.

1 hektoliter = 3.531 cu. feet.
1 hektoliter = 26.417 gallons.
1 hektoliter = 2.837 bushels.







1 Milliliter = 1 Cubic Centimeter.

1. The *liter* is used in measuring liquids; and the *hektoliter* is used in measuring grains, roots, and liquids in casks.

2. The *liter* is about 1.06 liquid quarts, or 0.9 of a dry quart; and the *hektoliter* is about 26½ gallons, or 2⅚ bushels.

14. How many liters, and what decimal of a liter, are expressed by 6.45^{l}?

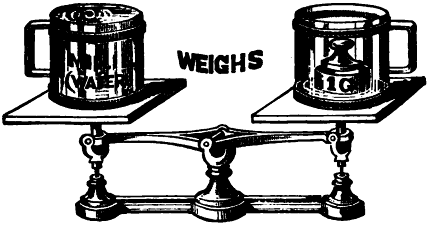
15. How many kiloliters, and what decimal of a kiloliter, are expressed by 9.750 ^{Kl}?

16. How many centiliters are 6 liters? 7.55 liters?

17. How many liters in 600 centiliters? How many kiloliters in 6000 liters?

211. WEIGHT MEASURES.

10 milligrams (^{mg})	are	1 **centigram**, ^{cg}.
10 centigrams	"	1 decigram, ^{dg}.
10 decigrams	"	1 GRAM, ^{g}.
10 grams	"	1 dekagram, ^{Dg}.
10 dekagrams	"	1 hektogram, ^{Hg}.
10 hektograms	"	1 **kilogram**, ^{Kg}, or ^{K}.
10 kilograms	"	1 myriagram, ^{Mg}.
10 myriagrams	"	1 quintal, ^{Q}.
10 quintals	"	1 **metric ton**, ^{T}.



Equivalents.

In weight

1 gram = 1 cu. centimeter, or 1 milliliter of water.
1 kilogram = 1 cu. decimeter, or 1 liter of water.
1 metric ton = 1 cu. meter, or 1 kiloliter of water.

1 gram = 15.432 grains troy.
1 gram = 0.03527 ounce av.
1 kilogram = 2.2046 pounds av.
1 metric ton = 1.1023 tons.

NOTE. — The weight of the gram is determined when the water is pure and at the temperature of its greatest density, which is 39.2° Fahrenheit.

1. The *gram* is used in weighing letters, gold, and jewels, and in mixing medicines; the *kilogram*, or, for brevity, *kilo*, is used in weighing common articles; and the *metric ton*, in weighing very heavy articles.

2. The *kilo* is about $2\frac{1}{4}$ pounds; and the *metric ton*, about $1\frac{1}{10}$ common tons.

3. Of the United States coinage, the nickel five-cent piece weighs 5 grams; two silver half-dollars, 25 grams; and 80 silver half-dollars, a kilo.

4. A letter sent for a single postage must not exceed the weight of three nickels, or 15 grams.

18. In 5.65^{g}, how many decigrams? How many centigrams?

19. In 5650^{mg}, how many centigrams? How many decigrams?

20. In 6.315^{Kg}, how many grams? In 4.670^{T}, how many kilos?

21. What decimal part of a kilo is a gram? What decimal part of a metric ton is a kilo?

22. At 20 cents a kilo, how many dollars will a metric ton of sugar cost?

23. A tank has the capacity of 5.250 kiloliters. How many metric tons of pure water will it hold?

24. I wish to weigh 8.75^{Kg}. How many silver half-dollars will serve for the weights?

REDUCTION OF UNITS.

212. The **Units** shown in the metric tables form a decimal system (Art. 19), to which apply the following

Principles.

1. *Ten units, or some multiple of ten units, of any denomination make one of the next larger unit.*

2. *A metric number may be changed from one denomination to another next smaller, or larger, by moving the decimal point one or more places to the right, or left, as the case may be.*

3. *Any denomination may be taken as the unit, the number at the right of the point being read as a decimal of the unit.*

213. The units of the metric and the common system may be readily compared by means of the equivalents which have been given, and by means of the following

COMPARATIVE TABLE.

Length.

1 inch = 2.54 centimeters.
1 foot = 30.48 centimeters.
1 rod = 5.029 meters.
1 mile = 1.6093 kilometers.

Surface.

1 sq. inch = 6.452 sq. centim.
1 sq. foot = 9.2903 sq. decim.
1 sq. yard = 0.8361 sq. meter.
1 sq. rod = 0.2529 are.
1 acre = 0.4047 hektare.
1 sq. mile = 2.59 sq. kilometers.

Volume.

1 cu. inch = 16.387 cu. centimeters.
1 cu. foot = 28.317 cu. decimeters.
1 cu. yard = 0.7646 cu. meter.
1 cord = 3.625 steres.

Capacity.

1 liq. quart = 0.9465 liter.
1 gallon = 3.785 liters.
1 dry quart = 1.101 liters.
1 bushel = 0.3524 hektoliters.

Weight.

1 grain troy = 0.648 centigram.
1 ounce troy = 31.1035 grams.

1 ounce av. = 28.35 grams.
1 pound av. = 0.4536 kilogram.
1 common T. = 0.9072 met. ton.

WRITTEN EXERCISES.

25. Express as meters and add 1365^{mm}, 497^{cm}, and 145.51^{m}.

26. Express as ares and add 15.16^{Ha}, 111.55^{a}, and 3615^{ca}.

27. From a range of wood containing 45 steres, I have sold 276 decisteres. How many steres remain?

28. How many liters in 6 casks, each containing 3.40^{Hl}?

29. Eight men shared equally 21.080 metric tons of sugar. What is each man's share worth at 20 cents a kilo?

30. From a farm containing 365.50^{Ha} there have been sold two small lots; the one containing 8.42^{Ha}, and the other 87.25^{a}. How much remains?

31. Change 125 meters to feet.

$$\begin{array}{r}
39.37 \text{ in.}\\
\underline{125}\\
19685\\
7874\\
\underline{3937}\\
12 \text{ in.})\,\underline{4921.25} \text{ in.}\\
410.10\tfrac{5}{12} \text{ ft.}
\end{array}$$

Solution. — As 1 meter = 39.37 inches, 125 meters must equal 125 times 39.37 inches, or 4921.25 inches. As 1 foot = 12 inches, there will be as many feet as 12 inches are contained times in 4921.25 inches, or $410.10\frac{5}{12}$ ft.

32. If your weight is 55 kilos, what is it in pounds avoirdupois?

33. A garden plat contains 306 square meters. How many square yards are there in it?

34. A farm is 450 hektares in extent. How many acres does it contain?

35. A barrel of flour weighs 196 pounds. What is its weight in kilos?

36. The produce of 7 acres was 210 bushels of wheat. What was it in hektoliters?

37. When butter is 35 cents a pound, how much should it be a kilo?

Solution. — At 35 cents a pound, a kilo, which is 2.2046 pounds, must cost 2.2046 times 35 cents, or 77+ cents.

38. At 65 cents a bushel, what should be the price of corn a hektoliter ?

39. What is the value of an eighth of a plantation of 600.58 hektares at $ 25 an acre ?

40. The distance by railroad between Boston and New Orleans is 1607 miles. What is it in kilometers ?

41. The dome of the capitol at Washington is 287 ft. 6 in. high, surmounted by a statue of Liberty 19 ft. 6 in. high. What is the whole height in meters ?

42. Bought a roll of carpeting of 65 yards at $ 1.20 a meter, and sold it at the same price a yard. How much did I make by the transaction ?

43. The capacity of a certain bin is 40.64 cubic meters. What is the value of the grain that can be put in it at 80 cents a bushel ?

QUESTIONS.

197. What is the metric system ? 198. What is a meter ? 199. An are ? 200. A stere ? 201. A liter ? 202. A gram ?

203. How are the names of the divisions of the metric units formed ?

205. Recite the table of metric measures of length. For what is the meter used ? The kilometer ?

206. How is a metric number written ? 207. How are metric numbers read ?

208. Recite the table of metric measures of surface. How is the square meter used ? The square kilometer ? The are and hektare ?

209. Name the measures of volume. How is the cubic meter used ? When does it take the name of the stere ?

210. Recite the table of measures of capacity. How is the liter used ? The hektoliter ?

211. Recite the table of measures of weight. For what is the gram used ? The kilogram ? The metric ton ?

212. How many units of one denomination make a unit of another in the metric system ? How may a metric number be changed from one denomination to another ? When any denomination is taken as the unit, how may the number at the right of the point be read ?

MEASUREMENTS.

SURFACES.

214. 1. How many square inches in a surface 8 inches long and 1 inch wide? In a surface 8 inches long and 2 inches wide?

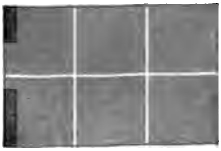
2. A path 12 feet long and 2 feet wide has how many square feet of surface in it?

3. A table is 6 feet long and 4 feet wide. How many square feet of surface has it?

215. A **Plane Figure** is a portion of a plane surface (Art. 163) bounded by lines.

216. The **Perimeter** of a plane figure is the sum of its bounding lines.

217. The **Area** of a plane figure is the surface included within its perimeter.



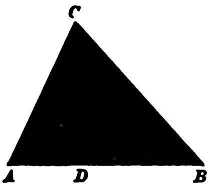
218. The **Dimensions** of a rectangle are its length and breadth.

A rectangle 3 inches long and 2 inches broad contains in one row 3 squares of 1 square inch each; and 2 such rows contain 2 times 3 square inches, or 6 square inches. That is,

The area of a rectangle is equal to the product of its length and breadth, taken in the same denomination.

Also,

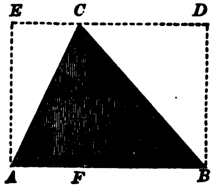
One of the dimensions of a rectangle is equal to the area divided by the other dimension.



219. A **Triangle** is a plane figure bounded by three straight lines.

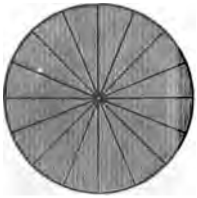
The *Base* of a triangle is the line upon which it stands; and the *Altitude* is its height above the base, or the base extended. Thus,

A B is the base, and *C D* the altitude, of the triangle *A B C.*



220. Draw the lines *A E, D B* perpendicular to the extremities of the base of the triangle *A C B,* and draw the line *E D* through *C,* parallel to *A B,* and it is evident that the triangle *A C B* is half the rectangle *A B D E,* of the same base and altitude. That is,

The area of a triangle is half the area of a rectangle of the same base and altitude.



221. A **Circle** may be regarded as consisting of a great number of triangles, whose bases form the circumference of a circle, and whose altitude is the radius of the circle. Hence,

The area of a circle is equal to half the product of the circumference by the radius.

222. The quotient of the circumference of a circle divided by the diameter, to the nearest ten-thousandth, is 3.1416. Hence,

The circumference is equal to the diameter multiplied by 3.1416; *the diameter is equal to the circumference divided by* 3.1416.

WRITTEN EXERCISES.

4. What is the area of a floor which is 52 feet 9 inches long and 26 feet wide?

$$\begin{array}{r} 52.75 \\ 26 \\ \hline 31650 \\ 10550 \\ \hline 1371.50 \end{array} \text{ sq. ft.}$$

Or,

1371 sq. ft. 72 sq. in.

Solution. — 52 ft. 9 in. = 52.75 feet. The product of the length by the width gives as the number of square feet of surface 1371.50 sq. ft., or 1371 sq. ft. 72 sq. in., which is the area required.

5. The area of a floor is 1371 square feet 72 square inches, and its width is 26 feet. What is its length?

52.75 ft. = 52 ft. 9 in.

$$\begin{array}{r} 26\,)\,1371.50 \\ 130 \\ \hline 71 \\ 52 \\ \hline 195 \\ 182 \\ \hline 130 \\ 130 \\ \hline \end{array}$$

Solution. — 1371 sq. ft. 72 sq. in. = 1371.50 sq. ft. As the area 1371.50 sq. ft. is the product of the length and width, the width must equal the quotient of the area 1371.50 divided by the length 26, which is 52.75 ft. = 52 ft. 9 in.

6. The base of a triangle is 46 ft. 3 in., and the altitude 35 ft. 6 in. What is its area?

7. The circumference of a circle is 314.16 feet, and its radius 50 feet. What is its area?

8. The diameter of a circle is 400 feet. What is its circumference?

9. The circumference of a circle is 1256.64 feet. What is its diameter?

10. How many yards of carpeting 1 yard wide will be required to carpet a room 18 ft. long and 15 ft. 6 in. wide?

11. A room that is 18 ft. 9 in. square requires 50 yards of carpeting. What is the width of the carpeting?

12. How many acres in a field 45 rods long and 48 rods wide?

13. The diameter of a circle is 15 meters. What is its circumference in feet?

14. What is the area of the gable end of a house 32 ft. wide, the ridge being 14 ft. 6 in. higher than the base of the gable?

15. How many hektares in a rectangular meadow 564.50 meters long and 260 meters wide?

16. What will it cost at 60 cents a square yard to concrete a walk 288 ft. long and 12 ft. 6 in. wide?

17. A horse is fastened to a stake by a chain 60 feet long. How many square rods of surface can the chain sweep over?

18. How many square feet of sheet zinc will be required to line a cistern 6 ft. deep, having a square bottom, of which each side is 2 ft. 6 in.?

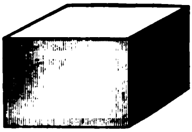
19. The capitol at Washington is 751 feet long and 348 feet wide. How many acres does it cover?

20. On laying the pavement of a court with stones 2 ft. 6 in. long by 9 in. wide, it is found that it requires 75 stones to form one strip extending the whole length of the court, and that 8¼ strips will exactly cover it. What is the area in square yards, and what is the cost of the pavement at 20 cents a square foot?

VOLUMES.

21. How many cubic feet in a beam 10 feet long, 1 foot wide, and 1 foot deep? In a beam 10 feet long, 1 foot wide, and 2 feet deep?

22. How many cubic feet in a block of marble 8 feet long, 2 feet wide, and 1 foot thick? In a block 8 feet long, 2 feet wide, and 2 feet thick?



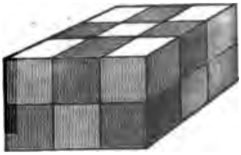
A Rectangular Volume.

223. A **Rectangular Volume** is a body bounded by six rectangles.

The **Dimensions** of a rectangular volume are its length, breadth, and thickness.

224. The **Contents** of a rectangular volume are the space contained within its bounding surfaces.

The dimensions of a rectangular volume determine its contents. Thus,

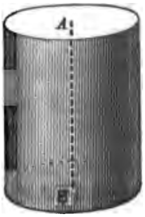


A rectangular volume 3 inches long, 3 inches wide, and 2 inches thick, contains in one layer 3 rows of 3 cubic inches, or 9 cubic inches, and 2 such layers, or 2 times 3 times 3 cubic inches, contain 18 cubic inches. That is,

The contents of a rectangular volume are equal to the product of its length, breadth, and thickness, taken in the same denomination.

Also,

One of the dimensions of a rectangular solid equals its contents divided by the product of the other two dimensions.



A Cylinder.

225. A **Cylinder** is a round body of uniform diameter, whose bases are equal and parallel circles.

The altitude of a cylinder is the straight line joining the centers of the two bases; as the line $A\ B$.

226. The *Contents of a cylinder are equal to the product of the area of the base by the altitude.*

The *Curved Surface of a cylinder is equal to the product of its circumference and altitude.*

WRITTEN EXERCISES.

23. How many cubic feet in a rectangular block 8 feet long, 4 feet 6 inches wide, and 2 feet 3 inches thick ?

Solution. — 4 ft. 6 in. $= \frac{9}{2}$ ft.; 2 ft. 3 in. $= \frac{9}{4}$ ft. The product of the three dimensions, $8 \times \frac{9}{2} \times \frac{9}{4} = 81$ cu. ft.

24. The contents of a rectangular block 8 feet long and $4\frac{1}{2}$ feet wide are 81 cubic feet. What is its thickness ?

$8 \times 4\frac{1}{2} = 36$

$81 \div 36 = 2\frac{1}{4}$

$2\frac{1}{4}$ ft. = 2 ft. 3 in.

Solution. — The contents of the block are the product of its three dimensions. The product of the length and width is 36. The given product of the three dimensions divided by 36 gives for the thickness $2\frac{1}{4}$ ft., or 2 ft. 3 in.

25. What are the contents of a trunk 4 feet long, $2\frac{1}{3}$ feet wide, and 18 inches deep ?

26. How many cubic feet of space in a room whose dimensions are each 10 feet 6 inches ?

27. A rectangular cistern, whose length is $13\frac{3}{8}$ feet and breadth 6 feet, contains $294\frac{1}{4}$ cubic feet of water. What is the depth of the water ?

28. A box is 2 meters long, 15 decimeters wide, and 1 meter deep. What is its capacity ?

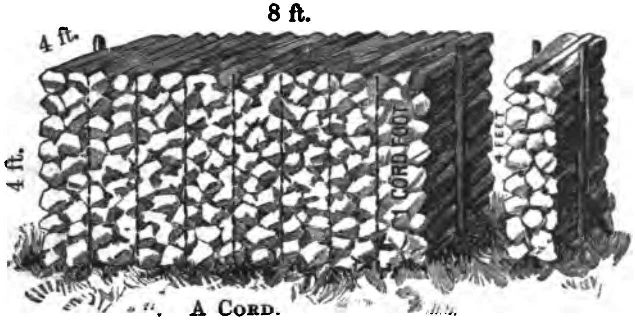
29. How many cubic yards of earth must be removed in excavating a cellar 60 feet long, 42 feet wide, and 8 feet 6 inches deep ?

30. A block is 3 meters long, 2 meters wide, and 1.45 meters thick. What are its contents ?

31. A circular well is 32 feet deep and 3 feet in diameter. How many cubic yards does it contain ?

32. A roller is 4 feet 6 inches long, and 6 feet 4 inches in circumference. How much surface will it pass over in revolving 36 times ?

WOOD MEASURE.


8 ft.
4 ft.
4 ft.
1 CORD FOOT
4 FEET

A Cord.

227. A **Range** of wood 8 feet long, 4 feet wide, and 4 feet high, makes a cord of wood (Art. 171).

A *Cord Foot* is 1 foot in length of this range, or 16 cubic feet.

WRITTEN EXERCISES.

33. How many cords of wood in a range 32 feet long, 8 feet high, and 4 feet wide?

34. How many cords of 4-foot wood in a range 64 feet long and 4 feet high?

35. A range of 4-foot wood is 28 feet long and 6½ feet high. How many cords does it contain?

36. A range of 4-foot wood is 56 feet long. How high must it be piled to contain 10 cords?

37. Wood is loaded upon a cart in 2 piles, 3 feet 6 inches wide and 4 feet 6 inches high. Allowing the wood to have been cut 4 feet long, how much wood is there on the cart?

38. A shed contains a pile of wood 30 feet long, 16 feet wide, and 12 feet high. What is the value of the wood at $5.25 a cord?

BOARD MEASURE.

228. A **Board Foot** is a square foot of board one inch thick; and

12 board feet make 1 cubic foot.

229. **Lumber,** or sawed timber, as boards, plank, joists, and the like, is estimated in board feet.

Hewn Timber is estimated either in board feet or cubic feet.

In the measurement of lumber, or hewn timber which tapers, half the width or half the thickness of the two ends is taken.

230. To find the contents of a piece of lumber in board feet,

Multiply the product of the length and width taken in feet by the thickness in inches.

NOTE. — Lumber less than an inch thick is measured as if it were an inch thick. In computations, therefore, thickness may be disregarded if an inch or less.

In the measurement of box boards, however, the standard of thickness is $\frac{5}{8}$ of an inch.

WRITTEN EXERCISES.

39. How many board feet in a joist 21 feet 6 inches long, 4 inches wide, and 3 inches thick?

21 ft. 6 in. $= \frac{43}{2}$ ft.

4 inches $= \frac{4}{12} = \frac{1}{3}$ ft.

$\frac{43}{2} \times \frac{1}{\cancel{3}} \times \cancel{3} = 21\frac{1}{2}$ bd. ft.

Solution. — Multiplying together the length and width expressed in feet, we have, as surface feet, $\frac{43}{2} \times \frac{1}{3}$, and, multiplying by the inches in thickness, have as board feet, $\frac{43}{2} \times \frac{1}{3} \times 3 = 21\frac{1}{2}$.

40. How many board feet in a board 16 feet long, 1 foot 6 inches wide, and $\frac{3}{4}$ inch thick?

41. Required the contents in board feet of a plank 20 feet long, 16 inches wide, and 2½ inches thick?

42. How many board feet in a piece of hewn timber 20 feet long, 10 inches thick, and whose width tapers from 18 to 16 inches?

43. What will be the cost of 44 spruce joists, each 18 feet long, 9 inches wide, and 3 inches thick, at $23 a thousand feet?

44. What is the value of 18 planks 2 inches thick, 24 feet long, 21 inches wide, at $35 a thousand feet?

MISCELLANEOUS EXERCISES.

45. A rectangular lot containing 9432 square feet has a frontage of 131 feet. What is its other dimension?

46. A wagon 8 feet long and 3 feet wide has wood piled on it 5¼ feet high. How much is the wood worth at $5.50 a cord?

47. How many board feet of plank 2 inches thick will be required to cover a platform 30 ft. 6 in. long and 20 ft. wide?

48. What must be paid, at the rate of $40 a thousand feet, for 6 boards 16 feet long, and in width tapering from 28 to 20 inches?

49. A cistern 2.50 meters long, 90 centimeters wide, and 110 centimeters deep, will hold how many hektoliters of water?

50. How many rolls of paper 12 yards long and 1 foot 8 inches wide will be required to paper a room 18 feet long, 12 feet wide, and 9 feet high, no allowance being made for windows and doors?

51. Allowing .8 of a bushel to a cubic foot, how many bushels of corn can be put in a bin whose inside measure is 4 feet long, 3 feet wide, and 4 feet deep?

52. Allowing 7½ gallons to a cubic foot, how much will a tank hold whose inside measure is 2 ft. 6 in. long, 2 ft. wide, and 1 ft. 9 in. deep?

53. Allowing for each square foot of surface of a brick wall twice as many bricks in number as the wall is inches thick, how many bricks are there in a wall 40 feet long, 12 feet 6 inches high, and 8½ inches thick?

54. If a ton of 2000 pounds of Lehigh coal fills 40 cubic feet, and of Lackawanna coal 45 cubic feet, how much of each kind will fill a bin 8 feet long, 5 feet wide, and 4 feet deep?

55. Water is flowing into a cistern whose rectangular base is 4840 square inches. How many cubic feet will have been supplied when the depth of water is 3⅓ feet, and what will be its weight at the rate of 1000 ounces a cubic foot?

56. How many cubic feet in the walls of a cellar, whose length is 33 feet, width 30 feet, and depth 9 feet, allowing the thickness of the walls to be 18 inches, and the lap of the one wall by the other at each corner of the cellar to be 18 inches?

QUESTIONS.

215. What is a plane figure?

216. What is the perimeter of a plane figure? 217. The area of a plane figure? 218. To what is the area of a rectangle equal?

219. What is a triangle? The base of a triangle? The altitude? 220. How is the area of a triangle found?

221. How may the circle be regarded? To what is the area of a circle equal? 222. To what is the circumference equal? To what is the diameter equal?

223. What is a rectangular volume? What are the dimensions of a rectangular volume? 224. The contents? 225. What is a cylinder? The altitude of a cylinder? 226. What are the contents of a cylinder? To what is the curved surface of a cylinder equal?

227. What range of wood makes a cord? How much of such a range is a cord foot?

228. What is a board foot? 229. How is lumber estimated? How is hewn timber estimated? 230. How are the contents of a piece of lumber found?

224. The contents of a room divided by the area of the floor will *give what?*

REVIEW.

ORAL EXERCISES.

231. 1. The top of your desk is 2 ft. long and 18 in. wide. How many yards around it?

2. It is $1\frac{7}{8}$ miles to the post-office. How many miles do I travel in making three round trips to it?

3. How many score in a gross?

4. My hens lay an egg apiece daily. How many dozen do 3 of them lay in 3 weeks 3 days?

5. If a ton of coal measures 40 cu. ft., how many tons will two 5-foot cubical bins hold?

6. How many degrees does the minute-hand of a clock move over in 45 minutes?

7. What is the perimeter of the largest surface measure?

8. What is the area of the entire surface of a 4-foot cube?

9. How wide must a yard of ribbon be to contain 144 sq. in.?

10. What will it cost to carpet a room 18 ft. square at $ 1 per square yard?

11. What costs a mile of wire at a cent a foot?

12. At 30 cents a gallon, what is my milk-bill for April? I take 2 quarts daily.

13. What will a 12-inch cube of marble cost at one cent per cubic inch?

14. $\frac{3}{4}$ of 48 is $\frac{9}{11}$ of what number?

15. If $\frac{1}{2}$ ounce of tea costs 5 cents, what will $2\frac{1}{2}$ pounds cost?

16. How many acres in my grandfather's farm, which is $\frac{1}{2}$ of a mile square?

17. If $3\frac{1}{2}$ dozen cost $ 3.50, what will $\frac{7}{12}$ dozen cost?

18. Bought an acre of land at $ 0.50 per square rod, and sold it so as to double my money. Required my gain?

19. Bought pencils for $ 1.25 a gross, and sold them for a cent each. How much did I gain on 10 gross?

20. How long must a field be that is 20 rods wide to contain 3 acres?

21. What will 7 lb. 15 oz. of sugar cost at 8 ¢ per pound?

22. A school contains 96 boys, and $\frac{7}{15}$ of the pupils are girls. How many pupils in the school?

23. The entire surface of a cube is 96 square inches. What is its edge?

24. An arc of 80 degrees is what fraction of a semi-circumference?

25. How many paper boxes 3 in. long, 2 in. wide, and 2 in. deep can be packed to a cubic foot?

26. How many writing-books of 24 pages each can be made from ½ a ream of paper, the sheets being of the proper size?

27. What cost 18 dozen blank-books at 33⅓ cents each?

28. I sailed for Europe May 17, and reached home again August 5. How long was I away?

29. School closes July 10 for a 6-weeks' vacation. When will it begin again?

30. What effect has moving the decimal point two places to the left, and why?

31. What is the circumference of a circle 10 inches in diameter?

32. How many 4-inch squares in a square foot?

33. How many days in the five shortest months of a leap year?

34. What costs a stere of wood at $ 0.75 per decistere?

35. What decimal of an acre is a plat of ground 15 rd. long and 4 rd. wide?

36. What fraction of March has passed at noon, March 25?

37. At $ 7 a week, what is my board-bill from December 15 to January 22?

38. How many strips of carpeting ¾ yd. wide will be required for a room 18 ft. square? How many yards?

39. Divide 0.48 by 0.2. Find their product.

40. A can do in 2 days the work which B can do in 3 days. If B works with A, in how many days will the work be done?

41. What costs a strip of rosewood 4 ft. long, 3 in. wide, and $1\frac{1}{2}$ in. thick at 60 cents a board-foot?

42. A note dated August 11 is payable in 33 days. On what day must it be paid?

43. A sidewalk a mile long contains just an acre. How wide is it?

44. If $\frac{1}{4}$ of a pound costs $\$\frac{1}{2}$, what will $\frac{3}{8}$ of a pound cost?

45. I give $\frac{1}{16}$ of my potatoes to Mr. Murphy for digging them. He sells his share for $\$8$, at 40 cents a bushel. What can I get for my share at the same rate?

46. Emma's birthday is Aug. 16, 1872. John is 4 y. 8 mo. 27 days older. When is his birthday?

47. Name the natural divisions of time.

48. A cylinder 10 inches long has $\frac{3}{4}$ of a square inch as the area of its base. What is its weight if each cubic inch weighs 2 ounces?

49. If your school-room is 15 feet high, and contains 12000 cubic feet, how many square feet are there in the floor?

WRITTEN EXERCISES.

50. How many times is the greatest common divisor of 1116 and 1364 contained in their least common multiple?

51. Divide the product of 0.008 and 0.06 by 0.16.

52. Change $\frac{3}{160}$ to a decimal.

53. Change $\frac{3}{7}$ to hundredths.

54. If a rail is 33 feet in length, how many rails will lay a double track of railroad 10 miles long?

55. What cost 17.65 tons of coal if $\$28$ pays for $3\frac{1}{5}$ tons?

56. An estate is valued at $\$14000$. The house is worth $\frac{2}{5}$ as much as the land. Required the value of each.

57. I have a square mile of land. I sell $\frac{2}{5}$ of it to one man, $\frac{1}{4}$ to another, and 100 A. to a third. How many acres have I left?

58. I have $570 in my safe, which is $80 less than ⅗ of what I have in the bank. How much money have I?

59. The runners in a 10-mile race go how many times around a park 320 feet long and 300 feet wide?

60. Change 20000 square inches to larger denominations.

61. How many yards of carpeting ¾ yd. wide will be required for a parlor 28 ft. by 15 ft.?

62. How many cubic yards of earth will be required to raise the surface of an acre one foot?

63. A brick is 8 in. long, 4 in. wide, and 2 in. thick. How many will be required to lay ¼ mile of 8-foot sidewalk, the bricks being laid flat?

64. How many will be needed if the bricks are laid with their sides uppermost?

65. How much larger is a square 20 rods in diameter than a circle of the same diameter?

66. How many tons of ice can be cut from ¼ of an acre of a pond, the ice being 12 inches thick, and a cubic foot weighing 930 ounces?

67. How many square yards in a piece of calico 30 inches wide and 45 yards long?

68. I gave $100 for a hektoliter of wine, and sold it for $5 a gallon. Did I gain or lose, and how much?

69. I have a piece of land 150 feet deep and fronting 200 feet on the street. I cut it into 50-foot lots, 150 feet deep, and fence each lot. What does my fencing cost at $0.16½ per rod?

70. How many times will the 5-foot driving-wheel of a locomotive revolve in going from Boston to Lawrence, 26 miles?

71. How many cords of wood can be piled in a shed 40 ft. long, 12 ft. wide, and 8¾ ft. high?

72. How many square feet in a lot of land 1 rd. 8 ft. long and ¾ rd. wide?

73. How much oil-cloth 8 feet wide is needed for a hall 55 ft. long and 32 ft. 8 in. wide?

74. How many acres in a street 60 feet wide and ¾ of a mile in length?

75. At $ 0.16⅔ per square yard, what will it cost to plaster a room 20 ft. square and 12 ft. high, allowing ⅙ for openings?

76. What will the flooring of the above room cost at $ 35 a thousand feet, 1/11 of the quantity bought being wasted?

77. $(17.28 \div 0.083\frac{1}{3}) \times \frac{7\frac{1}{2}}{3000} = ?$

78. What decimal of a day has gone at 3 P. M.?

79. A room 8 feet high is 16 feet long and 14 feet wide. How many yards of paper 2 feet wide will cover the walls?

80. A steamer reaches Boston at 3.15 P. M., Aug. 3, 1881, by the steamer's time, after a voyage of 7 d. 15 h. 30 min. Find the exact time of her sailing.

81. How many square yards of silk in 300 feet of 3-inch ribbon?

82. What cost 18 gal. 3 qt. 1 pt. at $ 0.45 per gallon?

83. Find the difference between 0.7⅜ and 0.7 + ⅜.

84. From an acre of land I sold 8 square rods, and also a piece 8 rods square. How much have I left?

85. A pile of wood containing 4½ cords is 5 ft. 6 in. high and 4½ feet wide. How long is it?

86. A room 30 feet long requires 80 yards of carpeting ¾ yd. wide. How wide is the room?

87. A field containing 5 acres is 600 feet long. Required the distance around it.

88. The 56 desks in a school-room are 2 feet by 18 inches. How many yards of 40-inch cloth will cover them?

89. How many 2-inch cubes will a cubic yard make?

90. How many feet of inch boards can be sawed from a stick of mahogany 15 feet long and 20 inches square, 0.05 being wasted in sawing?

91. From a lot 90 rods square I sold 90 square rods. What is the value of the remainder at $ 240 per acre?

92. From a piece of cloth containing $9\frac{1}{7}$ yards, $3\frac{3}{5}$ yards were cut. What part of the whole piece remained?

93. A garden is 12 rods long and 10 rods wide. At 75 cents a square yard, what will a concrete walk 6 feet wide, and extending around the garden inside the fence, cost?

94. A garden 240 feet long and 160 feet wide is enclosed by a tight board fence 6 ft. high. What will it cost to paint both sides of the fence at 5 cents per square yard?

95. How many days, hours, and minutes from 10.30 P. M., Feb. 4, to 3.40 A. M., May 11?

96. A regiment of 900 soldiers is to be clothed; each suit requires 9 yards of cloth $1\frac{1}{4}$ yd. wide. How many yards of flannel $\frac{3}{4}$ yd. wide will be required to line the suits?

97. I have a room to carpet that is 30 feet long and 27 feet wide. Which is cheaper, to buy yard-wide carpeting at $ 1.25, or carpeting $\frac{3}{4}$ yd. wide at $ 1, and how much?

98. If the weight of a hektoliter of wheat is 75 kilos, what weight of wheat in metric tons will fill a bin 2 meters long, 1.4 meters wide, and 1 meter deep?

99. What will it cost to shingle a house 112 ft. long, each side of the pitch-roof being 25 ft. wide, 10 shingles covering a square foot, the shingles costing $ 6.50 per thousand.

100. Bought $7\frac{1}{1}\frac{1}{2}$ dozen at $18\frac{3}{4}$ cents a dozen. Required the cost of 12 dozen.

101. Sold $14\frac{7}{8}$ tons of coal at $ $5\frac{5}{8}$ per ton, and $12\frac{3}{4}$ tons at $ $6\frac{1}{2}$ per ton, and charged 25 cents a ton for housing the coal. Find the amount of my bill.

102. From the product of $3\frac{1}{3}$ and $4\frac{1}{8}$ take half their difference.

103. If $17\frac{1}{2}$ gallons cost $ $19\frac{3}{8}$, what will $\frac{4}{5}$ of a gallon cost?

104. From a piece containing $\frac{7}{8}$ of a yard I sold $\frac{5}{8}$ of a yard at $ $\frac{3}{4}$ a yard, and gained $ $\frac{1}{8}$. What did the piece cost me?

PERCENTAGE.

232. 1. What is $\frac{1}{100}$ of 100? $\frac{3}{100}$? $\frac{7}{100}$? $\frac{13}{100}$? $\frac{51}{100}$?

2. What is $\frac{3}{100}$ of 500? $\frac{9}{100}$? $\frac{11}{100}$? $\frac{13}{100}$? $\frac{21}{100}$?

3. A farmer lost 15 sheep from a flock of 100. How many hundredths of the flock did he lose?

4. How many hundredths of $1 are 17 cents? 39 cents? 50 cents? ½ of a cent?

5. How many hundredths of anything is ¼ of it? ½ of it?

6. In a catch of 100 fish ⅗ were perch. How many hundredths of the whole were perch?

233. Per cent means *by the hundred.* Thus,

3 per cent means 3 of every 100, or 3 hundredths.

234. The **Sign** % is used for the words *per cent.* Thus,

3 % means 3 per cent, and 4½ % means 4½ per cent.

235. Percentage treats of computing in hundredths.

236. The **Rate** per cent is the number of hundredths.

237. The **Base** is the number of which the hundredths are taken.

238. The **Percentage** of a number is the part of it denoted by the rate per cent.

239. The **Rate** per cent, being a number of hundredths, is a *fraction,* and may be expressed in the form of a decimal or of a common fraction. Thus,

$\frac{1}{2}\% = 0.00\frac{1}{2} = \frac{\frac{1}{2}}{100} = \frac{1}{200}$
$5\% = 0.05 = \frac{5}{100} = \frac{1}{20}$
$12\frac{1}{2}\% = 0.12\frac{1}{2} = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$
$25\% = 0.25 = \frac{25}{100} = \frac{1}{4}$

$62\frac{1}{2}\% = 0.62\frac{1}{2} = \frac{62\frac{1}{2}}{100} = \frac{5}{8}$
$100\% = 1.00 = \frac{100}{100} = 1$
$137\frac{1}{2}\% = 1.37\frac{1}{2} = \frac{137\frac{1}{2}}{100} = \frac{11}{8}$
$225\% = 2.25 = \frac{225}{100} = \frac{9}{4}$.

EXERCISES.

Express decimally:

7. 5 %; 6 %; 7 %.

8. $\frac{3}{4}$ %; $2\frac{1}{2}$ %; $8\frac{1}{3}$ %.

9. 12 %; 125 %; 200 %.

10. $\frac{3}{10}$ %; $\frac{2}{5}$ %; $1\frac{1}{6}$ %; .07 %.

11. 50 %; $\frac{1}{2}$ %; 5 %; .05 %.

12. $\frac{3}{5}$ %; 60 %; 6 %; 600 %.

Express as common fractions in smallest terms:

13. 10 %; $12\frac{1}{2}$ %; 25 %.

14. $33\frac{1}{3}$ %; $37\frac{1}{2}$ %; 50 %.

15. $16\frac{2}{3}$ %; $62\frac{1}{2}$ %; $83\frac{1}{3}$ %.

16. $66\frac{2}{3}$ %; 75 %; 80 %.

17. 125 %; 150 %; 200 %.

18. 40 %; 60 %; $14\frac{2}{7}$ %.

Change the following fractions to hundredths:

19. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$.

20. $\frac{1}{10}$; $\frac{1}{11}$; $\frac{1}{12}$; $\frac{1}{16}$; $\frac{1}{20}$; $\frac{1}{25}$; $\frac{1}{30}$.

21. $\frac{1}{40}$; $\frac{1}{50}$; $\frac{2}{3}$; $\frac{2}{5}$; $\frac{2}{7}$; $\frac{2}{9}$; $\frac{2}{11}$; $\frac{2}{25}$; $\frac{4}{25}$.

22. $\frac{3}{4}$; $\frac{3}{5}$; $\frac{3}{7}$; $\frac{3}{10}$; $\frac{3}{16}$; $\frac{3}{20}$; $\frac{3}{40}$; $\frac{3}{50}$.

23. $\frac{4}{5}$; $\frac{4}{25}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{5}{4}$; $\frac{9}{8}$; $\frac{12}{10}$.

24. $\frac{3}{25}$; $\frac{3}{20}$; $\frac{3}{11}$; $\frac{7}{11}$; $\frac{1}{14}$; $\frac{1}{80}$.

The Base and the Rate given, to find the Percentage.

ORAL EXERCISES.

25. What is 5 % of $4?

Solution.—As 5 % is $\frac{5}{100}$, 5 % of $4 is $\frac{5}{100}$, or $\frac{1}{20}$, of $4, which is $0.20.

26. What is 4 % of $40? 6 % of $44? 3 % of $400?

27. What is 8 % of 200? Of 500? Of 700? Of 600?

28. What is 10 % of 90 yards? $12\frac{1}{2}$ % of 72 miles?

29. What is 20 % of a cubic yard? 5 % of an acre?

30. What is $16\frac{2}{3}$ % of a day? Of a foot? Of a gross?

31. What is $\frac{3}{4}$ % of a ton? 75 % of it? $\frac{3}{4}$ of it?

32. How many square inches in $66\frac{2}{3}$ % of a square foot?

33. What is the difference between $\frac{1}{2}$ of a mile and $\frac{1}{2}$ % of a mile?

240. The **Amount** is the base plus the percentage.

241. The **Difference** is the base less the percentage.

WRITTEN EXERCISES.

34. What is 37½ % of $ 8.24 ?

$\$8.24 \times 0.37\frac{1}{2} = \$3.09$

Or,

$\$\overset{1\cdot 0\,3}{\cancel{8}.\cancel{24}} \times \frac{3}{\cancel{8}} = \$3.09$

Solution. — As 37½ % is 0.37½, or ⅜, 37½ % of $ 8.24 is 0.37½ times $ 8.24, or ⅜ of $ 8.24, which is $ 3.09.

35. A's money is $ 2575. If by trading he should increase it by 40 %, how much would he then have?

36. A's income is $ 1890. If he should spend 83⅓ % of it, how much would he have left?

242. Rule for finding the Percentage.

Multiply the base by the rate per cent.

Let *b* represent the base, *r* the rate per cent, *p* the percentage, *a* the amount, and *d* the difference, we have the

FORMULAS: $p = b \times r, \quad a = b + p, \quad d = b - p.$

37. Find 28 % of 8500 tons.
38. Find 3 % of 6840 gal.
39. Find ⅛ % of 2584 miles.
40. Find 35 % of 3460 men.
41. Find 9¾ % of $ 5000.
42. Find 8 % of $ 645.50.
43. Find 6 % of $ 13.56.
44. Find 62½ % of $ 817.68.

45. Johnson bought a house for $ 4850, and paid down 15 %. How much did he then owe for it?

46. A merchant sold goods which cost him $ 9675.75, at a profit of 16 %. How much did he get for the goods?

47. Bought a bill of goods amounting to $ 186.80, and for cash payment obtained a deduction from it of 5 %. How much was the deduction?

48. How many persons are engaged in agriculture, when they constitute 24 % of a population of 61450 ?

49. How much bank currency could be bought for $ 4500, in coin, in 1864, when gold was at 285 %?

50. A man having $ 8550 bequeathed 33⅓ % to his wife and the remainder to his children. How much did he give his children?

51. A bought goods to the value of $ 345.75, and sold them to B at 15 % advance on his outlay, and B sold them to C at 15 % less than his outlay. How much did C give for them?

Base and Percentage given, to find the Rate per cent.

ORAL EXERCISES.

52. What per cent of 12 is 3?

Solution. — 3 is $\frac{3}{12}$ of 12; and $\frac{3}{12} = \frac{1}{4}$, or 0.25, or 25 %.

53. What per cent of $ 35 is $ 7? Is $ 14? Is $ 28?

54. What per cent of 24 is 6? Is 12? Is 18?

What per cent

55. Of 42 miles is 21 miles?

56. Of 75 tons is 45 tons?

57. Of 48 pounds is 30 pounds?

58. Of $ 6 is $ 4.50?

59. Of $ 64 is $ 40?

60. Of $ 96 is $ 36?

61. If 4 quarts of grain are given for grinding a bushel, what per cent is the cost of grinding?

62. John earns each week $ 7.50. He spends for board $ 2.50, and as much more for other things. What per cent of his earnings are his spendings?

WRITTEN EXERCISES.

63. What per cent of 128 is 16?

$16 \div 128 = 0.12\frac{1}{2}$, or $12\frac{1}{2}$ %

Or,

$\frac{16}{128} = \frac{1}{8} = .12\frac{1}{2}$, or $12\frac{1}{2}$ %

Solution. — 16 is $\frac{16}{128}$ of 128. As $\frac{16}{128} = .12\frac{1}{2}$, or $12\frac{1}{2}$ %, 16 is $12\frac{1}{2}$ % of 128.

64. What per cent of 600 bushels is 57 bushels?

243. Rule for finding the Rate per cent.

Divide the percentage by the base, extending the division to hundredths.

FORMULA. $r = p \div b$.

What per cent is

65. 182.75 of 2150?

66. $8 of $130?

67. 16 bu. of 62½ bu.?

68. $490 of $5000?

69. 600 of 720?

70. 70 tons of 500 tons?

71. $85 of $1700?

72. $57.375 of $765?

73. Of a farm containing 1640 acres there are 246 acres in meadow. What per cent is in meadow?

74. If the income on $1346 is $168.25, what is the rate?

Rate and Percentage given, to find the Base.

ORAL EXERCISES.

75. $25 is 5 % of my money. How much have I?

Solution. — As $25 is 5 % of my money, 1 % of my money is ⅕ of $25, or $5; 100 % of my money must be 100 times $5, or $500. Or, As $25 is 5 %, or $\frac{1}{20}$ of my money, $\frac{20}{20}$ of my money must be 20 times $25, or $500.

76. $6 is 20 % of what number? 40 rods is 12½ % of what number?

77. 1 pound 4 ounces is 25 % of how many pounds?

78. A number increased by 25 % of itself is 60. What is the number?

Solution. —As a number increased by 25 % of itself must be 125 %, or $\frac{5}{4}$ of itself, 60 must be $\frac{5}{4}$ of the number, and $\frac{4}{4}$, or the number, must be 48.

79. Sold a watch for $55, which was 10 % above its cost. What was its cost?

80. 42 is 33⅓ % less than what number? 33⅓ % more than what number?

WRITTEN EXERCISES.

81. $3550 is 25 % of what number?

$$\$\frac{3550}{\cancel{25}} \times \overset{4}{\cancel{100}} = \$14200$$

Or,
$$\begin{array}{r} \$3550 \\ 4 \\ \hline \$14200 \end{array}$$

Solution. — As $3550 is 25 % of an unknown number, 1 % of the unknown number is $\frac{1}{25}$ of $3550, or $\$\frac{3550}{25}$; 100 % of the unknown number must be 100 times $\$\frac{3550}{25}$, or $14200.

Or, as $3550 is 25 %, or $\frac{1}{4}$ of the number, the number must be four times $3550, or $14200.

82. $16.22 is 5 % of what number?

83. Having sold 40 % of my sheep, I have 177 left. What number had I at first?

244. Rule for finding the Base.

Divide the percentage by the rate per cent.

FORMULAS. $b = p \div r$. $b = a \div (1 + r)$. $b = d \div (1 - r)$.

Find the number of which

84. $125 is 8 %.

85. 0.108 of a ton is $\frac{3}{4}$ %.

86. 16 bu. is 24 %.

87. 7.80 yd. is $\frac{3}{4}$ %.

88. 21.6 rd. is $\frac{2}{3}$ %.

89. $14 is $\frac{4}{10}$ %.

90. I burned 1750 lb. of coal, or 25 % of my supply, in a single week. How much had I?

91. The expenses of a charity concert were 40 % of the receipts. The poor received $250. What were the expenses?

92. An agent's salary having been decreased 33⅓ % is now $1600. What was it at first?

93. A man owes $6750, which is 75 % as much as he is worth. How much is he worth?

94. The voters of a certain city number 16386, which is 20 % more than the number 3 years ago. What was the number then?

PROFIT AND LOSS.

95. At a gain of 20 % of the cost of an article, what part of the cost equals the gain?

96. How much is gained by selling goods at 10 % profit when the cost is $ 25?

97. How much is lost on goods which cost $ 40 by selling them at a loss of 12½ %?

98. A grocer buys tea at 45 cents a pound, and sells it at 60 cents a pound. What is the gain per cent?

99. Sold a cow which cost me $ 40 for $ 45. What was the gain per cent?

100. Sold a horse for $ 120, and gained 16⅔ %. What did it cost?

101. Sold goods for $ 108, and lost 10 %. What was their cost?

245. Profit and Loss, as commercial terms, express the gain or loss in business transactions.

246. The *base* of computation is the *cost*, the gain or loss is the *percentage;* and the cases which occur correspond to, and are solved like the preceding cases of percentage.

WRITTEN EXERCISES.

102. How much is made by selling flour at 20 % profit which cost $ 7.75 a barrel?

103. A dealer sold a stock of goods, damaged by fire, at an average of 66⅔ % less than cost. The cost being $ 18240, what was the loss?

104. Of a carload of fruit 25 % is spoiled. I sell the remainder for 16⅔ % advance on its cost. What is my profit or loss on the load if it cost $ 400?

105. A merchant sold a cask of molasses which cost him $ 69.60 at a profit of 15 %. What did he gain?

106. If I pay for goods $ 350.50, and sell them at 6½ % loss, how much shall I lose?

107. Sold a house which cost me $ 3500 at a gain of 8 %. What did I receive for it?

108. Sold a carriage for $ 210 at a loss of 16 %. What was the cost?

109. What per cent was gained by selling property that cost $ 2400 for $ 2550?

110. Bought hams for 8½ cents a pound. What per cent will be gained by selling them at 12 cents a pound?

111. Sold goods for $ 3312.70, and made 5½ %. What was the cost?

112. If I am compelled to lose 12½ % on damaged goods, how should I sell those that cost me $ 560?

113. When the cost of flour is $ 7.50 a barrel, and the expense of selling 6 %, at what price must it be sold to gain 5 %?

114. Of goods worth $ 1600, one fourth is sold at a profit of 15 %. For how much must the remainder be sold to gain 20 % on the whole?

115. Sold a watch for $ 28, and gained 12 %. What per cent would I have gained or lost if I had sold it at $ 24?

116. Bought a cask of molasses containing 120 gallons for $ 50. But, a fifth of the molasses having leaked out, for what must the remainder be sold a gallon to gain 10 % on the purchase?

117. By selling hay at $ 15 a ton I lose 10 %. At what price must I sell it to gain 15 %?

118. A merchant sold goods for $ 150 and lost 10 %, whereas he should have gained 30 % per cent. How much were they sold under their proper value?

119. Bought goods for $ 14500. Half of them I am obliged to sell at a loss of 20 %. If I sell the other half at a gain of 20 %, what shall I gain or lose on the whole?

COMMISSION.

120. How much should be received for selling $ 500 worth of goods if 3 % is allowed?

121. How much must be paid for selling $ 800 worth of goods at 5 %?

122. A collector of $ 700 was paid 2½ % for his services. How much did he receive?

247. Commission, or Brokerage, is the compensation or percentage allowed an agent for transacting business.

The agent may be known as a factor, broker, or commission merchant.

248. The commission is usually a certain per cent of the sum invested or collected by the agent, the investment or the collection being the *base*, and the commission the *percentage*.

WRITTEN EXERCISES.

123. What is the commission on the sale of $ 5678 worth of cotton cloth at 2½ %?

124. A broker negotiated the sale of $ 3500 United States securities at a brokerage of ¼ %. What was the brokerage?

125. A commission merchant sold goods to the amount of $ 7896.50. What was his commission at 2 %?

126. Find the commission on the sale of 368 barrels of flour for $ 6.50 a barrel, the rate of commission being 2½ %.

127. An agent invests $ 5000 for me in the purchase of land. His commission being ¼ %, how much shall I send him to cover all the cost?

128. My agent has sold goods for me amounting to $ 13500. His charges are: commission, 2½ %; guarantee, 2 %; cartage and storage $ 16.50. How much is due me?

129. I sent my agent $ 1500 to invest after deducting his commission of 2½ %. What sum can he invest, and what will be his commission?

Solution. — The remittance includes the investment and the commission. The investment is 100 % of the investment, and the commission is 2½ % of the investment. The remittance, therefore, or $ 1500, must be 102½ % of the investment. $ 1500 is 102½ % of $$\frac{1500}{102\frac{1}{2}} \times 100$, or $ 1463.41+. This, subtracted from the remittance, $ 1500, gives as the commission $ 36.59.

130. A commission merchant receives $ 650 to invest in goods after deducting his commission of 3 %. What will be his commission?

131. I have remitted to an agent $ 1426.80, with instructions to lay it out for me in flour at $ 6.50 a barrel, after deducting his commission of 2½ %. How many barrels can he buy?

132. An agent received a sum of money to lay out after deducting his commission of 2½ %. He laid out $ 1392. What was the sum he received?

133. A commission merchant sold on commission goods for $ 8134.75, and received $ 334.75, which included a charge for cartage, freight, and storage of $ 22.75. What was the rate of commission?

134. A lawyer collected 65% of a note of $ 950, and charged 6¼ % commission. Find his commission.

135. A factor received 5 % for buying wool. His commission amounted to $ 208.50. How much did he pay for the wool?

136. Sent my agent $ 4100 for the purchase of iron after taking out his commission of 2½ %. After he had bought the iron, I changed my business and telegraphed him to sell the iron at cost. He did so, taking a commission of 2½ % on the sale, and sent me the balance. How much did I lose by the transaction?

INSURANCE.

137. What is the cost of securing the payment to me of $1000 in case my house is destroyed by fire, when 2 % is paid to those taking the risk?

138. What will be the annual charge at 1½ % for taking the risk of $ 3000 against loss or damage on merchandise?

249. **Insurance** is security against loss.

250. The **Premium** is the sum paid for insurance.

251. The **Policy** is the writing containing the contract of the insurer with the insured.

252. The sum insured is the *base*, and the premium is the *percentage*.

WRITTEN EXERCISES.

139. What is the cost of insuring $3600 on a house for 5 years at 2%, and $1 for the policy?

140. What is the premium for insuring $5545 on merchandise for one year at 2¼ %?

141. What is the amount paid for insurance on ¾ of a ship valued at $ 68000 at 3 %, and $ 1 for the policy?

142. A factory is insured for $55000 at 2½ %. If the property should be burned, what loss would the insurer actually sustain?

143. A house was insured for ¾ of its value at 1½%, and the premium was $ 27. What was the value of the house?

144. Paid $73, including $1 for policy, for the insurance of $3600 in a house. What was the rate of insurance?

145. A man 44 years of age takes out a life-policy for $15000 for the benefit of his wife, at the yearly rate of $26.50 per $1000. Should his death occur at the age of 74, how much more would his widow receive than had been paid in yearly premiums?

MISCELLANEOUS EXERCISES.

146. The sum of ⅛%, 5%, 24% and 55% of a number is 60.25. What is the number?

147. An article is composed of 37 parts of pure silver and 3 parts of copper. What per cent of the whole is each of the components?

148. A clerk receiving a salary of $ 950 pays $ 275 a year for board, $ 180 for clothing, and $ 150 for other expenses. What per cent of his salary is left?

149. I bought 150 apples at 2 for a cent, and 150 at 3 for a cent. I sold them all at 5 for 2 cents. How much did I gain or lose?

150. A man bought 8 books at the rate of $ 10 a dozen, and sold them for $ 1.75 each. What per cent was gained?

151. Bought a bill of goods amounting to $ 1540, but a discount of 25% was allowed with 5% off for cash. How much did I pay?

152. If I should forward $ 603.75 to a broker in St. Louis for the purchase of flour, his brokerage being 5%, how many barrels of flour should he return at $ 5 a barrel?

153. Lost $ 17.25 by selling a watch 15% below cost. What was the cost?

154. For what sum must a store and its goods, valued at $ 25640, be insured so as, in case of its destruction, to recover the entire value of the building and goods and the premium of 2%?

155. If I purchase 15 pieces of cloth, of 35 yards each, at $ 4.25 a yard, for how much a yard must I sell the whole so as to gain 25%?

156. A man bought 80 tons of coal at $ 5 a ton, 10% of which went down in a boat and was lost. For how much a ton must the remainder be sold that he may lose nothing?

157. A man bought 4500 bushels of wheat at $ 1.20 a bushel. He sold 10% of it at 3% loss, 50% of it at 10% gain, and the *remainder* at 5% gain. What was gained by the entire *transaction?*

158. If you should sell a house for $ 6000, and lose 33⅓ %, for what should you sell another at the advance of 32 % for just enough to cover the loss upon the first house ?

159. In 1875 there were 104513 illiterate persons in Massachusetts out of a population of 1652000. What was the rate per cent of illiteracy ?

160. I receive a remittance of $ 13195 to be spent, after paying commission of 1½ %, in the purchase of coal. Required my commission.

161. My house cost me $ 12000. It is insured for ¾ of its value at ⅜ % premium. What is my actual loss in case it burns ?

162. The premium on an insurance of $ 930 was $ 23.25. What was the rate ?

163. An agent collects money at 2½ %, and pays his principal $ 4387.50. What was the amount of the collection ?

164. Leonard & Co. sell a lot of goods for me at auction to the amount of $ 11500. Their charges are as follows: commission, 2½ %; guarantee, 2½ %; advertising, $ 35; labor and storage, $ 17.25. How much is due me ?

165. Sold property for $ 1400, 25 % of which is gain. I found myself, however, able to collect only 90 % of the proceeds of the sale. What was my actual gain % ?

166. Giles lost 8⅘ % of his money in speculation, but had $ 920 left. How much had he at first ?

167. My room is 24 feet long; its width is 50 % of its length; how many yards of carpeting, ¾ yd. wide, will it require ?

168. A grocer after losing 11 % of his apples has 133.5 barrels left; if they cost him $ 2.50 per barrel, for what must they be sold that he may lose nothing on his purchase ?

169. My agent sells 500 barrels of flour at $ 10 per barrel and remits me $ 4750. What rate of commission did he charge for selling ?

170. My store is insured for $ 8000 at 1½ % premium, and my stock for $ 15000 at ⅜ %. If both are entirely consumed, what is the underwriters' loss ?

171. The difference between 24 % and 55 % of a number is 60.45. What is the number ?

172. Bought a range of wood 20 ft. long, 12 ft. high, and 4 ft. wide, at $ 5 per cord, and sold the whole for $ 50. Required the per cent of gain.

173. Paid $ 30 for my winter's wood, which was to have been 4 feet in length. It averaged, however, but 44 inches. Out of how much money was I cheated ?

174. In a school of 400 scholars there were 120 absences in 4 weeks; the school has 2 sessions 5 days in a week. What was the per cent of attendance ?

175. How must I mark cloth which cost $ 2.50 so as to gain 20 % and still fall 25 % from my marked price ?

176. A cubic foot of water weighs 62⅜ pounds and a cubic foot of ice 57½ pounds. Ice is what per cent lighter than an equal bulk of water ?

177. The population of Chicago in 1880 was 503620, an increase of 69 % in ten years; what was the city's population in 1870 ?

QUESTIONS.

233. What does per cent mean ? 235. Of what does percentage treat ? 236. What is the rate per cent ? 237. The base ? 238. The percentage of a number ?

239. How may the rate per cent be expressed ? 240. What is the amount ? 241. The difference ?

242. The base and rate being given, how is the percentage found ? 243. The base and percentage being given, how is the rate found ? 244. The rate and percentage being given, how is the base found ?

245. Define profit and loss. 246. What is the base of computation ? 247. What is commission, or brokerage ? 248. What is the base ? The percentage ?

249. What is insurance ? 250. The premium ? 251. The policy ?

INTEREST.

253. 1. When money is loaned for a year at 7%, what part of the money is the per cent?

2. How much must be paid for the use of $15 for 1 year at 5%? At 6%?

3. How much must be paid for the use of $20 at 7% for 1 year? For 2 years?

4. When $200 is borrowed for 2 years at 7% a year, what amount should the borrower pay at the end of that time?

254. **Interest** is the money paid for the use of money.

255. The **Principal** is the money for whose use interest is paid.

256. The **Amount** is the sum of the principal and the interest.

257. The **Rate** of interest is the number of hundredths of the principal taken as the interest for one year or other specified time.

NOTE 1.—The rate for one year and at 6% is to be understood in this book when no other time or rate is specified.

NOTE 2.—The rate of interest is regulated by law. The legal rates in the different States may be found in a table in the Appendix.

SIMPLE INTEREST.

258. **Simple Interest** is interest on the principal alone.

Interest is an application of percentage, the principal being the *base*, the annual rate multiplied by the time in years being the *rate per cent*, and the interest the *percentage*.

259. In the computation of interest it is customary to consider a year as consisting of 12 months of 30 days each.

General Method.

ORAL EXERCISES.

5. What is the interest of $ 50 for 1 year at 4 %?

Solution.—At 4 % 1 year's interest is .04 of the principal, and .04 of $ 50 is $ 2.

6. What is the interest of $ 60 for 1 year at 5 %?

7. What is the interest of $ 200 for 1 year at 6 %? For 3 years? For 5 years?

8. What is the interest of $ 200 for 2 years 6 months at 7 %?

Solution. — As at 7 % the interest of $ 200 for 1 year is $ 14, for 2 years 6 months, or 2½ years, it must be 2½ times $ 14, or $ 35.

9. What is the interest of $ 100 for 2 years 3 months at 8 %? For 3 years 1 month?

10. What is the amount of $ 400 for 2 years 9 months at 6 %? For 3 years 4 months?

WRITTEN EXERCISES.

11. What is the interest and what is the amount of $ 26.25 for 2 years 4 months at 7 %?

$$\begin{array}{r l}
\$\,26.25 & = \text{Principal} \\
.07 & = \text{Rate} \\
\hline
\$\,1.8375 & = \text{1 year's interest} \\
2\tfrac{1}{3} & \\
\hline
6125 & \\
36750 & \\
\hline
\$\,4.2875 & = \text{Interest} \\
26.25 & = \text{Principal} \\
\hline
\$\,30.54 & = \text{Amount}
\end{array}$$

Solution.— One year's interest is .07 of $ 26.25, or $ 1.8375; 2⅓ years' interest is 2⅓ × $ 1.8375, or, to the nearest cent, $ 4.29. Adding the principal to the interest, we have the amount, $ 30.54.

12. What is the interest and what is the amount of $ 1728 for 3 years 9 months at 6 %?

13. What is the interest of $144 for 1 year 8 months at % ?

14. What is the interest of $556 for 3 years 5 months days at 8 %?

$$\begin{array}{r}
\$\,556 \\
.08 \\ \hline
\$\,44.48 \\
41.2\frac{1}{3} \\ \hline
1482\frac{2}{3} \\
8896 \\
4448 \\
17792 \\ \hline
12)\ \$\,1834.058\frac{2}{3} \\ \hline
\$\,152.838+
\end{array}$$

Solution. — One year's interest is .08 of $556, or $44.48; the interest for 3 years 5 months 7 days, or 41.2⅓ months, or $\frac{41.2\frac{1}{3}}{12}$ years, is $\frac{41.2\frac{1}{3}}{12}$ times $44.48, or, to the nearest cent, $152.84.

15. What is the interest of $720 at 5 % for 1 y. 7 mo. 18 d.?

$$\begin{array}{r}
\$\,720 \\
.05 \\ \hline
12)\ \$\,36.00 \\ \hline
\$\,3.00 \\
19.6 \\ \hline
\$\,58.80
\end{array}$$

Solution. — One year's interest is .05 of $720, or $36; the interest for 1 y. 7 mo. 18 d., or 19.6 mo., or $\frac{19.6}{12}$ years, is $\frac{19.6}{12} \times \$36$, or $58.80. In this example, 1 year's interest being a multiple of 12, we divide by the denominator of the multiplier before multiplying by the numerator.

16. Find the amount of $1500 for 2 y. 6 mo. 15 d. at 6 %.

260. General Rule for Interest.

Multiply the principal by the rate, and this product by the time in years.

To find the amount, add the principal and the interest.

Let p represent the principal, r the rate per cent, i the interest, t the time, and a the amount, and we have the

FORMULAS. $i = p \times r \times t.$ $a = p + i.$

NOTE. — In interest *partial results* may be carried to four places of decimals. The *answers*, in business transactions, are deemed sufficiently exact if the mills are omitted, and when they are five or more, the cents are increased by 1.

17. What is the interest of $2464 for 2 y. 9 mo. 15 d. at 5 %?

18. What is the interest of $2503.75 for 3 y. 10 mo. 21 d. at 6 %?

19. What is the interest of $560.50 for 4 y. 10 d. at 7 %?

20. What is the interest of $ 97 16 for 1 y. 5 mo. at 6 %?

21. What is the interest of $156.80 for 3 y. 1 mo. 3 d. at 4 %?

22. What is the interest of $865 for 1 y. 9 mo. 24 d. at 8 %?

23. What is the interest of $890 for 5 y. 7 mo. 8 d. at 6 %?

24. What is the amount of $5000 for 3 y. 11 mo. 10 d. at 7 %?

Six per cent Method.

261. The **Interest** of any sum, at 6 per cent a year,

For 12 months, or 1 year, is .06 of the principal.
For 2 months, or $\frac{1}{6}$ year, is .01 of the principal.
For 1 month, or 30 days, is .00$\frac{1}{2}$ of the principal.
For $\frac{1}{5}$ month, or 6 days, is .001 of the principal.
For $\frac{1}{30}$ month, or 1 day, is .000$\frac{1}{6}$ of the principal.

262. Hence, as a convenient method of reckoning interest at 6 per cent,

Multiply $\frac{1}{2}$ *of* .01 *of the principal by the time in months.* Or,

Multiply .001 *of the principal by* $\frac{1}{6}$ *of the time in days.* Or,

Of the principal take 6 *times as many hundredths as years,* $\frac{1}{2}$ *as many hundredths as months, and* $\frac{1}{6}$ *as many thousandths as days.*

25. What is the interest of $926 for 3 years 11 months 15 days at 6%?

2) $926. = Principal
$4.63 = 1 mo.'s interest
47½ = Time in months
231½
3241
1852
$219.92½ = Required interest

Solution. — Two months' interest is .01 of the principal; 1 month's interest is ½ of .01 of the principal, $926, or $4.63; the interest for 3 y. 11 mo. 15 d., or 47½ mo., is 47½ × $4.63, or $219.92½.

26. What is the interest of $340 for 103 days at 6%?

$340 = Principal
$0.34 = Six days' interest
17⅙ = ⅙ time in days
5⅔
238
34
$5.83⅔ = Required interest

Solution. — Six days' interest is .001 of the principal, or $0.34; one day's interest is ⅙ of $0.34, and 103 days' interest is 103 × ⅙ of $0.34, or $\frac{103}{6}$ of $0.34, or 17⅙ × $0.34, or $5.84, to the nearest cent.

27. What is the interest of $1650 for 2 y. 7 mo. 18 d. at 6%?

Int. for 2 y. = .12 of principal.
" 7 mo. = .035 "
" 18 d. = .003 "
.158 "

$1650
.158
13200
8250
1650
$260.700

Solution. — Taking .06 for each year's interest, .00½ for each month's interest, and .000⅙ for each day's interest, we find that the interest for the given time is .158 of the principal, or .158 of $1650, or $260.70.

What is the interest at 6% of

28. $56.80 for 1 y. 8 mo. 17 d.?
29. $6000 for 4 y. 2 mo.?
30. $17.28 for 1 y. 11 mo. 3 d.?

31. $ 1850.75 for 9 mo. 24 d.?

32. $ 253.50 for 2 y. 4 mo. 7 d.?

33. $ 85.90 for 3 y. 6 mo. 27 d.?

34. $ 1992.25 for 93 days?

35. $ 15600 for 4 y. 7 mo. 19 d.?

36. What is the amount of $ 1400 for 2 years 6 months?

37. What is the amount of $ 7000 for 5 years 3 months?

263. For any other rate than 6 per cent we may, when more convenient than to apply the general rule (Art. 260),

Find the interest at 6 *per cent, and increase or diminish this interest by such part of itself as will give the interest at the required rate.*

That is, to find 4 % interest subtract from the 6 % interest ⅓ of itself; 4½ % interest, subtract from the 6 % interest ¼ of itself; 5 % interest, subtract from the 6 % interest ⅙ of itself; 7 % interest, add to the 6 % interest ⅙ of itself; and so on

38. What is the interest of $ 545 for 8 mo. 24 d. at 4 %?

39. What is the interest of $ 78.50 for 123 days at 5 %?

40. What is the interest of $ 64.70 for 2 y. 5 mo. at 7 %?

41. What is the interest of $ 1440 for 11 mo. 23 d. at 4½ %?

42. What is the interest of $ 9500 for 3 y. 6 mo. 17 d. at 7 %?

43. What is the interest of $ 600.80 for 2 y. 11 mo. 3 d. at 8 %?

44. What is the interest of $ 20000 for 63 days at 5 %?

45. What is the interest of $ 340.90 for 4 y. 7 mo. 11 d. at 7 %?

46. What is the interest of $ 15420 for 9 mo. 24 d. at 6½ %?

47. What is the interest of $ 374.75 for 3 y. 9 mo. at 8 %?

48. Ernest Williams borrowed, April 5, 1881, $525 at 7% interest, and kept it until May 16, 1882. What was the amount?

49. A note for $450.60, dated March 5, 1880, was paid Dec. 31, 1881, with interest at 7%. What was the amount?

Short Method.

264. The following method of computing 6% interest is often very convenient, especially for short times.

Find 60 days' interest by taking .01 of the principal. Then take such multiples or parts of this interest as the given time may require.

50. What is the interest of $2460 for 3 mo. 18 d.?

Solution.

Time, 108 d. $2460 = Principal.
Int. for 60 " = $24.60, or .01 of principal.
" 30 " = 12.30, or ½ of 60 days' int.
" 15 " = 6.15, or ½ of 30 " "
" 3 " = 1.23, or ⅒ of 30 " "
" 3 mo. 18 d. = $44.28, or the sum of the above.

51. What is the interest of $480 for 84 days at 7½%?

Solution.

Time, 84 days $480 = Principal.
Int. for 60 " = $4.80, or .01 of principal.
" 20 " = 1.60, or ⅓ of 60 days' int.
" 4 " = .32, or ⅕ of 20 " "
" 84 days at 6% = $6.72, or the sum of the above.
Int. at 1½% = 1.68, or ¼ of 6% int.
Int. at 7½% = $8.40, or $6.72 + $1.68.

52. What is the interest of $66.42 for 4 mo. 12 d. at 5%?

53. What is the interest of $800 for 3 mo. 3 d. at 7%?

54. What is the interest of $130.50 for 45 days at 8%?

55. What is the interest of $7500 for 2 mo. 21 d. at 6%?

56. What is the interest of $225 from Feb. 3, 1880, to May 9, 1881, at 6%?

57. What is the amount of $163.20 from Dec. 12, 1881, to March 27, 1882, at 10%?

58. What is the amount of $900.65 from Sept. 16, 1881, to Nov. 8, 1882?

59. What is the interest of $4000 from May 12, 1879, to June 24, 1880, at 4½%?

60. What is the amount of $653.63 from Feb. 11, 1880, to Nov. 9, 1882, at 7%?

61. A bill of goods amounting to $4498.25 was paid at the end of 60 days, with interest at 5%. What was the amount?

By any of the preceding methods find the interest

	Of	For	At
62.	$248	6 mo. 18 d.	3½%.
63.	$845	13 y. 2 mo. 13 d.	4%.
64.	$245.80	2 y. 5 mo. 7 d.	4½%.
65.	$960	3 y. 7 mo. 9 d.	5%.
66.	$849.50	8 y. 4 mo. 12 d.	6%.
67.	$2846	3 y. 5 mo. 10 d.	6½%.
68.	$180	1 y. 9 mo. 15 d.	7%.
69.	$948.39	3 y. 11 mo. 6 d.	7½%.
70.	$862	4 y. 7 mo. 22 d.	8%.
71.	$1500	1 y. 3 mo. 27 d.	9%.
72.	$8400	2 mo. 17 d.	7.3%.
73.	$9398	1 mo. 18 d.	10%.
74.	$479.85	106 days	5%.
75.	$948.25	89 days	4½%.
76.	$84.32	45 days	4%.
77.	$961.18	111 days	6%.

Find the amount

	Of	From	To	At
78.	$ 549.82	Dec. 14, '80	May 5, '81	9 %.
79.	$ 856.84	Aug. 17, '81	Apr. 4, '82	8 %.
80.	$ 1248	Jan. 24, '81	Mar. 31, '81	6 %.
81.	$ 960.50	Mar. 5, '81	Sept. 9, '82	4½ %.
82.	$ 849.25	May 5, '81	Aug. 11, '81	5 %.
83.	$ 562.75	Aug. 15, '80	Dec. 29, '82	7 %.
84.	$ 476.84	Sept. 30, '81	May 5, '82	7½ %.
85.	$ 942	Aug. 31, '81	Dec. 30, '81	3½ %.
86.	$ 1728	Jan. 16, '80	Oct. 11, '81	8 %.
87.	$ 945.96	June 4, '81	Sept. 10, '81	9 %.
88.	$ 200	May 9, '82	Aug. 5, '82	10 %.
89.	$ 816.42	Nov. 4, '80	May 1, '82	7 %.
90.	$ 945.55	May 19, '80	Oct. 19, '84	7 %.
91.	$ 624.87	Sept. 4, '81	Dec. 15, '81	5 %.

EXACT INTEREST.

265. In the computation of *Exact Interest,* as by the United States on its securities, for parts of a year, the actual number of days in each calendar month included in the time is counted, and each day's interest made a 365th of a year's interest. That is, to compute exact interest,

Multiply the interest of the principal for one year at the given rate by the exact number of days in the time, and divide by 365.

92. What is the interest on a United States Treasury certificate for $ 500 from April 1, 1881, to July 15, 1881, at 4 %?

$$\$\,500 \times .04 = \$\,20$$

$$\$\,\frac{20 \times \overset{21}{\cancel{105}}}{\underset{73}{\cancel{365}}} = \$\,5.75$$

Solution. — The exact time from April 1, 1881, to July 15, 1881, is 105 days. The interest of $ 500 for 1 year at 4 % is $ 20; and the interest for 105 days must be $\frac{105}{365}$ of $ 20, which is $ 5.75$\frac{25}{73}$, or, to the nearest cent, $ 5.75.

93. What is the exact interest on a note for $ 3000 from Feb. 15, 1880, to June 5, 1880, at 5%?

94. What is the exact interest on a $ 1000 bond from Nov. 1, 1881, to March 15, 1882, at 4½%?

95. A note for $ 225.50 was given March 16, 1881, bearing exact interest from date at 6%. What sum should discharge the note Jan. 13, 1882?

PROBLEMS IN INTEREST.

Principal, Interest, and Time given, to find the Rate.

96. At what rate must $ 450 be on interest to yield $ 81 in 3 years?

$$\$\,450 \times .01 \times 3 = \$\,13.50$$

$$\$\,81 \div \$\,13.50 = 6$$

Solution. — As the interest of $ 450 for 3 years at 1% is $ 13.50, $ 81 is the interest at as many per cent as there are times $ 13.50 in $ 81, or 6.

97. The interest of $ 250 for 1 year 3 months is $ 28.12½. What is the rate per cent?

266. Rule to find the Rate of Interest.

Divide the given interest by the interest of the principal for the given time at 1 *per cent, and the quotient will be the rate.*

Representing the principal by p, the interest by i, the time by t, and the rate by r, as in Art. 260, we have

FORMULA. $r = i \div (p \times t)$.

98. If $ 1400 yields $ 126 in 1 year 6 months, what is the rate?

99. At what rate must $ 1000 be on interest to yield $ 282 in 4 years 8 months 12 days?

100. At what rate must $ 416 be on interest to yield $ 88.64 in 3 years 16 days ?

101. At what rate must $ 1600 be on interest to yield $ 46.20 in 66 days ?

102. At what rate will $ 241.20 amount to $ 250.58 in 6 months 20 days ?

103. At what rate will $ 480 yield $ 52.20 in 2 y. 5 mo. ?

104. What is the rate of interest if $ 640 gains $ 10.56 from August 12 to October 18 ?

105. In 1 y. 3 mo. 15 d. $ 960 amounts to $ 1084. What is the rate ?

106. At what rate will $ 444 gain $ 156.695 in 6 y. 5 mo. ?

107. At what rate must I invest a trust fund of $ 25000 to secure a semi-annual income of $ 500 ?

Principal, Interest, and Rate given, to find the Time.

108. In what time will $ 450 yield $ 94 50 at 6 % ?

$ 450 × .06 = $ 27
$ 94.50 ÷ $ 27 = 3½
3½ y. = 3 y. 6 mo.

Solution. — As $ 27 is the interest of $ 450 for 1 year at 6 %, $ 94.50 is the interest for as many years as there are times $ 27 in $ 94.50, or, 3½ years, equal to 3 years 6 months.

109. The interest of $ 140 is $ 49 at 7 %. How long has it been on interest ?

267. Rule to find the Time.

Divide the given interest by the interest of the principal for one year, and the quotient will be the time.

FORMULA. $t = i \div (p \times r)$.

110. How long must $ 98 be on interest to gain $ 23.48 at 8 % ?

111. How long must $ 75 be on interest to gain $ 6.25 at 6 % ?

112. How long must $3600 be on interest to gain $46.20 at 7 %?

113. How many days must $875 bear interest to gain $7 at 6 %?

114. How long must $9080 be on interest to gain $794.50 at 3½ %?

115. In what time will $750, on interest at 6 %, gain $750, or double itself?

116. I deposited $540 in a bank paying 4 % simple interest until it amounted to $700. How long did it remain?

117. Principal, $892; rate, 10 %; interest, $187. Required the time.

118. In what time will $12000 yield $2500 at 4½ %?

119. In what time will a trust fund of $4500 amount to $6000 at 3½ %?

Interest, or Amount, Time, and Rate given, to find the Principal.

120. What principal at 6 % will gain $94.50 in 3 years 6 months?

$$\$1 \times .06 \times 3\tfrac{1}{2} = \$0.21$$
$$\$94.50 \div \$0.21 = 450$$

Solution. — As the interest of $1 at 6 % for 3 years 6 months is $0.21, $94.50 must be the interest of as many dollars as $0.21 is contained times in $94.50, or $450.

121. What sum of money at 7 % interest will amount to $320 in 4 years?

$$\$1 \times .07 \times 4 = \$0.28$$
$$\$1 + \$0.28 = \$1.28$$
$$\$320 \div \$1.28 = 250$$

Solution. — As the amount of $1 at 7 % for 4 years is $1.28, $320 must be the amount of as many dollars as $320 is times $1.28, or $250.

122. What principal on interest at 6 % will gain $6.25 in 1 year 4 months 20 days?

268. Rule to find the Principal.

If the interest is given, divide it by the interest of $ 1 *at the given rate for the given time, and the quotient will be the principal.*

Or,

If the amount is given, divide it by the amount of $ 1 *at the given rate for the given time, and the quotient will be the principal.*

FORMULAS. $p = i \div (r \times t)$; $p = a \div (1 + r \times t)$.

123. What principal at 7% interest will gain $ 46.20 in 66 days?

124. What sum at 4% interest will amount to $ 318 in 1 year 6 months?

125. What sum at 5% interest will amount to $ 734.20 in 5 months 10 days?

126. What principal at 6% interest will give a quarterly income of $ 210?

127. How large a sum in the savings-bank at 5% interest will give a yearly income of $ 1200?

128. The interest on a note for 2 y. 6 mo. at 7% was $ 118.23. What was the face of the note?

129. What sum must be invested in stock paying 3¼ % semi-annually to yield $ 924 per year?

130. How much must I invest in U. S. 4% bonds to pay the college expenses of my son, $ 560 per year, with the income?

131. A gentleman owns stock in a manufactory which pays annually 9%. He receives quarterly $ 324. What sum has he invested?

132. Mr. V. is said to have an income of $ 5400 per day. If his income from railroad stock, paying 8%, is equal to his income from government securities, paying 4%, what is he worth?

PARTIAL PAYMENTS.

269. A Promissory Note.

\$872$\frac{75}{100}$. *Boston, Dec. 11, 1881.*

One year after date I promise to pay Noble Towne, or order, eight hundred seventy-two $\frac{75}{100}$ dollars.

Value received.

Charles S. Davis.

270. A **Promissory Note** is a written promise to pay absolutely a specified sum of money for value received.

271. The **Promisor,** or **Maker,** of the note is the person who makes the promise to pay.

272. The **Promisee,** or **Payee,** is the person to whom the maker of the note promises to pay the money.

273. The **Face** of a note is the sum named in it.

274. A **Negotiable Note** is one payable to the bearer, or to the payee's order.

275. The **Indorser** of a note is the person who writes his name on its back as security for the payment of the note.

276. The **Holder** of a note is the person who owns it.

277. **Partial Payments** are payments in part of a note or debt.

278. **Indorsements** are records of the partial payments with their dates made on the back of the note.

279. A note *matures,* or is legally payable, on the third day after the time named in the note has expired.

280. A note draws interest from maturity at the legal rate, unless it contains the words "with interest." In such case interest accrues from the date of the note. Thus,

The preceding note draws 6 % interest from Dec. 14, 1882, and the following note draws 7 % interest from Dec. 28, 1881.

281. Form of a Demand Note.

$500\frac{75}{100}$. *Albany, Dec. 28, 1881.*

On demand we promise to pay Joseph Garland, or order, five hundred $\frac{75}{100}$ dollars, with interest at 7 %.

Value received. *A. Manly & Co.*

282. The Supreme Court of the United States has adopted for finding the amount due on a note on which partial payments have been made the following, called

The United States Rule.

Find the amount of the principal to the time when the payment, or the sum of the payments, equals or exceeds the interest due. Then subtract such payment or payments from the amount, and, with the remainder as a new principal, proceed as before to the time of settlement.

133.

$ $304\frac{84}{100}$. PROVIDENCE, Dec. 8, 1876.

On demand, I promise to pay J. B. Anthony, or order, three hundred four $\frac{84}{100}$ dollars, with interest at 6 %. Value received.

WILLIAM C. THOMAS.

INDORSEMENTS: Sept. 25, 1877, $ 60; July 4, 1878, $ 90; **Aug. 1, 1879, $ 10; Dec. 2, 1879, $ 100. What was due Jan. 7, 1881?**

Solution.

Principal		$ 304.84
Int. from Dec. 8, 1876, to Sept. 25, 1877, 9 mo. 17 d.		14.58
Amount		$ 319.42
1st payment		60.00
New principal		$ 259.42
Int. from Sept. 25, 1877, to July 4, 1878, 9 mo. 9 d.		12.06
Amount		$ 271.48
2d payment		90.00
New principal		$ 181.48
Int. from July 4, 1878, to Aug. 1, 1879, 12 mo. 28 d.		11.74
Int. from Aug. 1, 1879, to Dec. 2, 1879, 4 mo. 1 d.		3.66
Amount		$ 196.88
3d payment, less than int. due	$ 10	
4th payment	100	
		110.00
New principal		$ 86.88
Int. from Dec. 2, 1879, to Jan. 7, 1881, 13 mo. 5 d.		5.72
Amount due Jan. 7, 1881		$ 92.60

134.

$ 600. SPRINGFIELD, Jan. 6, 1880.

For value received, I promise to pay James Dennis & Co., or order, six hundred dollars, on demand, with interest at 7 per cent. BENJAMIN POOL, JR.

INDORSEMENTS: April 6, 1880, $ 50; Nov. 21, 1880, $ 60.50; March 31, 1881, $ 150. What was due June 30, 1881?

135. A note for $ 750, dated Oct. 12, 1880, had two indorsements, — Dec. 27, 1880, $ 325; Aug. 7, 1881, $ 25. What was due July 1, 1882, at 6 % ?

136.

$ 1500. NEW ORLEANS, March 10, 1880.

Six months after date, we jointly and severally promise to pay John Hyde fifteen hundred dollars, with interest at 5 per cent. Value received.

JOSEPH RAYMOND.
LOUIS BERNARDIN.

INDORSEMENTS: Nov. 25, 1880, $ 45; July 20, 1881, $ 500; Jan. 30, 1882, $ 600. What was due May 15, 1882?

137. May 16, 1881, I gave my note, on demand, with interest at 7 %, for $ 563.50; Sept. 26, 1881, I paid $ 250. What was due May 16, 1882?

138. You borrow, Feb. 9, 1880, of Charles E. Lowe, $ 3000 by note, with interest at 5 %, and pay $ 1000 March 9, 1881, and $ 800 Nov. 24, 1881; write the note and the indorsements on it in proper form, and find the balance due Mr. Lowe Jan. 3, 1882.

139. On a note for $ 1200, dated Aug. 7, 1880, drawing 4 % interest, there were paid, May 13, 1881, $ 300; Nov. 23, 1882, $ 275. What was due Jan. 1, 1883?

140. What is due May 15, 1881, on a $ 6000 note drawing 7½ % interest, on which $ 400 was paid Aug. 11, 1879, $ 700 Dec. 15, 1880, the note being dated Jan. 1, 1879?

141. A note for $ 500, given Jan. 1, 1879, at 10 % interest, has on it two indorsements of $ 100 each, paid on the first day of each year. What was due June 19, 1881?

142. The face of a note is $ 2400; its date, Aug. 12, 1880; rate of interest, 4½ %; indorsements: Sept. 12, 1881, $ 25; Oct. 12, 1881, $ 700; settled, Feb. 15, 1882. What was due?

143. What sum will discharge a note Nov. 10, 1881, for $ 1728 at 9 %, dated Nov. 23, 1878, which is indorsed as follows: May 15, 1879, $ 248; Aug. 28, 1880, $ 301; May 30, 1881, $ 300?

283. Business men, when settlement is made within a year after interest begins, often make use of the following, called

The Merchants' Rule.

Find the amount of the note or debt from the time of its beginning to draw interest to the time of settlement; also, the amount of each payment from its date to the settlement; and then subtract the sum of the amounts of the payments from the amount of the note or debt.

144.

$850. PHILADELPHIA, Jan. 2, 1882.

For value received, I promise to pay John S. Moreland, or bearer, eight hundred fifty dollars, on demand, with interest at 6 per cent.

ARTHUR AYER.

INDORSEMENTS: March 18, 1882, $200; May 2, 1882, $150; Aug. 18, 1882, $300. What was due Dec. 2, 1882?

Solution.

Amount of $850 for 11 mo.		$896.75
" $200 for 8 mo. 14 d.	$208.47	
" $150 for 7 mo.	155.25	
" $300 for 3 mo. 14 d.	305.20	$668.92
		$227.83

145.

1164\frac{50}{100}$. ST. LOUIS, July 6, 1881.

For value received, I promise to pay to the order of Simeon H. Wright eleven hundred sixty-four $\frac{50}{100}$ dollars, with interest at 7%.

ALFRED SHAW.

INDORSEMENTS: Sept. 21, 1881, $250; Nov. 22, 1881, $315; March 6, 1882, $100; May 17, 1882, $200. What was due on settlement, July 6, 1882?

COMPOUND INTEREST.

146. Alfred Nickerson deposits in a savings-bank $600, with the understanding that at the end of every 6 months he is to receive interest on his deposit at the yearly rate of 4%. How much interest is due him at the end of 6 months?

147. If he does not choose to draw this interest, it will be placed to his credit with his original deposit. Of what sum should he receive the interest for the next 6 months?

148. How much interest will be due him from the bank for the second 6 months?

149. Suppose that he allows all his money to remain in the bank, on how much will he receive interest for the third period of 6 months?

284. Compound Interest is the interest on both the principal and its unpaid interest added to it at stated intervals.

285. Interest may become due, and made a part of the principal, or compounded, according to agreement, at the end of each year, half-year, or quarter, or any other period of time.

150. What is the compound interest and the amount of $500 for 2 y. 7 mo. 12 d. at 6%?

Solution.

Principal for 1st year	$500.00
Interest " "	30.00
Principal for 2d year	$530.00
Interest " "	31.80
Principal for 7 mo. 12 d.	$561.80
Interest " " "	20.79
Compound amount for 2 y. 7 mo. 12 d.	$582.59
Given principal	500.00
Compound interest for 2 y. 7 mo. 12 d.	$82.59

151. What is the compound interest of $750 for 4 years at 5 %?

286. Rule for Compound Interest.

Find the amount of the given principal for the first period of time. Using this amount as a principal, find its amount for the second period, and so on for the entire time. The last amount, less the given principal, will be the compound interest.

NOTE. — When the interest is compounded half-yearly, the rate must be considered one half the yearly rate, and when quarterly, one fourth the yearly rate. Interest compounds annually if nothing is said to the contrary.

152. What is the compound interest of $600 for 3 years 6 months at 5 %?

153. What is the compound interest of $320 for 2 y. 9 mo. at 7 %?

154. What is the compound interest of $500 for 4 y. 4 mo. 15 d. at 4 %?

155. What is the amount of $1000 for 2 years at 6 %, compounded half-yearly?

156. What is the amount of $1200 for 1 y. 6 mo. at 4 %, compounded quarterly?

157. Willard Aldrich deposits $200 in a savings-bank paying 4 % interest, compounding half-yearly. He withdraws his money after three dividends have been declared. How much has he?

158. Charles Underhill borrows $2000 for 1 y. 8 mo. 24 d., paying simple interest at 6 %. He lends it immediately at 7 % compound interest. What does he gain?

159. What is the compound interest of $300 for 4 y. 8 mo. 12 d. at 8 %, interest compounding semi-annually?

287. The computation of compound interest may be abridged by means of the following

TABLE

SHOWING THE AMOUNT OF $1 AT COMPOUND INTEREST, FROM 1 TO 20 YEARS, AT 1½, 2, 2½, 3, 3½, 4, 5, 6, 7, 8, 9, AND 10 PER CENT.

Yrs.	1½ per cent.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.
1	1.015000	1.020000	1.025000	1.030000	1.035000	1.040000
2	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600
3	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864
4	1.061363	1.082432	1.103813	1.125509	1.147523	1.169859
5	1.077283	1.104081	1.131408	1.159274	1.187686	1.216653
6	1.093442	1.126162	1.159693	1.194052	1.229255	1.265319
7	1.109843	1.148686	1.188686	1.229874	1.272279	1.315932
8	1.126491	1.171660	1.218403	1.266770	1.316809	1.368569
9	1.143388	1.195093	1.248863	1.304773	1.362897	1.423312
10	1.160539	1.218994	1.280085	1.343916	1.410599	1.480244
11	1.177947	1.243374	1.312087	1.384234	1.459970	1.539454
12	1.195616	1.268242	1.344889	1.425761	1.511069	1.601032
13	1.213550	1.293607	1.378511	1.468534	1.563956	1.665074
14	1.231753	1.319479	1.412974	1.512590	1.618695	1.731676
15	1.250229	1.345868	1.448298	1.557967	1.675349	1.800944
16	1.268982	1.372786	1.484506	1.604706	1.733986	1.872981
17	1.288017	1.400241	1.521618	1.652848	1.794676	1.947901
18	1.307337	1.428246	1.559659	1.702433	1.857489	2.025817
19	1.326946	1.456811	1.598650	1.753506	1.922501	2.106849
20	1.346849	1.485947	1.638616	1.806111	1.989789	2.191123

Yrs.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.050000	1.060000	1.070000	1.080000	1.090000	1.100000
2	1.102500	1.123600	1.144900	1.166400	1.188100	1.210000
3	1.157625	1.191016	1.225043	1.269712	1.295029	1.331000
4	1.215506	1.262477	1.310796	1.360489	1.411582	1.464100
5	1.276282	1.338226	1.402552	1.469328	1.538624	1.610510
6	1.340096	1.418519	1.500730	1.586874	1.677100	1.771561
7	1.407100	1.503630	1.605781	1.713824	1.828039	1.948717
8	1.477455	1.593848	1.718186	1.850930	1.992563	2.143589
9	1.551328	1.689479	1.838459	1.999005	2.171893	2.357948
10	1.628885	1.790848	1.967151	2.158925	2.367364	2.593742
11	1.710339	1.898299	2.104852	2.331639	2.580426	2.853117
12	1.795856	2.012197	2.252192	2.518170	2.812665	3.138428
13	1.885649	2.132928	2.409845	2.719624	3.065805	3.452271
14	1.979932	2.260904	2.578534	2.937194	3.341727	3.797498
15	2.078928	2.396558	2.759031	3.172169	3.642482	4.177248
16	2.182875	2.540352	2.952164	3.425943	3.970306	4.594973
17	2.292018	2.692773	3.158815	3.700018	4.327633	5.054470
18	2.406619	2.854339	3.379932	3.996019	4.717120	5.559917
19	2.526950	3.025600	3.616527	4.315701	5.141661	6.115909
20	2.653298	3.207136	3.869684	4.660957	5.604411	6.727500

Note 1. — When the time extends beyond the limits of the table, find the amount for a convenient length of time, and use this amount for a new principal.

Note 2. — If the interest is compounded half-yearly, take one half the given rate and twice the number of years; and if compounded quarterly, take one fourth the given rate and four times the number of years.

160. What is the compound interest of $400 for 15 years 6 months at 6%?

Solution.

Amt. of $1 for 15 y. at 6%, from table	$2.396558
Amt. of $400 for 15 y. at 6% = $2.396558 × 400 =	$958.623
Int. of $958.623 for 6 mo.	28.759
Amount of $400 for 15 y. 6 mo. at 6%.	$987.382
Int. of $400 for 15 y. 6 mo. = $987.38 — $400 . .	$587.38

161. What is the amount of $500 for 20 years at 7% compound interest?

162. What is the compound interest of $120 for 14 years at 8% compound interest?

QUESTIONS.

254. What is interest? 255. The principal? 256. The amount? 257. The rate?

258. What is simple interest? 260. What is the general method of computing simple interest? 262. What is the six per cent method?

264. How may the six per cent method be shortened? 265. What is the method for finding the exact interest for parts of a year?

266. How do you find the rate of interest, the principal, interest, and time being given?

267. The time, the principal, interest, and rate being given? 268. The principal, the interest or amount, time, and rate being given?

270. What is a promissory note? 273. The face of a note? 277. What are partial payments? Indorsements?

282. What is the United States Rule for partial payments?

284. What is compound interest? 286. What is the process of computing compound interest?

DISCOUNT.

288. 1. What sum put at interest at 5% will in 5 years ount to $ 40 ?

Solution. — 5 years' interest is .25, or $\frac{1}{4}$ of the principal. The ount is $\frac{5}{4}$ of the principal. As $ 40, then, must be $\frac{5}{4}$ of the prin-al, $\frac{4}{4}$ of the principal is 4 × $\frac{1}{5}$ of $ 40, or $ 32.

2. What sum on interest at 6% will become $ 36 in 3$\frac{1}{3}$ ars ?

3. When an article whose list price is $ 25 is sold for cash 10% off, how much is the deduction ?

4. If I borrow $ 200 for 4 months, and pay the interest at % in advance, how much is deducted from the debt for the terest ?

289. **Discount** is the sum deducted from a debt or price.

TRUE DISCOUNT.

5. Fred Wood has this day bought a horse of me, agreeing pay me $ 525 for it in 1 year without interest. If I prefer, will pay me cash, provided I make him an equitable dis-nt from the price. I propose a discount of $ 15. He ob-ts, saying that I can then take the proposed price, $ 510, it at interest at the current rate, 5%, and at the end of the r I shall have more than the original $ 525 which he agreed pay. What discount ought I in justice to make ?

290. The **Present Worth** of a debt, payable at a future e without interest, is the sum which will amount to e debt when it becomes due if put at interest at the rrent rate.

291. **True Discount** is the difference between the face of debt and its present worth, and is equal to the interest the *present worth* of the debt for the given time.

WRITTEN EXERCISES.

6. What is the present worth of a debt of $25.44 due one year hence, the current rate of interest being 6 %? What is the true discount?

$$\$1 \times 1.06 = \$1.06$$
$$\$25.44 \div 1.06 = \$24$$
$$\$25.44 - \$24 = \$1.44$$

Solution. — As the amount of $1 for 1 year at 6% is $1.06, the present worth of $1.06, due 1 year hence, is $1; and as the present worth of $1.06 is $1, the present worth of $25.44 must be as many dollars as $1.06 is contained times in $25.44, or $24. $25.44 − $24 = $1.44, the true discount.

NOTE. — The process is the same as finding the principal (Art. 268), the debt being the *amount*, the present worth the *principal*, and the true discount the *interest*.

7. What is the true discount of $192 due 4 years hence, money being worth 7 %?

292. Rule to find the Present Worth and True Discount.

Divide the given debt by the amount of $1 *for the given time and rate, and the quotient will be the present worth.*

Subtract the present worth from the debt, and the difference will be the true discount.

8. What is the present worth of $3450, due in 1 year 6 months, without interest, the current rate being 7 %?

9. What is the true discount of $172.86, due in 3 years 4 months, money being worth 6 %?

10. What is the present worth of $360, due in 90 days, the current rate being 4 %?

11. What is the difference between the true discount and the interest on $5000 for 2 years 6 months at 7 %?

12. What must be the face of a note, due in 2 years 7 months 15 days, with interest, to exactly cancel a debt of $347.25, due in the same time, without interest, money being worth 6 %?

COMMERCIAL DISCOUNT.

293. Commercial Discount is a certain percentage deducted from the price of an article, or from the face of a bill, for cash payment.

294. The **Net Price** of an article is the selling, or list price, less the discount.

13. When goods whose list price is $125 are sold at 5% off, what is the commercial discount?

14. What is the net cash price of a carriage billed at $350, on 30 days, or 6% off for cash?

15. What is the discount on a bill of goods invoiced at $1344.50, sold on 30 days, at 2% off for cash?

16. What is the net cash value of books amounting as per bill to $460.50, less a discount of 10% and 5% off for cash?

17. Paid $433 for goods after a discount of 6% had been made from the list price. What was the list price?

18. What is the difference in net cash value between a bill of $1600, less a discount of 25% and 5% off the remainder, and the bill less a discount of 30%?

19. Find the amount of the following bill:

BOSTON, Oct. 19, 1881.
TAYLOR & AMES,
Bought of LEE & SHEPARD.

200 Harper's Geography	95¢	10% off.
50 Harrington's Graded Speller	18½¢	7½% off.
3 Cases School Slates	$6.35	20% off.
25 Webster's School Dictionary	1.12	12½% off.

A discount of 2½% was allowed for cash payment.

20. Bought of A. L. Davenport 62 yards Brussels carpeting at $1.87½; 118½ yards 3-ply carpet at 90¢; 1 set parlor furniture, $285; 2 sets black walnut chamber furniture at $125 and $140. Received a discount of 5%. What was the amount of my bill?

BANK DISCOUNT.

Form of a Discountable Note.

$\$564\frac{50}{100}$. *New York, July 10, 1882.*

Sixty days after date, I promise to pay to the order of Edward Bradford five hundred sixty-four $\frac{50}{100}$ dollars, at the Merchants' National Bank.

Value received. *George Horton.*

295. The above note may be transferred by the indorsement of the payee (Arts. 272 and 275) to any person to whom he may order it paid; or if the payee writes only his signature on the back, the note becomes payable to, and may be collected by, the bearer or holder.

Should the holder of this note wish to turn it into cash, he can take it at any time before its maturity to a bank. If the bank officers are satisfied that the maker and the indorsers are responsible parties, they will pay to the holder the face of the note less the interest on it from the day of discount to the day of maturity, retaining the note as security for the money advanced. On the day the note matures, the maker of the note, who has been previously notified that the note is held by the bank, appears at the bank and pays the note.

296. Bank Discount is an allowance made to a bank by the holder of a note for its payment before maturity.

It is the simple interest of the face of the note from the day of discount to the day of maturity, paid in advance.

297. The **Proceeds** of a note discounted are its face less the discount.

298. **Three Days of Grace** are generally allowed by law after a note is nominally due, — that is, after the expiration of the time named in the note for its payment, — before it is legally due.

NOTE 1. — A note matures, or becomes legally due, on the last day of grace, or on the day before when the last day of grace is Sunday or a legal holiday.

NOTE 2. — The time when a note is due is usually indicated by writing the date when nominally due and the date of maturity with a line between. Thus, Aug. $^{8}/_{11}$, 1882.

299. The **Term of Discount** is the time from the day of discount to the day of maturity.

NOTE 1. — When a note is discounted at date, the term of discount is the time specified in the note plus three days of grace. When the note is discounted after date it is generally customary to use as the term of discount the exact number of days from the day of discount to the day of maturity.

A note is understood to be discounted at date if nothing is said to the contrary.

NOTE 2. — When a note is payable in a certain number of months, calendar months are understood, and the note is nominally due on the corresponding day of the month, or on the last day of the month when there is no corresponding day. Thus,

A 3-mo. note dated Dec. 31, 1881, is due March $^{31}/_{\text{April } 3}$, 1882, but a 2-mo. note of the same date is due Feb. $^{28}/_{\text{March } 3}$, 1882.

300. Rule for finding the Proceeds of a Note.

Find the bank discount by computing the simple interest of the face of the note from the day of discount to the day of maturity.

The bank discount subtracted from the face of the note gives the proceeds.

NOTE. — When an interest-bearing note is discounted, the discount must be computed on the *amount due at maturity.*

WRITTEN EXERCISES.

21. What are the proceeds of a 90-day note for $842 discounted at date at 6%?

Solution.

Term of discount	93 days.	Face of note	$ 842.00
Interest for	60 "	. . . .	$ 8.42
"	30 "	. . . .	4.21
"	3 "	. . . .	.42
		Bank discount	$ 13.05
		Proceeds	$ 828.95

22.

$ 960. BOSTON, Oct. 19, 1881.

Four months after date, I promise to pay to the order of George Horton nine hundred sixty dollars at the People's National Bank. Value received. JOSEPH E. LIBBEY.

The above note was discounted Nov. 12, 1881, at 9 %. Required the proceeds.

Solution.

4 mo. 3 d. after Oct. 19, 1881 = Feb. 22, 1882, day of maturity.

From Nov. 12, 1881, to Feb. 22, 1882 = 102 d., term of discount.

Interest of $ 960 for 102 d. at 9 % = $ 24.48, bank discount.

$ 960 – $ 24.48 = $ 935.52, proceeds required.

Find the bank discount and proceeds

23. Of a 4-month note for $ 875 at 6 %.

24. Of a 6-month note for $ 85.60 at 7 %.

25. Of a 90-day note for $ 600 dated Jan. 4, 1881, and discounted Feb. 3, 1881, at 6 %.

26. Required the proceeds of a 4-month note for $ 5000 discounted 15 days after date at 8 %.

27. What are the proceeds of a 6-month note for $ 10500, bearing interest at 6 %, if discounted 60 days after date at

Find the proceeds of notes, using the following conditions:

	Face.	Date	Time to run.	Day of Discount.	Rate.
28.	$ 485.96	Jan. 14	4 mo.	Feb. 27	6 %
29.	$ 966	Feb. 12	3 mo.	Mar. 8	7 %
30.	$ 1024	Apr. 1	2 mo.	Apr. 15	7½ %
31.	$ 287	May 5	60 d.	May 27	8 %
32.	$ 648.50	July 10	90 d.	Date	5 %
33.	$ 984	June 15	5 mo.	Aug. 4	9 %
34.	$ 328	Sept. 20	6 mo.	Nov. 27	4 %
35.	$ 696	Aug. 25	30 d.	Sept. 1	4½ %
36.	$ 842.50	Oct. 31	1 mo.	Nov. 1	3½ %
37.	$ 500	Sept. 7	3 mo.	Oct. 10	6 %
38.	$ 8643	Nov. 12	75 d.	Dec. 15	7 %
39.	$ 242.16	Jan. 17	4 mo.	Feb. 10	10 %
40.	$ 800	Dec. 27	90 d.	Jan. 15	3 %
41.	$ 560	Mar. 5	3 mo.	May 1	8 %
42.	$ 576	Dec. 11	6 mo.	Mar. 18	5 %
43.	$ 898.96	Aug. 8	4 mo.	Oct. 27	9 %

To find the Face of a Note to yield given Proceeds.

44. For what sum must a 90-day note be written that the proceeds may be $ 500, discounted at 6 % ?

Bank discount of a $ 1 note for 93 days = $ 0.0155
Proceeds of a $ 1 note = $ 1 – $ 0.0155, or $ 0.9845
Face of note required = $ 500 ÷ $ 0.9845, or $ 507.87+

Solution. — As the proceeds of a $ 1 note for 93 days are $ 0.9845, to give $ 500 proceeds the face of the note must be as many dollars as $ 500 is times $ 0.9845, or $ 507.87+.

45. What must be the face of a 60-day note which, discounted at 7 %, will give as proceeds $ 1500 ?

46. The proceeds of a 4-month note, discounted at 6 %, are $ 293.85. What was its face ?

301. Rule for finding the Face of a Note.

Divide the given proceeds by the proceeds of $ 1 for the given rate and term of discount.

47. The proceeds of a 60-day note, discounted at 7 %, are $ 444.48¾. What was its face ?

48. A merchant discounted a bill payable in 6 months, by deducting the interest for the time without grace at 10 %, and received as the cash proceeds $ 1520. What was the face of the bill ?

49. Receiving a 90-day note, I had it discounted at once at 6 %, and received as proceeds $ 828.95. What was the face of the note ?

50. For what amount must a note be payable in 8 months, so that when discounted at 7½ % the proceeds may be $ 483.56?

MISCELLANEOUS EXERCISES.

51. I owe a debt of $ 924, payable without interest April 18, 1882. What shall be discounted for payment to-day, Oct. 6, 1881, money being worth 5 % ?

52. Find a year's interest of the present worth of $ 540, due 12 months hence without interest, money being worth 8 %?

53. At what date must a $ 1200 note have begun to draw interest which at 6 % amounted to $ 1380, Oct. 15, 1880 ?

54. Wood owes Davis $ 5811. He pays him with a 60-day note. For what sum should the note be written to pay the exact debt if discounted at 1½ % a month ?

55. A 4-month note is dated August 22. On exactly what day must it be paid to save a protest ? Why ?

56. What is the present worth of $ 477.71, due 4 years hence, without interest, money being worth 6 % ?

57. What is the true discount on $ 900, due in 72 days, the current rate being 7 % ?

58. What is the difference between the interest and the true discount of $ 576, due 16 months hence, at 6 % ?

59. What are the proceeds of a note for $ 368 payable in 90 days, discounted at bank at 6 % ?

60. On what month and day will a note for 60 days, dated Jan. 31, 1882, become legally due ?

61. A man was offered $ 3675 in cash for his house, or $ 4235 in 3 years without interest. He accepted the latter offer. Did he gain or lose, and how much, money being worth 7 % ?

✗ **62.** Wishing to borrow $ 500 at a bank, for what sum must my note be drawn at 30 days to obtain that amount, discount being 6 % ?

63. What must be the face of a note, due in 45 days, that, when discounted at a bank charging 7 % interest, will enable me to take up my note for $ 750, that has been on interest at 7½ % for 3 months and 15 days ?

64. Pratt, Davis, & Co. sold an acre of land, which cost them $ 400, at 5 cents per square foot, taking in payment a 6 mo.-note which they immediately get discounted at the Maverick Bank, at 5 %. What were their profits ?

QUESTIONS.

289. What is discount? 290. What is the present worth of a debt? 291. What is true discount? 292. How is the present worth found? The true discount?

293. What is commercial discount? 294. What is the net price of an article?

296. What is bank discount? 297. What are the proceeds of a note?

298. When is a note said to mature? 299. What is the term of discount? 300. How are the proceeds of a note found? 301. How do you find the face of a note to yield given proceeds?

274. What is a negotiable note? 275. What responsibility does a person incur by indorsing a note?

STOCK INVESTMENTS.

302. 1. A company start business with $10000. Into how many shares of $100 each can this be divided?

2. How much do 10 shares of $100 each represent?

3. What is the value of five shares of $100 each at a discount of 20%?

4. When $100 shares sell at $120 each, how much is the advance on the original value?

5. When a $100 share sells at $125, what per cent is the selling price above the original value?

6. If you own 10 shares of $100 each, and receive as the profits $60, what per cent are the profits?

303. A **Share** is one of the equal parts into which the capital of a corporation is divided.

The share is usually of the original value of $100, and may be so considered unless otherwise denoted.

304. **Bonds** are the interest-bearing notes of governments or corporations.

The interest on bonds is usually paid quarterly or semi-annually.

A *Coupon* is the interest certificate attached to a bond.

305. **Bonds** are commonly named according to their rate of interest and date of maturity. Thus,

U. S. 4½'s '91, means United States Bonds bearing 4½ per cent interest and payable in 1891.

306. **Stocks** are the shares of companies and the bonds of governments and corporations.

307. The **Par** of stocks is their face value. Thus,

When a stock is quoted at 105, it is worth 105% of its face value.

)8. The **Market Value** of stocks is the price at which sell. Stocks are at a *premium* when the market value *ove par*, and at a *discount* when the market value is *› par.*

)9. A **Stock Certificate** is a document signed by the ›rs of a corporation specifying the number of shares ›d by the holder.

.0. A **Dividend** is a sum divided among the stock-›rs as the profits of the business.

.1. An **Assessment** is a sum required of the stock-›rs to meet the losses or expenses of the business.

.2. The market value of leading stocks and bonds at nercial centers is given in the daily papers. The fol-ıg is an extract:

Stock Quotations.

›n Pacific	118½	U. S. 4's reg.	116
Y. Central	146⅜	U. S. 4's coup.	116¼
ie R. R.	47¾	$ 1000 Mass. 5's '94	116⅞
n. Bell Telephone . . .	132	$ 5000 Pacific 6's '95	130
:tinental Mills	90¼	$ 5000 Ogd's & Lake C. 6's . .	97
ıange Bank	145	$10000 N. Y. Central 6's '83 .	106½
tern R.R.	42¼	$ 5000 Hartford & Erie 7's . .	63

.3. **Brokerage** is computed on the par value of stocks, :he usual rate is either ¼ % or ⅛ %.

.4. *The rules of percentage and interest already given / to stocks, the premium, discount, dividend, and assess-being always a percentage of the par.*

What must be paid for $ 5000 Union Pacific Railroad s at 114, brokerage ¼ %?

ıtion. — 114 % + ¼ % = 114¼ %, and 114¼ % of $ 5000 = ².50, Ans.

8. What will 20 shares of New Jersey Central Railroad cost, at 102¼, brokerage ¼ %?

✗ **9.** How much, including brokerage at ⅛ %, must be paid for $ 30000 of U. S. 4's at 113½ ?

10. When gold is at 101½, what is the value in currency of $ 1250 in gold?

11. When the cost of 20 shares of the Atlantic National Bank, including ¼ % brokerage, is $ 2200, what is the market value per share?

Solution.

$ 100 × 20 = $ 2000, par value.
¼ % of $ 2000 = $ 5.00, the brokerage.
$ 2200 − $ 5 = $ 2195, the market value.
$ 2195 ÷ 20 = $ 109.75, the market value of 1 share.

12. When the cost of 25 shares of the Adams Express Company, including brokerage at ⅛ %, is $ 3206.25, what is the market value per share?

✗ **13.** When gold is at 101¾, what is the value in gold of $ 126.75 in currency?

14. How should Pacific Railroad bonds be quoted when 10 hundred-dollar bonds cost, including brokerage ¼ %, $ 1302.50?

15. What income will be realized from investing $ 1905 in 6 % stock bought at 95, allowing ¼ % for brokerage?

Solution.

$ 1905 ÷ .95¼ = $ 2000, par value.
$ 2000 × .06 = $ 120, yearly income.

16. What will be the income from investing $ 2650 in State 5's at 105¾, brokerage ¼ %?

✓ **17.** How much can be realized yearly from an investment of $ 6900 in a 4½ % stock, bought at 86, brokerage ¼ %?

18. Which will yield the greater income in amount, City 6's, at 105¾, purchased for $ 2650, or State 5's at 104¼, purchased for $ 3135, brokerage in both cases being ¼ %?

19. How much must be invested in 5 % stock, purchased at 103, to afford an income of $ 800 ?

Solution.

$ 800 ÷ .05 = $ 16000, par value of stock.
$ 16000 × 1.03 = $ 16480, amount to be invested.

20. How much must I invest in Government 4½'s at 105½, to secure annually $ 900 ?

21. How much must be invested in 7 % railroad bonds at 108¾, brokerage ¼ %, to afford an income of $ 1050 per annum ?

22. What sum must be invested in U. S. 5 % bonds of $ 500 each, at 108¾, brokerage ¼ %, to secure a yearly income of $ 2500 ?

23. When a 6 % stock is at 95¾, brokerage ¼ %, what rate of income on the investment will the stock yield ?

Solution. — The annual income of a share of 6 % stock is $ 6. If the cost is $ 96, the income is $\frac{6}{96}$, or $\frac{1}{16}$, or 6¼ %, of the cost.

24. What is the rate of income on Railroad 5's at 110, no allowance for brokerage ?

25. Which will yield the greater per cent income, Railroad 6's bought at 120, or 5 % stock bought at 105 ?

26. What per cent income will land-grant 8's purchased at 125 yield ?

27. How much must be paid for a 5 % stock that the investment shall yield 6 % ?

Solution. — A 5 % stock yields $ 5 on an investment of $ 100. But if the $ 5 is 6 % of the investment, 1 % of it is $\frac{1}{6}$ of $ 5, and 100 % of the investment 100 × $\frac{1}{6}$ of $ 5, or $ 83⅓.

28. How much must I pay for Railroad 6's, that my investment shall yield 7 % ?

29. At what price must I purchase 8 % stock that the investment shall pay 6 % ?

EXCHANGE.

315. 1. How can you pay a creditor in New Orleans $800 without actually sending him the money?

2. What will an order of John Hall of Chicago on Abram Brown of New York for $300 cost, payable at sight, if ¼% premium is charged for it?

3. What will an order for $250, payable at sight, cost, if purchased at ½% discount?

316. A **Draft** is a written order for the payment of money, made in one city or town, and payable in another.

317. The *Drawer* is the maker of a draft; the *Drawee*, the party to whom it is addressed; and the *Payee*, the party to whom it is payable.

318. A **Sight Draft** is a draft payable on presentation to the drawee.

319. A **Time Draft** is a draft payable at a time named after presentation, or after date.

320. Form of a Sight Draft.

$1000. *Rochester, Jan. 5, 1882.*

At sight pay to the order of William P. Johnson one thousand dollars, and charge to the account of

Thomas Prince & Co.

To Andrew Alston, Philadelphia.

321. If the drawee agrees to pay the money specified in the draft, on its presentation he writes his name under the word "Accepted" across its face. This act is called *the Acceptance* of the draft.

322. Exchange is the method of making payments by means of drafts.

The exchange is at *par* when a draft sells for its face.

Three days of grace are usually allowed on time drafts.

DOMESTIC EXCHANGE.

323. Domestic Exchange is between persons in the same country.

WRITTEN EXERCISES.

4. What is the cost of a sight draft for **\$452 at $1\frac{1}{2}$%** discount?

Solution.

$\$452 \times 0.015 = \$6.78$, discount.
$\$452 - \$6.78 = \$445.22$, cost of the draft.

5. What is the cost of a draft on New York for **\$1164 at** 1 % premium?

6. How much must be paid for a draft on St. Louis for \$4000 at $2\frac{1}{2}$% discount?

7. What is the cost of a draft of the Girard National Bank, Philadelphia, on the National Bank of Commerce, Boston, for \$2517.70 at $\frac{1}{8}$% premium?

8. Find the face of the draft at $1\frac{1}{2}$% discount that can be bought for \$445.22.

Solution.

$\$1 - \$0.015 = \$0.985$, cost of \$1 of draft.
$\$445.22 \div 0.985 = \$452$, face of the draft.

9. How large a draft at $\frac{1}{8}$% premium can be bought for \$2520.84?

10. Bought a draft, at $2\frac{1}{2}$% discount, for \$3900. What was its face?

11. A merchant in Mobile bought a draft on New York at 1 % premium for $ 1175.64. What was the face of the draft?

12. A man in Trenton bought a draft on Richmond at ⅝ % discount, for $ 447.18¾. What was the face of the draft?

13. What must be paid for a draft of $ 1000, on Hartford, at 30 days, interest at 6 %, when exchange is at 2 % premium?

$$\$1 \times 1.02 = \$1.02$$
$$\$1.02 \times 1000 = \$1020$$
$$\$1000 \times 0.0055 = \$5.50$$
$$\$1020 - \$5.50 = \$1014.50$$

Solution. — The cost of $ 1 at sight at 2 % premium is $ 1.02, and of $ 1000 is $ 1020. As the draft has 33 days to run, interest at the given rate for that time, amounting to $ 5.50, must be deducted; and the proceeds, $ 1014.50, will be the cost of the draft.

14. What must be paid for a draft of $ 1500, at 60 days, at 7 %, exchange being at ½ % discount?

15. I wish to obtain a draft on Boston for $ 3000, at 60 days, interest at 6 %, exchange being at 1 % premium. What must I pay for it?

16. What is the face of a 30 days' draft, at 2 % premium, which can be bought for $ 2029, interest at 6 %?

Solution.

$$\$1 + \$0.02 = \$1.02, \text{ cost of } \$1 \text{ at sight.}$$
$$\$1 \times 0.0055 = \$0.0055, \text{ int. of } \$1 \text{ for 33 days.}$$
$$\$1.02 - \$0.0055 = \$1.0145, \text{ cost of } \$1 \text{ of exchange.}$$
$$\$2029 \div 1.0145 = \$2000, \text{ face of the draft.}$$

17. How large a draft on Philadelphia, at par, at 30 days, can be bought for $ 3978, interest 6 %?

18. What is the face of a 60 days' draft at ½ % discount, which can be bought for $ 491.37½, money being worth 7 %?

19. How large a 30 days' draft can I buy for $ 2998.50, interest at 6 %, and exchange at 1 % premium?

FOREIGN EXCHANGE.

324. Foreign Exchange is between persons in different countries.

In foreign exchange drafts or bills are expressed in the money of the country in which they are payable.

325. English or **Sterling Money** is expressed in pounds, shillings, pence, and farthings.

TABLE.

4 farthings (far.) are 1 penny, *d.*
12 pence " 1 shilling, *s.*
20 shillings " 1 pound, £.

Also,

10 florins (fl.) are 1 pound, £.

326. The **Pound Sterling** is represented by a gold coin, the *Sovereign* (sov.), whose value is $ 4.8665.

327. French Money is expressed in *francs* and *centimes*; a franc being 100 centimes.

A franc has the value of $ 0.193, and about 5.18 francs are equivalent to a dollar.

328. The **Money of the German Empire** is expressed in *marks* (reichmarken) and *pennies* (pfennige); a mark being 100 pennies.

A mark is equivalent to $ 0.238, and 4 marks are about 95 cents.

NOTE. — For the value of other foreign coins as fixed by the U. S. government refer to the table in the Appendix.

329. Sterling Bills, or drafts on England, Ireland, and Scotland, are quoted at the exchange value of a sovereign or pound sterling in United States dollars.

Exchange on Paris, Antwerp, and Geneva is quoted at a certain number of francs per dollar; and on Bremen, Hamburg, Frankfort, and Berlin at a certain number of cents per 4 reichsmarks. Thus,

Foreign exchange may be quoted as follows: Sterling sight 4.86½ @ 4.87, 60 days 4.85; Francs sight 5.18 @ 5.18⅛, 60 days 5.15⅛ @ 5.15¾; and Reichsmarks sight 94½ @ 94⅝, 60 days 93⅞ @ 94½.

330. Bills of exchange, or drafts, on foreign countries are usually made in sets of three of the same tenor and date, named first, second, and third of exchange. Any one of the set being paid, the others are void.

20. Find the cost in Boston of the following bill drawn on London, exchange at 4.85.

£160 8s. *Boston, Dec. 15, 1881.*

At sight of this First of Exchange, second and third of the same date and tenor unpaid, pay to the order of Samuel W. Walker one hundred sixty pounds eight shillings sterling, value received, and charge the same to account of *Kidder, Peabody, & Co.*

To Messrs. Manson & Clews,
London, England.

Solution. — £160 8s. = £160.4; $4.85 × 160.4 = $777.94.

21. When exchange on London is at 4.85, what will be the *face* of a draft that can be bought for $777.94?

22. Bought a set of exchange on England for £1320 10*s.* at 4.87½. What was the cost?

23. Find the value in New York of a set of exchange on Paris for 2380 francs at 5.15.

24. Andrew Taylor, of Providence, wishes to remit 1500 francs to Antwerp. What will be the cost of a draft for that sum, exchange at 5.19?

25. When exchange on Paris is 5.20, how many francs of exchange will $3195 buy?

26. How much must be paid for a set of exchange on Hamburg for 1304 reichsmarks, exchange at 95?

27. When exchange on Berlin is 95½, what must be the face of a draft that $1420.20 will purchase?

28. Find the cost of a set of exchange on London at 60 days for £1254 15*s.* 6*d.*, exchange being quoted at 4.87½.

29. What will be the face of a draft in francs that can be bought for $1042.50, exchange being 5.21¼?

QUESTIONS.

316. What is a draft? 317. Who is the drawer? The drawee? The payee?

318. What is a sight draft? 319. A time draft?

321. How is a draft accepted? 322. How many days' grace are usually allowed on time drafts?

322. What is exchange? 323. Domestic exchange?

324. What is foreign exchange? 325. How is English money expressed? Recite the table. 326. What is the value of a pound or sovereign?

327. How is French money expressed? What is the value of a franc? 328. What is the money of the German Empire? What is the value of a mark?

329. How is exchange on London quoted? On Paris, Antwerp, and Switzerland? On Bremen, Hamburg, Frankfort, and Berlin?

330. How are drafts on foreign countries usually made?

AVERAGE OF PAYMENTS.

331. 1. How long should $1 be kept to equal the use of $2 for 1 month?

2. In how many months will the interest of $6 balance the interest of $18 for 4 months at the same rate per cent?

Solution. — $6 being but $\frac{1}{3}$ of $18 will require three times as many months to gain as much interest as $18 at the same rate, or 3 × 4 months, or 12 months.

3. The interest of $15 for 2 months is balanced by the interest of $1 in how many months? Of $3? Of $5?

4. If I should be allowed the use of $50 for 3 months, how long in return should I lend $25?

332. **Average,** or **Equation of Payments,** is the process of finding when several debts, due at different times, may be paid at one time without loss to either debtor or creditor.

333. The **Average,** or **Equated Time,** is the date of payment.

WRITTEN EXERCISES.

5. July 1, A owes B $100; of which $20 is due in 2 months, $40 in 3 months, $30 in 4 months, and $10 in 5 months. When may the $100 be equitably discharged by a single payment?

Solution.

A is entitled to 2 months' use of $20 = 40 months' use of $1
" " 3 " " $40 = 120 " "
" " 4 " " $30 = 120 " "
" " 5 " " $10 = 50 " "

A is entitled to $\overline{330}$ months' use of $1, which is equivalent to the use of $100 for $\frac{1}{100}$ of 330 mo., or $3\frac{3}{10}$ mo., or 3 mo. 9 d., which, added to July 1, gives the equated time, Oct. 10.

Or, briefly,

$$\begin{array}{rcl} 20 \times 2 \text{ mo.} &=& 40 \text{ mo.} \\ 40 \times 3 \text{ "} &=& 120 \text{ "} \\ 30 \times 4 \text{ "} &=& 120 \text{ "} \\ \underline{10} \times 5 \text{ "} &=& \underline{50} \text{ "} \\ 100 &) & \underline{330} \text{ "} \\ && 3\tfrac{3}{10} \text{ mo., or 3 mo. 9 d.} \end{array}$$

July 1 + 3 mo. 9 d. = Oct. 10.

6. What is the average time of paying $200 due April 1, $200 due May 11, and $400 due June 30?

Solution.

April 1, $200 is due.
May 11, or 40 days after April 1, $200 is due.
June 30, or 90 " " $400 "

$$\begin{array}{lcl} 200 && \\ 200 \times 40 \text{ days} &=& 8000 \text{ days} \\ \underline{400} \times 90 \text{ days} &=& \underline{36000} \text{ "} \\ 800 &) & \underline{44000} \text{ "} \\ && 55 \text{ days} \end{array}$$

April 1 + 55d. = May 26, the average time.

334. Rule to find the Average Time of Payment.

Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit. This added to the date from which the credits were reckoned will give the average time of payments.

NOTE. — When the cents are 50 or more, reckon them as one dollar; but if less than 50, disregard them. Also, when in any result the fraction of a day is ½ or more, reckon it one day; otherwise, disregard it.

7. I have purchased goods of A. B. Blake, as follows: January 3, a bill of $ 150 on 30 days; January 15, a bill of $ 125 on 3 months; and February 1, a bill of $ 200 on 60 days. Find the average time of payment.

8. May 7, A owes B $ 100, of which $ 40 is to be paid in 3 months, and $ 60 in 5 months. Find the average time of payment.

9. John Oldham owes Henry Smith $ 1000, $ 250 of which is due now, $ 350 in 2 months, and the remainder in 6 months. What is the average time of payment?

10. May 16, 1881, Joseph Milton owes $ 169.85, payable in 40 days, $ 200.15 in 60 days, and $ 150 in 90 days. Find the equated time.

11. Three bills are due as follows: April 15, $ 200; May 1, $ 310.50; June 1, $ 160.25. What is the average date of payment?

12. Find the average time for paying $ 800 due in 30 days, $ 500 in 60 days, and $ 120 in 90 days.

335. When there is a common term of credit, we may *find the average time, without regard to that term, and then add it to the result.*

13. Albert Thayer bought merchandise as follows on 60 days: July 5, 1881, $ 600; July 15, $ 400; August 10, $ 500. Find the average time of payment.

14. Bought the following bills on 4 months: September 9, 1880, $ 140; October 9, $ 160; November 6, $ 200. What is the average time for payment?

15. Three 60-day notes bear date as follows: April 11, 1881, $ 450; April 30, $ 600; May 16, $ 400. What is the average date of maturity?

16. Bought goods on 6 months' credit as follows: July 2, 1881, $ 225; August 4, $ 360; September 10, $ 500; September 24, $ 320. When shall a note to settle for the whole be made payable?

REVIEW.

ORAL EXERCISES.

336. 1. I have $ 120. If I pay away 25 % of it, what sum shall I have left?

2. Sold a horse which cost me $ 225 at 10 % profit. What did I get for him?

3. Bought a watch for $ 75 and sold it for $ 84. What was the gain per cent?

4. When goods are sold for $\frac{5}{4}$ of their cost, what is the gain per cent?

5. If $\frac{3}{4}$ of $\frac{4}{5}$ of the cost of my horse is the cost of my chaise, what per cent is the cost of the horse more than the cost of the chaise?

6. A knife which cost me 31¼ cents was sold for 25 cents. What was the loss per cent?

7. At 2½ %, what is the premium on an insurance of $ 3000?

8. An agent received $ 40.50 for selling goods at 5 % commission. What was the amount of goods sold?

9. What will be the interest of $ 550 at 6 % for 2 years 6 months?

10. What principal at 6 % interest in 2 years 6 months will give $ 66 interest?

11. What principal in 5 years at 5 % will amount to $ 50?

12. The interest of $ 550 for 2 years 6 months is $ 66. What is the rate per cent?

13. What is the market value of 6 shares of stock at 112?

14. What is the cost of a draft for $ 400 at 2¼ % premium?

15. At what price must a 4 % stock be bought for a 5 % investment?

16. How much must be paid for a 10 % stock that 8 % may be realized on the investment?

WRITTEN EXERCISES.

17. 2½ is what % of 3¾?

18. If 35 % of my money is $ 2359, what is my money?

19. Of goods worth $ 1200, one fourth is sold at a profit of 15%. For how much must the remainder be sold to gain 17%?

20. Bought a horse for $ 360, which was 20 % less than his real value, and sold him for 30 % more than his real value. Required the selling price.

21. A class of 50 pupils miss 75 words in spelling 10 each. What per cent of words were correctly spelled?

22. What per cent of the year 1880 expired at midnight, June 15?

23. Bought a bill of goods which, at 5 % off for cash, amounted to $ 232.75. How much was the discount?

24. Bought 12 Webster's Unabridged Dictionaries at $ 9.50, 10 % off, and 15 Longfellow's Poems at $ 1.50, 5 % off. Paid cash, and received an additional discount of 3 %. Required the cost.

25. At 7¼ % interest, how much is due, July 5, 1881, on a note for $ 325, dated Jan. 7, 1880?

26. How long must $ 922 be on interest at 5 % to gain $ 53.78½?

27. For what sum must a note be made payable in 60 days, so that when discounted at 6 % the proceeds may be $ 593.70?

28. At what rate of interest will $ 640 amount to $ 774.40 in 3 years 6 months?

29. In what time will $ 6000 at 8 % gain an interest equal to ⅔ of itself?

30. A merchant bought a bill of goods amounting to $ 1550, on 30 days' credit, but could have bought the same for cash at a discount of 5 %. What was the difference?

31. What is the difference between the true discount and the simple interest, both at 5 %, on $ 6415.50 for 3 years 6 months?

32. How much more is the compound interest than the simple interest of $1300 for 4 years at 7 % ?

33. On a note for $2000, dated Jan. 1, 1880, at 6 % interest, there was paid, July 1, 1880, $500. Required the balance due Jan. 1, 1882.

34. A man makes a difference in his income of $82.50 by transferring a 4 % stock at 92 to a 5 % stock at 110. What amount was transferred ?

35. Invested $26250 in bonds at 87½, and sold the same at 91. What was the gain ?

36. Find the difference between the income derived from $9080 invested in 3 % stock at 85⅛, and that derived from $9800 invested in 5 % bonds at 122½.

37. A owes B $460, of which $100 is to be paid in 50 days, $130 in 40 days, and the remainder in 140 days. Find the average time.

38. A merchant bought a draft on St. Louis for $2660, at 60 days, paying $2570.89. What was the rate of exchange ?

39. A note for $500, dated Oct. 8, 1880, and bearing interest at 6 %, is indorsed as follows : Nov. 4, 1881, $30 ; Jan. 30, 1882, $250. What will be due July 1, 1882 ?

40. I have purchased goods to the amount of $800, on a credit of 4 months. At the end of 2 months I pay $100, and at the end of 3 months I pay $200. How long after the expiration of the 4 months ought the balance in equity to remain unpaid ?

REVIEW QUESTIONS.

235. What is percentage ? 254. What is interest ? 296. What is bank discount ? 277. What are partial payments ? 284. What is compound interest ?

270. What is a promissory note ? 304. What are bonds ? What is a coupon ? 306. What are stocks ? 309. A stock certificate ?

316. What is a draft ? 322. What is exchange ? 329. How are sterling bills, or drafts, quoted ? 332. What is average, or equation of payments ? 333. What is the average, or equated time ?

RATIO AND PROPORTION.

RATIO.

337. 1. Thomas has 20 books and John 5. Thomas has how many times as many as John?

2. Peter is 16 years old and his brother 8. How do their ages compare?

3. What part of 27 feet is 9 feet?

4. How does 20 compare with 5? 24 with 6? 27 with 9?

5. What is the relation of $35 and $7? Of 51 miles and 17 miles? Of 7 pounds and 42 pounds?

338. **Ratio** is the relation of two like numbers shown by their quotient.

It is determined by dividing the first by the second. Thus,

The ratio of 15 to 5 is $15 \div 5$, or 3.

339. Ratio is usually indicated by :, which is an abbreviated form of ÷. Thus,

$18 : 6$ expresses the ratio of 18 to 6.

340. The **Terms** of a ratio are the two numbers compared. The *Antecedent* is the first term of a ratio, the *Consequent* is the second term, and the two terms together are called a *Couplet.*

341. An **Inverse Ratio** is a ratio formed by inverting the terms of a given ratio. Thus,

$8 : 9$ is the inverse of $9 : 8$.

342. A **Simple Ratio** is the ratio of two numbers. Thus,

$21 : 3 = 7$ is a simple ratio.

343. **A Compound Ratio** is the product of two or more simple ratios. It is usually indicated by means of the brace. Thus,

$$\left.\begin{array}{r} 8 : 2 \\ 9 : 3 \\ 10 : 5 \end{array}\right\}, \text{ or } 8 \times 9 \times 10 : 2 \times 3 \times 5 \text{ is a compound ratio,}$$

equal to the simple ratio 720 : 30.

A compound ratio may be changed to a simple ratio by *multiplying antecedents together for a new antecedent, and consequents for a new consequent.*

A ratio, like a fraction, is simply an indicated division (Art. 108). The principles of common fractions are equally applicable to ratios, the antecedent being the numerator and the consequent the denominator (Arts. 111, 114).

344. Principles of Ratio.

1. *The ratio is equal to the antecedent divided by the consequent.*

2. *The consequent is equal to the antecedent divided by the ratio.*

3. *The antecedent is equal to the consequent multiplied by the ratio.*

4. *Multiplying or dividing both the antecedent and the consequent by the same number does not change the ratio.*

EXERCISES.

Find the ratios of

6. 65 : 15.
7. 25 : 625.
8. $256 : $228.
9. $5 : \frac{3}{4}$.
10. 2.25 : 0.75.
11. $3\frac{1}{4} : 13$.
12. $\frac{5}{6} : \frac{4}{7}$.
13. 63 : 72.
14. $6\frac{1}{2} : 7\frac{3}{4}$.

15. What is the inverse ratio of 65 : 15?

16. What is the inverse ratio of 25 : 625?

17. Which is *the greater* ratio, 12 to 13 or 25 to 27?

18. If 6.25 is the antecedent and 5 the ratio, what is the consequent?

19. If 27 is the consequent and ½ the ratio, what is the antecedent?

20. What is the value of the compound ratio $\left.\begin{matrix}4:10\\8:\ \ 6\end{matrix}\right\}$?

21. What is the ratio compounded of $(6:8) \times (16:10) \times (12:9)$?

PROPORTION.

22. The ratio of 21 to 7 is what number? Of 51 to 17?

23. Name two numbers having the same ratio as 51 to 17.

24. What number has the same relation to 17 as 21 has to 7?

25. If 12 yards of cloth cost $40, what part of $40 will 8 yards cost?

26. How does the ratio of 3 yards to 12 yards compare with the ratio of $10 to $40?

345. A **Proportion** is an equality of ratios. Thus,

$$12 : 3 = 40 : 10 \text{ is a proportion.}$$

The equality of ratios may be indicated either by = or : :. Thus,

$$8 : 2 = 16 : 4, \text{ or } 8 : 2 :: 16 : 4.$$

means 8 to 2 equals 16 to 4, or 8 is to 2 as 16 is to 4.

346. Each term of a proportion is called a *Proportional;* the first and fourth terms are called *Extremes;* and the second and third terms, *Means.*

When the two means are the same number, that number is a *Mean Proportional* between the two extremes. Thus,

In $12 : 6 = 6 : 3$, 6 is a mean proportional between 12 and 3.

347. In the proportion

$$6 : 3 = 4 : 2,$$

as the ratios are equal, we have

$$\frac{6}{3} = \frac{4}{2}.$$

Changing these fractions to a common denominator, we have

$$\frac{6 \times 2}{6} = \frac{3 \times 4}{6}.$$

As these fractions are equal and their denominators alike, their numerators must be equal, or $6 \times 2 = 3 \times 4$. But 6 and 2 are the extremes, and 3 and 4 the means. Hence the following

348. Principles of Proportion.

1. *In a proportion the product of the means is equal to the product of the extremes.*
2. *Either extreme is equal to the product of the means divided by the other extreme.*
3. *Either mean is equal to the product of the extremes divided by the other mean.*

WRITTEN EXERCISES.

Find the missing term represented by x in the following proportions:

27. $14 : 7 = 18 : x$.

28. $5 : 20 = x : 60$.

29. $x : 8 = 65 : 13$.

30. $648 : 243 = 24 : x$.

31. $\frac{1}{6} : x :: 4 : 8$.

32. \$45 : \$24 : : 15 yd. : x.

33. x : \$9 : : 60 men : 18 men.

34. 5 tons : $\frac{1}{2}$ ton : : x : \$7.50.

SIMPLE PROPORTION.

349. A **Simple Proportion** is an equality between two simple ratios.

It applies to the solution of questions in which three terms of a proportion are given to find the fourth.

NOTE. — Of the given terms two must be of the same kind, and constitute a ratio; and the other must be of the same kind as the required term, and constitute with it another ratio equal to the first.

WRITTEN EXERCISES.

35. If 37 yards of cloth cost \$ 111, what will 19 yards cost?

$$37 : 19 = \$111 : \$x$$

$$\frac{19 \times \$\overset{3}{\cancel{111}}}{\cancel{37}} = \$57$$

Solution. — As 37 yards must evidently have the same ratio to 19 yards that \$ 111, the cost of 37 yards, has to the cost of 19 yards, or the answer, we arrange the terms so as to express the equality of these ratios. Or,

As the fourth term is to be dollars, we make \$ 111 the third term. The fourth term is to be less than the third term, because 19 yards will cost less than 37 yards. Hence the second term must be smaller than the first, and the first ratio is 37 : 19. Dividing the product of the means by the given extreme, we have as the answer \$ 57. Or,

If 37 yards cost \$ 111, 1 yard costs $\frac{1}{37}$ of \$ 111, and 19 yards 19 times as much, or $\frac{19}{37}$ of \$ 111, or \$ 57.

36. If 12 barrels of apples cost \$ 51, what will 30 barrels cost?

37. If the rent of 183 acres of land is \$ 273, what will be the rent of 61 acres?

38. What number of men will be required to perform in 16 days a piece of work that would take 30 men 48 days?

39. If 24 men can mow a field in 15 days, how many days will it take 20 men to do it?

350. Rule for Simple Proportion.

Make that number which is of the same kind as the answer the third term.

If from the nature of the question the answer is to be larger than the third term, make the larger of the remaining numbers the second and the smaller the first term; but if the answer is to be smaller than the third term, make the second term smaller than the first.

Divide the product of the means by the given extreme, and the quotient is the fourth term, or answer.

40. When $ 120 are paid for 15 barrels of flour, what will 79 barrels cost?

41. If 7 gallons of molasses cost $ 5.88, what will 27 gallons cost?

42. If a man travel 319 miles in 11 days, how far will he travel in 47 days?

43. If 27 men can do a piece of work in 12 days, how long will it take 36 men to do it?

44. Find the cost of 7 sheep when 98 cost $ 441.

45. What time should 24 men take to perform a piece of work which 18 men can perform in 15 days?

46. A garrison of 2100 men has provisions for 9 months, but receives a reinforcement of 600 men. How long will the provisions last?

47. If $4\frac{7}{8}$ bushels of oats cost $ $2\frac{7}{8}$, what will $19\frac{1}{2}$ bushels cost?

48. 74 men had provisions for 35 days, but after five days 20 men were sent away. How long will the provisions last the remaining 54 men?

49. If 6336 stones of $3\frac{1}{4}$ feet in length will make a certain quantity of wall, how many similar stones of $2\frac{2}{3}$ feet in length will make a like quantity?

50. If $3\frac{3}{4}$ tons of coal cost \$ 27.50, what will $4\frac{5}{8}$ tons cost?

51. A certain piece of work was to have been performed by 288 men in 72 days, but, a number of them having been sent away, it was performed in 108 days. What was the number of men sent away?

52. If $3\frac{1}{2}$ cords of wood cost \$ $11.37\frac{1}{2}$, what will $12\frac{1}{8}$ cords cost?

COMPOUND PROPORTION.

351. A Compound Proportion is an equality between a compound and a simple ratio. Thus,

$$\left.\begin{matrix} 3 : 4 \\ 5 : 8 \end{matrix}\right\} = 45 : 96 \text{ is a compound proportion.}$$

It applies to the solution of questions which would require several simple proportions.

WRITTEN EXERCISES.

53. If 4 men can earn \$ 64 in 8 days, how much can 12 men earn in 3 days?

$$\left.\begin{matrix} 4 : 12 \\ 8 : \ \ 3 \end{matrix}\right\} = \$64 : \$x$$

Or,

$$\frac{\overset{3}{\cancel{12}} \times 3 \times \overset{8}{\cancel{64}}}{\cancel{8} \times \cancel{4}} = 72$$

Solution. — As the answer sought is in dollars, we make the \$64 the third term, and x, representing the answer, the fourth term. If the answer depended only on the number of men, it would be larger than the third term, as 12 men will earn more than 4 men; hence the first ratio is 4 to 12. But if the answer depended only on the number of days worked it would be smaller than the third term, as less can be earned in 3 days than in 8 days; hence the second ratio is 8 to 3.

Dividing the product of the means by the product of the given extremes, we have \$ 72 as the answer.

Or, if 4 men can earn \$ 64 in 8 days, 12 men in the same time can earn $\frac{12}{4}$ of \$ 64, and in 3 days $\frac{3}{8}$ of $\frac{12}{4}$ of \$ 64, or \$ 72.

352. Rule for Compound Proportion.

Make that number which is like the answer the third term.

Form a ratio of each pair of the remaining numbers of the same kind according to the rule for simple proportion, as if the answer depended on them alone. Divide the product of the means by the product of the given extremes, and the quotient is the fourth term, or answer.

54. If 3 men can make 108 pairs of shoes in 2 days, how many pairs can 2 men make in a week?

55. If $ 250 yields $ 175 interest in 7 years, how long will it take $ 500 to yield $ 360 at the same rate?

56. If 24 men can reap 76 acres in 6 days, how many men will reap 114 acres in 9 days?

57. How many acres can 10 men plow in 14 hours, if 5 men plow 6 acres in 10½ hours?

58. Two cogged wheels, one of which has 15 cogs and the other 28, work in each other. If the first turns 16 times in 7½ seconds, how often will the other turn in 4 seconds?

59. If 15 men are fed for 7 days when flour is $ 8 a barrel, what must be the price when 10 men are fed 8 days at the same cost?

○ **60.** If a man travels 117 miles in 15 days, employing only 9 hours, how far would he go in 20 days, traveling 12 hours a day?

61. If 96 horses eat 192 tons of hay in one winter, how many tons will 150 horses eat in 6 winters?

62. If a man, walking 12 hours each day, travels 250 miles in 9 days, in how many days, walking 10 hours each, at the same rate, would he travel 400 miles?

○ **63.** If the expenses of a family of 8 persons amount to $ 84 in 16 weeks, how long will $ 200 support a family of 6 persons?

64. If a pasture of 16 acres will feed 6 horses for 4 months, how many acres will feed 12 horses for 9 months?

65. If 180 bricks, 8 inches long and 4 inches wide, are required for a walk 20 feet long and 6 feet wide, how many bricks will be required for a walk 100 feet long and 4 feet wide?

66. If 34 men can saw 90 cords of wood in 6 days, when the days are 9 hours long, how many cords can 8 men saw in 36 days when they are 12 hours long?

67. If 12 men in 15 days can build a wall 30 feet long, 6 feet high, and 3 feet thick, working 12 hours a day, in what time will 30 men build a wall 300 feet long, 8 feet high, and 6 feet thick, working 8 hours a day?

68. If a loaf which sells for 20 cents when wheat is $4 a bushel, weighs 3 pounds, what is the price of wheat when a 12-cent loaf weighs 2½ pounds?

69. If a bin 8 ft. long, 4½ ft. wide, and 2½ ft. deep, holds 67½ bu., how deep must another bin be made, that is 18 ft. long and 3⅚ ft. wide, to hold 450 bu.?

70. If the annual salary of a man who works 8 hours a day, 48 weeks in the year, is $1200, how much ought a conductor to receive per month who works 14 hours daily the year round?

QUESTIONS.

338. What is ratio? 338. How is it determined? 340. What are the terms of a ratio?

341. What is an inverse ratio? 342. A simple ratio? 343. A compound ratio? 344. What are the principles of ratio?

345. What is a proportion? 346. What is each term of a proportion called? What is a mean proportional? 348. What are the principles of proportion?

349. What is a simple proportion? 350. Which number is made the third term? How are the terms arranged? How, then, is the required extreme found?

351. What is a compound proportion? 352. Which term is made the third term? How is each pair of the remaining numbers arranged? How, then, is the required term found?

PARTNERSHIP.

353. 1. Two men share between them $35, the one receiving $3 as often as the other $4. How much does each receive?

2. What number is $\frac{3}{7}$ of 35? $\frac{4}{7}$ of 35?

3. A and B are in business together. A put in $3000 and B $5000. They gain $800. What are their respective shares of it?

4. Divide $800 into two parts having the ratio of 3 to 5.

5. Divide 54 oranges between two boys in the ratio of 4 to 5.

354. **Partnership** is the association of two or more persons in business.

355. The **Company**, or **Firm**, is the association, and the **Partners** are the members.

356. The **Capital**, or **Stock**, is that which is invested in the business, and the **Dividend** is the profits shared by the partners.

The profits or losses are usually shared according to the terms of the agreement, or contract, made when the partnership is formed.

In the absence of a special agreement, dividends are in proportion to the capital invested and the time during which it is invested.

WRITTEN EXERCISES.

6. A, B, and C engage in trade. A furnishes $200, B $250, and C $350. They gain $100.80. What is each partner's share of *the gain?*

Solution.

$200 + $250 + $350 = $800, the capital.

A's stock = $\frac{200}{800} = \frac{1}{4}$; $\frac{1}{4}$ of $100.80 = $25.20, A's gain.

B's " = $\frac{250}{800} = \frac{5}{16}$; $\frac{5}{16}$ of $100.80 = $31.50, B's gain.

C's " = $\frac{350}{800} = \frac{7}{16}$; $\frac{7}{16}$ of $100.80 = $44.10, C's gain.

As A's stock is $\frac{1}{4}$ of the entire capital, B's stock $\frac{5}{16}$, and C's $\frac{7}{16}$, A must have $\frac{1}{4}$, B $\frac{5}{16}$, and C $\frac{7}{16}$ of the gain. Hence A's gain is $25.20, B's $31.50, and C's $44.10.

7. A, B, and C engage in trade. A puts in $6000, B $9000, and C $5000. They gain $1680. What is each partner's share of the gain?

357. To find the gain or loss of partners,

Rule.

Take for each partner such a part of the gain or loss as his stock is of the entire capital.

NOTE. — The rule applies to the distribution of the assets of bankrupts and other like apportionments.

8. A, B, and C engage in trade, investing capital to the amount of $1280, $1760, and $1920, respectively. Their profits were $2790. How were they divided?

9. A bankrupt owes three creditors, A, B, and C, $1750, $2100, and $2650, respectively. His assets are $4225. What should they each receive?

10. Hall and Bishop gain by trade $728. Hall put in $1200, and Bishop $1600. What is the gain of each?

11. B puts on board of a ship 400 barrels of flour, C 600, and D 400; but when at sea it was found necessary to throw 360 barrels overboard. How much of the loss should fall to each?

12. A, B, and C hire a pasture for $300. A puts in 8 horses, B 6, and C 10. How many dollars should each pay?

13. A, B, and C engage in trade. A puts in $ 1400, B $ 600, and C 125 barrels of flour. They gained $ 180; of which C took $ 60 as his part. What will A and B receive, and what was the value of C's flour a barrel?

358. When the capital of the partners is employed for unequal times,

Find the product of each partner's stock multiplied by the time it was invested, and divide the gain or loss in proportion to the products.

14. A, B, and C engage in trade. A puts in $ 300 for 7 months, B $ 500 for 8 months, and C $ 200 for 12 months. They gain $ 170. What is each man's share of the gain?

Solution.

A's $ 300 for 7 months = $ 2100 for 1 month.
B's $ 500 for 8 months = 4000 for 1 month.
C's $ 200 for 12 months = 2400 for 1 month.
The entire stock is the same as $ 8500 for 1 month.

$\frac{2100}{8500} = \frac{21}{85}$; $\frac{21}{85}$ of $ 170 = $ 42, A's gain.
$\frac{4000}{8500} = \frac{40}{85}$; $\frac{40}{85}$ of $ 170 = $ 80, B's gain.
$\frac{2400}{8500} = \frac{24}{85}$; $\frac{24}{85}$ of $ 170 = $ 48, C's gain.

As $ 300 for 7 months is the same as $ 2100 for 1 month, $ 500 for 8 months the same as $ 4000 for 1 month, and $ 200 for 12 months the same as $ 2400 for 1 month, the entire stock is $ 2100 + $ 4000 + $ 2400 = $ 8500 for 1 month.

Hence A's gain will be $\frac{21}{85}$ of the whole gain, or $ 42; B's, $\frac{40}{85}$, or $ 80; and C's, $\frac{24}{85}$, or $ 48.

15. A, B, and C had a joint stock of $ 2400. A's part was $ 750, and continued in trade 4 months; B's was $ 850, and continued 8 months; the remainder was C's, and continued in trade throughout the year. They lost $ 640. What was each man's share of it?

16. A, B, and C were in partnership. A had in the business $ 5000 for 8 months, B $ 4000 for 12 months, and C $ 3000 for 15 months. The profits were $ 1330. How much is each partner's part of the profits?

17. A and B rent a pasture for $ 46.80. A puts in 30 horses for 33 days, and B 21 horses for 42 days. How much ought each to pay of the rent?

18. A, B, and C form a partnership. A furnishes $ 500 for 9 months, B $ 700 for 1 year, and C $ 400 for 15 months. They lose $ 300. What is each man's share of the loss?

19. A and B are in partnership. A put in $ 6000, and at the end of 6 months put in $ 4000 more; B put in $ 12000, and at the end of 8 months took out $ 6000. They trade 1 year, and gain $ 2160. What is each man's share of the gain?

20. A and B enter into partnership for 1 year. A had $ 500 in the business during the first 4 months, and $ 300 more during the remainder of the year; whereas B had only $ 400 during the first 6 months, but $ 900 during the last 6 months. They gained $ 2400. What was each man's share of the gain?

21. Jan. 1, 1881, Wood goes into business with a capital of $ 6000. March 1, Furbush joins him with $ 5000. July 1, they take Davis into the partnership with $ 4000 capital, agreeing to pay him 6 % interest for his money, and give him an annual salary of $ 1200. The profits were $ 3160, out of which Davis was paid and the balance divided between the other partners. Find each man's share at the end of the year.

22. Jan. 1, 1882, A and B form a partnership for a year. A furnishes $ 2000, and B $ 3000. May 1, they take C into the firm with a capital of $ 5000. August 1, A receives a legacy of $ 4000, which he adds to the capital. Oct. 1, B withdraws $ 1000 of his capital. At the end of the year the firm's net gains were $ 5850. Divide it equitably among the partners.

INVOLUTION AND EVOLUTION.

INVOLUTION.

359. 1. What is the product of 5 used twice as a factor?

2. Of what number are 5 and 5 the factors?

3. What is the product of 5 used three times as a factor?

4. Of what number are 5, 5, and 5 the factors?

5. What is the product of .3 used three times as a factor? Of $\frac{1}{4}$ used three times as a factor?

360. A **Power** of a number is the product arising from taking the number a certain number of times as a factor.

The *First Power* of a number is the number itself;

The *Second Power* of a number is the product arising from taking the number twice as a factor;

The *Third Power* of a number is the product arising from taking the number three times as a factor; and so on. Thus,

The first power of 3 is 3.
" second " 3 is 3 × 3, or 9.
" third " 3 is 3 × 3 × 3, or 27.
" fourth " 3 is 3 × 3 × 3 × 3, or 81.

The second power is also called the *Square* of the number, as the area of a square is the product of two equal factors.

The third power is called the *Cube*, as the volume of a cube is the product of three equal factors.

361. The **Exponent** of a power is a small figure placed at the right and above a number. Thus,

25^2 means the second power, or square, of 25;
3.1^3 means the third power, or cube, of 3.1;
$(\frac{3}{4})^4$ means the *fourth* power of $\frac{3}{4}$.

362. **Involution** is the process of finding powers.

WRITTEN EXERCISES.

6. What is the third power of 12?

Solution. — $12^3 = 12 \times 12 \times 12 = 1728$.

7. Find the squares and cubes of the first nine numbers.

363. Rule for Involution.

Use the given number as many times as a factor as there are units in the exponent of the required power.

Find the powers indicated by the exponents:

8. 23^2.
9. 16^3.
10. 13^4.
11. $(\frac{3}{4})^2$.
12. $(2\frac{1}{2})^3$.
13. 3.6^2.
14. 25^3.
15. 11^5.
16. 0.15^3.
17. 3.5^2.
18. $(14\frac{1}{4})^2$
19. 0.07^3.

EVOLUTION.

20. What are the factors of 9? Of 25? Of 49?
21. What are the two equal factors of 16? Of 64? Of 81?
22. What are the three equal factors of 27? Of 64?

364. A **Root** of a number is one of the equal factors which produce it.

The *Second,* or *Square, Root* of a number is one of the *two* equal factors which produce it;

The *Third,* or *Cube, Root* of a number is one of the *three* equal factors which produce it;

The *Fourth Root* of a number is one of the *four* equal factors which produce it; and so on.

365. A **Perfect Power** is a number whose exact root can be found, and an **Imperfect Power** is a number whose root *cannot* be exactly found.

366. The **Radical Sign**, $\sqrt{\ }$, is used to indicate a root. Thus,

$\sqrt[2]{16}$ means the second, or square, root of 16.
$\sqrt[3]{25}$ means the third, or cube, root of 25.

The number in the opening of the sign, called the *Index* of the root, denotes the name of the root.

The index of the square root may be understood. Thus, $\sqrt{81}$, or $\sqrt[2]{81}$, may indicate the second, or square, root of 81.

367. Evolution is the process of finding roots. It is the reverse of involution.

SQUARE ROOT.

368. To *extract the square root* of a number is to find one of the two equal factors which produce it.

369. *The square of a number contains twice as many figures as the root, or twice as many less one.* Thus,

$1^2 = 1.$ $9^2 = 81.$
$10^2 = 1'00.$ $99^2 = 98'01.$
$100^2 = 1'00'00.$ $999^2 = 99'80'01.$

370. *If a power is separated into periods of two figures each, beginning at the decimal point, the number of periods will show the number of figures in the root.* Thus,

The square root of 2'35.'92'96 contains two integral and two decimal figures.

371. *The square of the highest order of units in the root is found in the highest period of the power.* Thus,

9 tens^2, or 90^2 = 81 hundreds, or 81'00.
9 hundreds^2, or 900^2 = 81 ten-thousands, or 81'00'00.
9 thousands^2, or 9000^2 = 81 millions, or 81'00'00'00.

372. The parts which make up a second power may be learned by a careful inspection of the process of multiplication by which the power is produced. For example, let us square 36, keeping its tens and ones and their products distinct and separate. Thus,

$$\begin{array}{rcl}
36 = & 30 + 6 = & \text{tens} + \text{ones}.\\
\underline{36} = & \underline{30 + 6} = & \underline{\text{tens} + \text{ones}.}\\
216 = & (30 \times 6) + 6^2 = & (\text{tens} \times \text{ones}) + \text{ones}^2.\\
\underline{108} = & \underline{30^2 + (30 \times 6)} = & \underline{\text{tens}^2 + (\text{tens} \times \text{ones}).}\\
1296 = & 30^2 + 2\,(30 \times 6) + 6^2 = & \text{tens}^2 + 2\ \text{tens} \times \text{ones} + \text{ones}^2.
\end{array}$$

That is, the square of any number composed of tens and one equals

373. *The square of the tens, plus two times the tens times the ones, plus the square of the ones, or* $t^2 + 2\,t \times o + o^2$.

WRITTEN EXERCISES.

23. Find the square root of 1296.

$$\begin{array}{rl}
 & 12'96\ (30 + 6\\
30^2 = & 900 = t^2\\
2\,t = 2 \times 30 = 60 & \big|\ \overline{396} = 2\,t \times o + o^2\\
 & \big|\ \underline{360} = 2\,t \times o\\
 & 36 = o^2\\
o^2 = 6 \times 6 = & \underline{36}
\end{array}$$

Solution. — This power contains two periods; hence its root has two figures, tens and ones (Art. 370). The square of the tens is in the highest period (Art. 371). Taking out of the 12 hundreds the largest "tens^2" possible, 900, and placing its root, 3 tens, or 30, at the right, there remains 396, which must be the "2 tens × ones + ones^2" (Art. 372). The "ones^2" being but a small part of the 396, we may treat this number as the approximate product of the "2 tens × ones." Dividing this product, 396, by one of its factors, "2 tens," or 60, we have 6 as the other factor, the "ones." Taking from 396 the "2 tens × ones," or 60 × 6, or 360, there remains 36, which contains the "ones^2." Taking the ones^2, or 6^2, from 36, nothing remains. *Hence we conclude* that 30 + 6, or 36, is the root required.

Geometrical Explanation of Square Root.

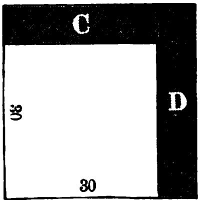
374. As the length of a square is the square root of its area, the method of extracting the square root of a number may be illustrated by the process of finding the length of a square, its area being given.

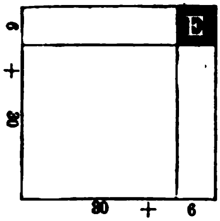
It is required to find the length of a square containing 1296 square inches.

Solution.

A square containing 1296 sq. in. cannot be 40 in. long, for a 40-inch square contains 1600 sq. in. It must be more than 30 in. long, for a 30-inch square contains but 900 sq. in. The length of the given square must therefore be between 30 and 40 inches.

Removing from the given square, *A*, a 30-inch square, *B*, containing 900 sq. in., there remains a surface containing 396 sq. in., largely made up of the rectangles *C* and *D*, whose length is evidently that of the square, *B*. It is obvious that the width of these rectangles added to the length of the square, *B*, will give the required length of the given square, *A*. Now the width of a rectangle is found by dividing its area by its length (Art. 218). The length of each of these rectangles, *C* and *D*, is 30 in., and their united length 2 × 30 in., or 60 inches. Dividing their approximate area, 396 sq. in., by their length, 60 in., we have as their probable width 6 inches.

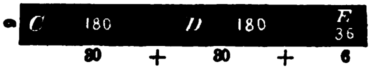




Removing the rectangles *C* and *D*, there remains the little square, *E*, whose length is evidently the width of the rectangles removed.

Combining the two rectangles and the little square, we find their united length to be 60 + 6, or 66 in. Multiplying their length and width together, we find their area to be 66 × 6, or 396 sq. in., the exact area of that portion of the given square, *A*, remaining after the removal of the square, *B*.

We therefore conclude that the length of the given square is 36 inches.



The work may be expressed thus:

Area. **Width.**
12'96 sq. in. (30 in. + 6 in. = 36 in.
900 "
Length.
30 in. × 2 = 60 in | 396 "
6 "
66 " | 396 "

The process may be shortened by the omission of the ciphers, and proved by involution.

24. Extract the square root of 3998.64.

39'98.'64 (63.234+
36
123 | 398
| 369
1262 | 2964
| 2524
12643 | 44000
| 37929
126464 | 607100
| 505856
101244

375. Rule for finding the Square Root of a Number.

Beginning at the decimal point, separate the given number into periods of two figures each.

Find the greatest square in the left period, and place its root at the right; subtract the square of this root from the first period, and to the remainder annex the next period for a dividend.

Divide this dividend, omitting the last figure, by double the root already found, and annex the quotient to the root and also to the divisor.

Multiply the divisor as it now stands by the last root figure, and subtract the product from the dividend.

If there are more periods to be brought down, proceed in the same manner as before.

NOTE 1. — If 0 occurs in the root, annex 0 to the divisor and another period to the dividend, and proceed as before.

NOTE 2. — If there is a remainder after using all the periods, we can only approximate to the root. But nearer and nearer approximations can be obtained by annexing and using successive periods of decimal ciphers.

25. What is the square root of 49.434961 ?

```
        49.'43'49'61 (7.031
        49
 1403  | 4349
       | 4209
 14061 |  14061
       |  14061
```

Here in the process, as 0 occurs in the root, we annex 0 to the divisor, 14, and annex the next period to the corresponding dividend.

Find the square root

26. Of 9216.
27. Of 27225.
28. Of 182329.
29. Of 717409.
30. Of 948676.
31. Of 6.7081.
32. Of 4.2025.
33. Of 1866.24.
34. Of 0.009409.
35. Of 0.05625.
36. Of 0.9409.
37. Of 77841.
38. Of 16.2409.
39. Of 14.8996.
40. Of 39.0625.

41. What is the square root of 538 to the nearest hundredth?

42. What is the value of $\sqrt{71}$ to the nearest thousandth?

43. What is the value of $\sqrt{0.002}$ to the nearest ten-thousandth?

376. To extract the root of a fraction,

First change it to smallest terms; then, if both terms are perfect squares, take the root of each; otherwise change the fraction to a decimal, and find the root.

Mixed numbers may be changed either to improper fractions or to mixed decimals.

44. What is the square root of $\frac{49}{529}$?

Solution. — $\sqrt{\frac{49}{529}} = \frac{\sqrt{49}}{\sqrt{529}} = \frac{7}{23}.$

Find the value of the following:

45. $\sqrt{\frac{3721}{7569}}.$

46. $\sqrt{\frac{1899}{10339}}.$

47. $\sqrt{60\frac{1}{16}}.$

48. $\sqrt{37\frac{36}{49}}.$

49. $\sqrt{49\frac{4}{9}}.$

50. $\sqrt{\frac{450}{2048}}.$

51. $\sqrt{72\frac{1}{4}}.$

52. $\sqrt{\frac{30}{32}}.$

53. $\sqrt{\frac{3}{4} + \frac{5}{6} + \frac{6}{9}}.$

54. $\sqrt{981\frac{7}{9}}.$

55. $\sqrt{146\frac{5}{8}}.$

56. $\sqrt{81\frac{9}{25}}.$

57. What is the square root of $\frac{7}{8}$ to the nearest thousandth?

58. What is the square root of $\frac{11}{16}$ to the nearest thousandth?

59. A general has an army of 226576 men. How many must he place rank and file to form them into a square?

60. A gentleman has 3 fields, one containing 3 acres 1 square rod, another 5 acres 69 square rods, and the third 6 acres 91 square rods. He wishes to exchange them for a square field of equal area. Find the side of the square field?

CUBE ROOT.

377. To *extract the Cube Root* of a number is to find one of the three equal factors which produce it.

378. *The cube of a number contains three times as many figures as the root, or three times as many less one or two.* Thus,

$1^3 = 1.$ $10^3 =$ 1′000. $100^3 =$ 1′000′000.
$4^3 = 64.$ $25^3 =$ 15′625. $243^3 =$ 14′348′907.
$9^3 = 729.$ $99^3 =$ 970′299. $999^3 =$ 997′002′999.

379. *If a third power is separated into periods of three figures each, beginning with the decimal point, the number of periods will show the number of figures in the root.* Thus,

The cube root of 4′080′659.′192 contains one decimal and three integral figures.

380. *The cube of the highest order of units in the root is found in the highest period of the power.* Thus,

9 tens^3, or 90^3 = 729 thousands, or 729′000.
9 hundreds^3, or 900^3 = 729 millions, or 729′000′000.

381. Taking any number composed of tens and ones, as 36, separate it into its tens and ones, cube it, keeping the products distinct, and we have

$$\begin{array}{r@{\;=\;}r}
36 & 30 + 6 \\
36 & 30 + 6 \\ \hline
216 & (30 \times 6) + 6^2 \\
108 & 30^2 + (30 \times 6) \quad\;\; \\ \hline
1296 & 30^2 + 2\,(30 \times 6) + 6^2 \\
36 & 30 + 6 \\ \hline
7776 & (30^2 \times 6) + 2\,(30 \times 6^2) + 6^3 \\
3888 & 30^3 + 2\,(30^2 \times 6) + (30 \times 6^2) \quad\;\; \\ \hline
46656 & 30^3 + 3\,(30^2 \times 6) + 3\,(30 \times 6^2) + 6^3
\end{array}$$

That is, the cube of any number that can be separated into tens and ones equals

382. *The cube of the tens, plus three times the product of the square of the tens and the ones, plus three times the product of the tens and the square of the ones, plus the cube of the ones.*

This statement may be represented by the following formula in which initial letters are used, and the sign of multiplication is the period.

$$t^3 + 3\,t^2 . o + 3\,t . o^2 + o^3.$$

WRITTEN EXERCISES.

61. Find the cube root of 405224.

$$\begin{array}{rr|l}
t^3 + 3\,t^2 . o + 3\,t . o^2 + o^3 = & & 405'224\ (70 + 4 \\
t^3 = & & \underline{343\,000} \\
3\,t^2 = 70^2 \times 3 \quad = & 14700 & 62224 = 3\,t^2 . o + 3t . o^2 + o^3 \\
3\,t . o = 70 \times 3 \times 4 = & 840 & \\
o^2 = \qquad\quad 4^2 = & 16 & \\
3\,t^2 + 3\,t . o + o^2 \quad = & \overline{15556} & 62224 = (3\,t^2 + 3\,t . o + o^2) \times o
\end{array}$$

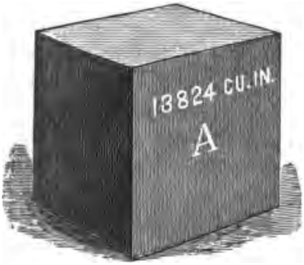
Solution. — This power has two periods; hence its root has two figures, tens and ones (Art. 379). The cube of the tens is in the highest period (Art. 380). The greatest tens^3 in 405 thousands is 343000, which, subtracted, and its cube root placed at the right, leaves 62224, which equals "$3\,t^2 \times o + 3\,t \times o^2 + o^3$." 62224 consists principally of $3\,t^2 \times o$; hence, if we divide it by $3\,t^2$, or $70^2 \times 3 = 14700$, we shall have the ones, which we find to be 4. Finding $3\,t \times o$, or $70 \times 3 \times 4 = 840$, and o^2, or $4^2 = 16$, and adding them to 14700, we have $14700 + 840 + 16 = 15556$, which equals $3\,t^2 + 3\,t \times o + o^2$. Multiplying this by the ones, 4, we have 62224, or $3\,t^2 \times o + 3\,t \times o^2 + o^3$. Hence we conclude that 74 is the cube root required.

Geometrical Explanation of Cube Root.

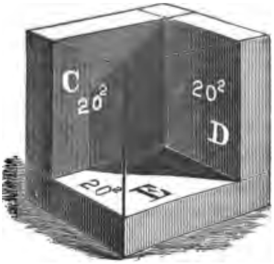
383. As the length of a cube is the cube root of its contents, the method of extracting the cube root of a number may be illustrated by the process of ascertaining the length of a cube, its contents being given.

62. It is required to find the length of a cube containing 13824 cu. in.

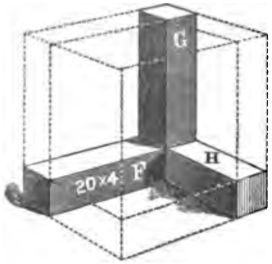
The length of a cube containing 13824 cu. in. cannot be as much as 30 in., for a 30-inch cube contains 27000 cu. in. It must be more than 20 in., for a 20-inch cube contains only 8000 cu. in. The length of the given cube, then, must be between 20 and 30 inches.



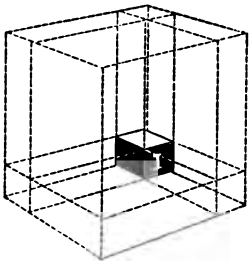
Removing from the given cube, *A*, a 20-inch cube, *B*, containing 8000 cu. in., there remains a solid containing 5824 cu. in. An inspection of this solid shows that it is largely made up of three rectangular solids, *C*, *D*, *E*, having square faces corresponding in area to the faces of the cube removed. It is evident that the thickness of these solids added to the length of the cube removed will be the length of the given cube. Now the thickness of a rectangular solid is found by dividing its contents by the area of its face (Art. 224). We find the area of one of the square faces of



each of these rectangular solids to be 20^2, or 400 sq. in., and the area of their combined square faces to be $20^2 \times 3$, or 1200 sq. in. Dividing the approximate contents of these solids, 5824 cu. in., by this area of their square faces, 1200 sq. in., we have as their probable thickness 4 inches.

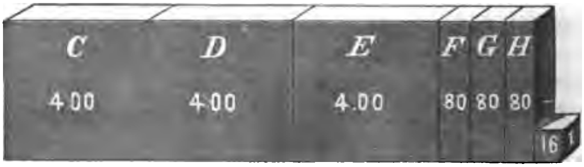


Removing the three rectangular solids, *C*, *D*, *E*, there remain three smaller rectangular solids, *F*, *G*, *H*, whose length is evidently that of the cube *B* removed, and whose width is the thickness of the larger rectangular solids. The area of one face of one of these solids is 20 × 4, and the area of one face of all of them is 20 × 4 × 3, or 240 square inches.



Removing these three smaller solids, there remains the little cube *I*, whose face corresponds in area to the end of one of the smaller rectangular solids removed, or 4^2, or 16 square inches.

Combining the area of the faces of the six rectangular solids with that of a face of the little cube, we have a total area of 1200 + 240 + 16, or 1456 sq. in. Multiplying the area of one face of these seven solids by their thickness, 4, we have 4 × 1456, or 5824 cu. in., which is the number of cubic inches remaining of the original cube, *A*, after the removal of the cube *B*. We therefore conclude that the length of the given cube is 24 inches.



The work is expressed thus:

$$\begin{array}{rr|ll}
 & & \textbf{Contents.} & \textbf{Length.}\\
 & & 13'824 \text{ cu. in.} & (20 \text{ in.} + 4 \text{ in.} =\\
\textbf{Area.} & & 8000 \text{ ''} & [24 \text{ in.}\\
20^2 \times 3 = & 1200 \text{ sq. in.} & 5824 \text{ ''} & \\
20 \times 4 \;\; \times 3 = & 240 \text{ ''} & & \\
4^2 = & 16 \text{ ''} & & \\ \cline{2-2}
 & 1456 \text{ ''} & 5824 \text{ ''} & \\ \cline{2-3}
\end{array}$$

63. What is the cube root of 14348907?

$$\begin{array}{rr|l}
 & & 14'348'907 \; (243\\
 & 2^3 = & 8\\ \cline{3-3}
20^2 \times 3 = & 1200 & 6348\\
20 \times 4 \times 3 = & 240 & \\
4^2 = & 16 & \\ \cline{2-2}
 & 1456 & 5824\\ \hline
240^2 \times 3 = & 172800 & 524907\\
240 \times 3 \times 3 = & 2160 & \\
3^2 = & 9 & \\ \cline{2-2}
 & 174969 & 524907\\ \hline
\end{array}$$

384. Rule for finding the Cube Root of a Number.

Beginning at the decimal point, separate the given power into periods of three figures each.

Find the greatest cube in the left period, and place its root at the right. Subtract the cube of this root from the left period, and to the remainder annex the next period for a dividend.

Annex a cipher to the root already found, and take three times its square for a trial divisor. Divide the dividend by this trial divisor, and place the quotient as the next root figure.

Multiply the number last squared by the last root figure, and add three times the product and the square of the last root figure to the trial divisor for a complete divisor.

Multiply the complete divisor by the last root figure, subtract the product from the dividend, and to the remainder annex a new period.

Form a second trial divisor, and proceed as before until all the periods have been used.

NOTE. — Note 2, under the rule for square root, applies likewise to cube root.

Extract the cube root of the following numbers:

64. 91125.
65. 421875.
66. 571787.
67. 912.673.
68. 3796416.
69. 12977875.
70. 60236.288.
71. 101847563.
72. 258474853.
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74. What is the cube root of 8.144865728 ?

8.′144′865′728′ (2.012
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Here, as 0 occurs in the root, we annex 00 to the trial divisor, 1200, and bring down to the corresponding dividend another period.

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84. What is the cube root of $\frac{729}{4096}$?

Solution. — $\sqrt[3]{\frac{729}{4096}} = \frac{\sqrt[3]{729}}{\sqrt[3]{4096}} = \frac{9}{16}$.

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QUESTIONS.

360. What is a power? The first power? The second power? The third power? 361. The exponent of a power? 362. What is involution?

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369. The square of a number contains how many times as many figures as its root? 378. The cube of a number contains how many times as many figures as its root?

MENSURATION.

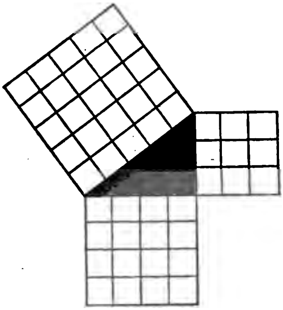
386. Mensuration treats of the measurement of lines, surfaces, and solids or volumes.

RIGHT-ANGLED TRIANGLES.

A Right-angled Triangle.

387. A **Right-angled Triangle** has one right angle.

The *Hypothenuse* is the side opposite the right angle, and the *Perpendicular* is the side perpendicular to the base.



388. It will be seen from the diagram that

The square of the hypothenuse is equal to the sum of the squares of the other two sides. Hence,

389. To find the hypothenuse,

Take the square root of the sum of the squares of the other two sides.

390. To find the base or perpendicular,

Take the square root of the difference of the squares of the hypothenuse and the other side.

1. The base of a right-angled triangle is 8, and the perpendicular 6. What is the hypothenuse?

Solution. — $8^2 + 6^2 = 64 + 36 = 100$; $\sqrt{100} = 10$.

2. The hypothenuse is 30, and one of the sides is 18. What is the other side?

Solution. — $30^2 - 18^2 = 900 - 324 = 576$; $\sqrt{576} = 24$

3. What must be the height of a ladder to reach to the top of a house 20 feet high, the bottom of the ladder being placed 15 feet from the base of the house?

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QUADRILATERALS.

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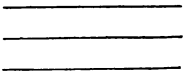
392. **Parallel Lines** are lines in the same plane having the same direction.



Parallel Lines.

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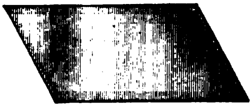
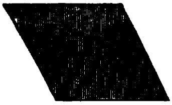
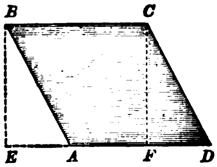


A Rhomboid.



A Rhombus.

A *Rectangle* (Art. 166) is a right-angled parallelogram; a *Rhomboid* is a parallelogram having no right angles; and a *Rhombus* is a rhomboid having equal sides.



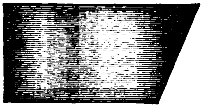
394. It will be seen by the diagram that the rhombus *A B C D* is equal to the rectangle *E B C F* of the same base and altitude (Art. 218). Hence,

The area of a parallelogram is equal to the product of the base and altitude.

9. What is the area of a parallelogram whose base is 36 feet and altitude 15 feet?

✓ **10.** The base of a rhombus is 16 feet and its height 12 feet. What is its area?

11. What is the difference in the area of two floors, the one being 37 feet long and 27 feet wide and the other 40 feet long and 20 feet wide?



A Trapezoid.

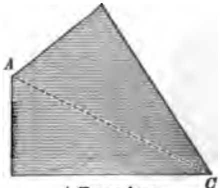
395. A **Trapezoid** is a quadrilateral having only two of its sides parallel.

396. *The area of a trapezoid is equal to the product of half the sum of the parallel sides and the altitude.*

✓ **12.** What is the area of a trapezoid, the longer of the two parallel sides being 120 feet, the shorter 100 feet, and the altitude 85 feet?

13. What is the area of a plank whose length is 6 meters, the width of one of the parallel ends being 60 centimeters and the other 40 centimeters?

14. The parallel sides of a field are 131 and 243 yards, and the breadth 220 yards. How many acres does it contain?



A Trapezium.

397. A **Trapezium** is a quadrilateral having no two of its sides parallel.

A *Diagonal* is a straight line joining any two angles of a plane figure not adjacent, as the line *A C*.

398. It will be seen from the above diagram that a diagonal divides a trapezium into two triangles. Hence,

The area of a trapezium is equal to the product of the diagonal and half the sum of the perpendiculars drawn to the diagonal from the vertices of opposite angles.

NOTE 1. — Any plane figure bounded by straight lines is called a *Polygon*, and may be divided into triangles; and the sum of the areas of the triangles will be the area of the figure.

NOTE 2. — For the *Circle* see Art. 221.

15. The diagonal of a trapezium is 16 feet, and the perpendiculars upon it from the opposite angles are 7 feet and 5 feet. Find the area.

Solution. — $\frac{7 \text{ ft.} + 5 \text{ ft.}}{2} \times 16 = 96$ sq. ft.

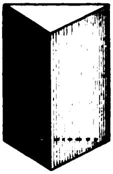
x **16.** What is the area of a trapezium whose diagonal is 65 feet, and the length of the perpendiculars let fall upon it from opposite angles is 14 feet and 18 feet?

x **17.** How many square yards of paving are there in a trapezium whose diagonal is found to measure 126 feet 3 inches, and the perpendiculars upon it 58 feet 6 inches and 65 feet 9 inches?

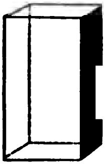
PRISMS.

399. A **Prism** is a body having two equal parallel polygons as bases and the other faces parallelograms.

A prism is *triangular*, *quadrangular*, *pentagonal*, etc., according as its bases have *three* sides, *four* sides, *five* sides, etc.



A Triangular Prism.



A Quadrangular Prism.

400. *The contents of a prism are equal to the product of the area of the base by the altitude or length.*

NOTE. — For the *Cylinder* see Art. 225.

18. What are the contents of a triangular prism whose length is 15 feet and the area of its triangular base is 6 square feet?

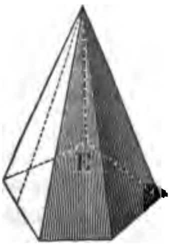
19. What are the contents of a quadrangular prism whose length is 6 meters, and whose base is 18 by 20 centimeters?

20. The altitude of a pentagonal prism is 20 feet 6 inches, and the area of its base 1075.30 square inches. What are its contents in cubic feet?

PYRAMIDS AND CONES.

401. A **Pyramid** is a body whose base is any polygon, and whose sides are triangles meeting at a point called the vertex of the pyramid.

A pyramid, like a prism, is *triangular*, *quadrangular*, *pentagonal*, etc., according to the form of its base.

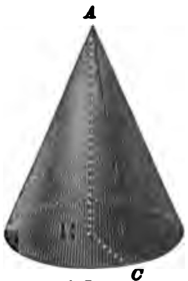


A Pyramid.

402. A **Cone** is a body whose base is a circle, and whose convex surface tapers uniformly to a point called the vertex of the cone.

The *altitude* of a pyramid or cone is the shortest distance from the vertex to the center of the base; as *A B.*

The *slant* height is the shortest distance from the vertex to the perimeter of the base; as *A C.*



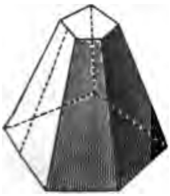
A Cone.

403. *The contents of a pyramid or cone are equal to the product of its base by one third of the altitude.*

✓**21.** What are the contents of a triangular pyramid whose altitude is 14 feet 3 inches, and the area of whose base is 14.70 square feet?

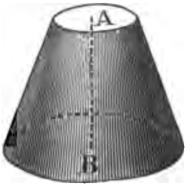
✓**22.** What are the contents of a cone whose altitude is 15.06 meters, the circumference of the base being 12.5 meters?

✓**23.** The largest of the Egyptian pyramids is square at its base, and measures 693 feet on a side. Suppose its other sides to meet at a point 500 feet above the base. What are the contents of the pyramid in cubic feet?



Frustum of a Pyramid.

404. A **Frustum** of a pyramid or a cone is the part which remains after cutting off the top by a plane parallel to the base.



Frustum of a Cone.

405. *The contents of the frustum of a pyramid or cone are equal to the sum of the areas of the two bases plus the square root of their product, multiplied by a third of the altitude.*

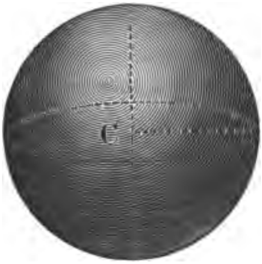
✓ **24.** What are the contents of the frustum of a square pyramid whose altitude is 30 feet, and whose side at the base is 20 feet and at the top 10 feet?

Solution.

$$20 \times 20 = 400;\ 10 \times 10 = 100;\ 400 \times 100 = 40000.$$
$$\sqrt{40000} = 200;\ 200 + 400 + 100 = 700.$$
$$30 \div 3 = 10;\ 700 \times 10 = 7000 \text{ cu. ft.}$$

✓ **25.** What are the contents of a column whose altitude is 28 feet 6 inches, and whose diameter at the larger end is 3 feet and at the other 2 feet 6 inches?

26. How many cubic feet in a square stick of timber whose length is 18 feet 8 inches, and whose side at the larger end is 27 inches and at the smaller is 16 inches?



A Sphere.

406. A **Sphere** is a body bounded by a curved surface, all parts of which are equally distant from a point within called the center.

The *Diameter* of a sphere is any straight line drawn through its center, and terminating both ways in the surface; and the *Circumference* is the greatest distance around the sphere.

407. *The surface of a sphere is equal to the product of* 3.1416 *by the square of the diameter;* and

The contents of a sphere are equal to the product of $\frac{1}{6}$ *of* 3.1416 *by the cube of the diameter.*

27. What is the surface of a sphere whose diameter is 25 inches?

✓**28.** What number of square meters of gold-leaf will gild a globe 18 centimeters in diameter?

29. How many cubic feet of gas will be required to fill a spherical balloon whose diameter is 50 feet?

✓**30.** How many cubic miles of volume has the moon, allowing its diameter to be 2100 miles?

SIMILAR SURFACES.

408. Similar Surfaces are surfaces having the same form, without regard to size.

409. *Similar surfaces are to each other as the squares of their corresponding dimensions.* Hence,

The corresponding dimensions of similar surfaces are to each other as the square roots of their areas.

31. A triangle whose base is 12 feet has an area of 72 square feet. What is the base of a similar triangle whose area is 32 square feet?

Solution. — $72 : 32 = 12^2 : x^2$, or $72 : 32 = 12^2 : 8.^2$ Ans. 8 ft.

32. A side of one of two similar triangles is 12 feet, and the corresponding side of the other is 8 feet. If the larger of the triangles has an area of 72 square feet, what is the area of the other?

33. The areas of two similar rectangular fields are, respectively, 9 and 6.25 acres, and the first is 120 rods long. What is the length of the second?

34. The diameters of two circles are, respectively, 100 feet and 50 feet. How much larger is the one than the other?

35. If a pipe whose diameter is 1.5 inches fills a cistern in 5 hours, in what time will another whose diameter is 35 inches fill it?

36. If it costs $ 30 to pave the bottom of a cellar whose width is 16 feet, what will it cost to pave a similar one whose width is 24 feet?

37. A gentleman has a park, in the form of a right-angled triangle, containing 105.55 square meters, the longest side of which is 15 meters. He wishes to lay out another in the same form, whose longest side shall be 3 times the length of the first. Required the area.

SIMILAR SOLIDS.

410. Similar Solids are solids having the same form without regard to size.

411. *Similar solids are to each other as the cubes of their corresponding dimensions.* Hence,

412. *The corresponding dimensions of similar solids are to each other as the cube root of their contents or volumes.*

38. If a cone 3 feet in height contains 1539 cubic feet, what are the contents of a similar cone 2 feet in height?

Solution.—$3^3 : 2^3 = 1539 : x$, or $27 : 8 = 1539 : 456$.

39. Two similar bins contain, respectively, 400 and 600 bushels. If the first is 4 feet deep, how deep is the second?

40. A sugar loaf which is 12 inches high weighs 16 pounds. How many inches may be broken from the base, that the rest may weigh 8 pounds?

41. If a sphere 6 inches in diameter weighs 16.50 kilograms, what is the weight of a sphere of the same material 12 inches in diameter?

42. Suppose the diameters of the earth and the moon to be, respectively, 8000 and 2000 miles, how many times larger is the earth than the moon?

43. If the weight of a well-proportioned man 5 ft. 6 in. in height, is 140 pounds, what should be the weight of Barnum's Chinese giant who is 8 ft. 3 in. in height?

REVIEW.

ORAL EXERCISES.

413. 1. If 6 barrels of flour cost $33, what will 11 barrels cost?

2. If 9 tons of coal cost $ 54, how many cords of wood at $4 a cord would cost as much as 5 tons of coal?

3. If 6 horses in a certain time consume $14\frac{3}{4}$ bushels of oats, how many bushels will 10 horses consume in the same time?

4. If $\frac{2}{3}$ of a barrel costs $ $4\frac{2}{3}$, what will $\frac{5}{8}$ of a barrel cost?

5. If A can do a piece of work in 3 days, and B can do the same in 4 days, what part of the work can they together do in 1 day?

6. If A and B can in 1 day perform $\frac{2}{15}$ of a piece of work, how long will it take them to do the whole?

7. Divide 75 into two numbers that shall be to each other as 7 to 8?

8. The first three terms of a proportion are 7, 8, and 9. What is the fourth term?

9. How long will 40 bushels of oats last 3 horses, if they consume 8 bushels in 2 weeks?

10. If 7 horses consume 3 tons of hay in 6 weeks, how many tons will 6 horses consume in 4 weeks?

11. A and B run a race in which A gains 3 rods in running 11 rods. How far must he run to gain 7 rods?

12. A box of tea lasted a man and his wife 9 months. When the man was absent it would last his wife 12 months. How long would it have lasted the man alone?

13. Smith and Jones hire a pasture together. Smith pays $16 and Jones $20. Smith puts in 12 sheep. How many does Jones put in?

14. A furnished 2 loaves for a lunch, and B furnished 3, while C contributed 30 cents to be divided between A and B. How much should each receive?

Multiply the number last squared by the last root figure, and add three times the product and the square of the last root figure to the trial divisor for a complete divisor.

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WRITTEN EXERCISES.

49. I am offered $7000 cash for my house or $7700 in a year without interest. I can get 10 % for my money. Which is the better offer?

50. I buy butter for 32 cents, and am obliged to sell it for 28 cents. What % do I lose?

51. A suit of clothes marked to sell at $40 is sold at 10 % discount, and yet the seller makes 25 %. What did it cost him?

52. What is $33\frac{1}{3}$ % of the square of the cube root of $\frac{27}{64}$?

53. What is the entire surface of a 14-foot cube?

54. When Washington was inaugurated there were 13 States in the Union, and, March 4, 1881, there were 38. Required the rate per cent of increase.

55. I lost $16\frac{2}{3}$ % of my property by fire and 20 % by failures. How much had I originally if $5700 remained?

56. I buy books at 80 cents, 20 % off, and sell for $1.00, 15 % off, and 5 % off the remainder. How much do I gain?

57. Received a consignment of 2000 barrels of flour, which I sell at $8.50, paying $74 storage and $27 cartage. My commission is $1\frac{1}{2}$ %. How much do I remit?

58. My insurance at $\frac{1}{4}$ % cost me a premium of $62.40, and is $\frac{3}{4}$ of the value of my house. What do I lose in case of its total destruction by fire?

59. Received a remittance of $25375 for the purchase of cotton at $12\frac{1}{2}$ ¢ per pound after deducting my commission of $1\frac{1}{2}$ %. How many pounds do I purchase?

60. I buy 6 % stock at 112. What per cent do I receive on my investment?

61. What sum will cancel a note of $892, Dec. 24, 1881, dated May 30, 1878, drawing $4\frac{1}{2}$ % interest?

62. Sold two pianos for $600 each, gaining 20 % on one and losing 20 % on the other. Required my gain or loss.

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Solution. — $30^2 - 18^2 = 900 - 324 = 576$; $\sqrt{576} = 24$

3. What must be the height of a ladder to reach to the top of a house 20 feet high, the bottom of the ladder being placed 15 feet from the base of the house?

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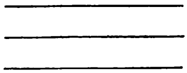

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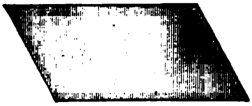
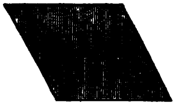
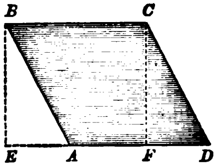


A Rhomboid.



A Rhombus.

A *Rectangle* (Art. 166) is a right-angled parallelogram; a *Rhomboid* is a parallelogram having no right angles; and a *Rhombus* is a rhomboid having equal sides.



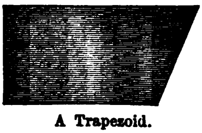
394. It will be seen by the diagram that the rhombus $ABCD$ is equal to the rectangle $EBCF$ of the same base and altitude (Art. 218). Hence,

The area of a parallelogram is equal to the product of the base and altitude.

9. What is the area of a parallelogram whose base is 36 feet and altitude 15 feet?

10. The base of a rhombus is 16 feet and its height 12 feet. What is its area?

11. What is the difference in the area of two floors, the one being 37 feet long and 27 feet wide and the other 40 feet long and 20 feet wide?



A Trapezoid.

395. A **Trapezoid** is a quadrilateral having only two of its sides parallel.

396. *The area of a trapezoid is equal to the product of half the sum of the parallel sides and the altitude.*

12. What is the area of a trapezoid, the longer of the two parallel sides being 120 feet, the shorter 100 feet, and the altitude 85 feet?

94. What is the entire area of a cube containing 592704 cubic inches?

95. If 8 men can make 300 pairs of shoes in 6 days, how many men must be added to their number in order that twice as much may be done in half the time?

96. There are two numbers in the ratio of 6 to 11, and the larger is $167\frac{1}{5}$. What is the smaller?

97. Reduce the ratio $\frac{3}{12} : \frac{5}{8}$ to the smallest integral terms.

98. If $4\frac{5}{16}$ pounds of coffee cost $1.38, what will $11\frac{3}{8}$ pounds cost?

99. If 24 men can build 405 yards of wall in 24 days, how many men will it require to build it in 8 days?

100. A and B are in partnership for one year. January 1, A put in $2000, but B did not put in any until April 1. What did he then put in to have an equal share with A at the end of the year?

101. A and B start from the same place, and travel the same road. A goes 5 days before B, at the rate of 20 miles a day. B follows at the rate of 25 miles a day. In what time and at what distance will he overtake A?

102. A cistern has 3 pipes: the first will empty it in 2 hours, the second in 3 hours, and the third in 4 hours. In what time would they together empty the cistern?

103. A certain piece of labor was to have been performed by 144 men in 36 days, but a number of them having been sent away, the work was performed in 48 days. What number of men was sent away?

104. If 15 carpenters can build a bridge in 60 days by working 15 hours a day, how long will it take 20 men to build the bridge by working only 10 hours a day?

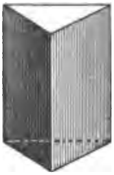
105. Divide $2000 among A, B, and C, so that for every $3 given to A, B is to receive $5, and C $8. What sum did they each receive?

106. How many inches in the diagonal of a square whose side is 24 feet?

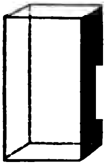
PRISMS.

399. A **Prism** is a body having two equal parallel polygons as bases and the other faces parallelograms.

A prism is *triangular*, *quadrangular*, *pentagonal*, etc., according as its bases have *three* sides, *four* sides, *five* sides, etc.



A Triangular Prism.



A Quadrangular Prism.

400. *The contents of a prism are equal to the product of the area of the base by the altitude or length.*

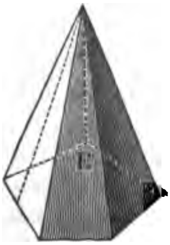
NOTE. — For the *Cylinder* see Art. 225.

18. What are the contents of a triangular prism whose length is 15 feet and the area of its triangular base is 6 square feet?

19. What are the contents of a quadrangular prism whose length is 6 meters, and whose base is 18 by 20 centimeters?

✓ **20.** The altitude of a pentagonal prism is 20 feet 6 inches, and the area of its base 1075.30 square inches. What are its contents in cubic feet?

PYRAMIDS AND CONES.



A Pyramid.

401. A **Pyramid** is a body whose base is any polygon, and whose sides are triangles meeting at a point called the vertex of the pyramid.

A pyramid, like a prism, is *triangular*, *quadrangular*, *pentagonal*, etc., according to the form of its base.

EXAMINATION QUESTIONS

FOR TESTING PROFICIENCY, FOR PROMOTIONS, AND FOR SUPPLEMENTARY PRACTICE. ARRANGED FROM PAPERS USED IN VARIOUS CITIES.

FUNDAMENTAL RULES.

414. 1. Write in words the following: $4238 - 758 = 145 \times 24$.

2. What is the difference between 24 times 325 and 36 times 245?

3. How many more times is 16 contained in 192 than it is in 64?

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2. The dividend is 50000, the quotient is 136, and the remainder 360. What is the divisor?

3. Divide 149184 by 84, and write the quotient in figures and in words.

4. Two men, who are 1224 miles apart, travel towards each other; one 32 miles a day and the other 36 miles a day. In how many days will they meet?

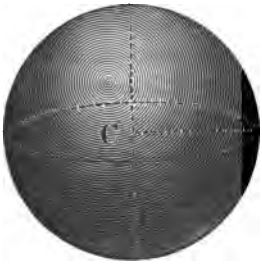
✓ **24.** What are the contents of the frustum of a square pyramid whose altitude is 30 feet, and whose side at the base is 20 feet and at the top 10 feet?

Solution.

$$20 \times 20 = 400;\ 10 \times 10 = 100;\ 400 \times 100 = 40000.$$
$$\sqrt{40000} = 200;\ 200 + 400 + 100 = 700.$$
$$30 \div 3 = 10;\ 700 \times 10 = 7000 \text{ cu. ft.}$$

✓ **25.** What are the contents of a column whose altitude is 28 feet 6 inches, and whose diameter at the larger end is 3 feet and at the other 2 feet 6 inches?

26. How many cubic feet in a square stick of timber whose length is 18 feet 8 inches, and whose side at the larger end is 27 inches and at the smaller is 16 inches?



A Sphere.

406. A **Sphere** is a body bounded by a curved surface, all parts of which are equally distant from a point within called the center.

The *Diameter* of a sphere is any straight line drawn through its center, and terminating both ways in the surface; and the *Circumference* is the greatest distance around the sphere.

407. *The surface of a sphere is equal to the product of 3.1416 by the square of the diameter;* and

The contents of a sphere are equal to the product of $\frac{1}{6}$ *of 3.1416 by the cube of the diameter.*

27. What is the surface of a sphere whose diameter is 25 inches?

✓**28.** What number of square meters of gold-leaf will gild a globe 18 centimeters in diameter?

29. How many cubic feet of gas will be required to fill a spherical balloon whose diameter is 50 feet?

✓**30.** How many cubic miles of volume has the moon, allowing its diameter to be 2100 miles?

SIMILAR SURFACES.

408. Similar Surfaces are surfaces having the same form, without regard to size.

409. *Similar surfaces are to each other as the squares of their corresponding dimensions.* Hence,

The corresponding dimensions of similar surfaces are to each other as the square roots of their areas.

31. A triangle whose base is 12 feet has an area of 72 square feet. What is the base of a similar triangle whose area is 32 square feet?

Solution. — $72 : 32 = 12^2 : x^2$, or $72 : 32 = 12^2 : 8.^2$ Ans. 8 ft.

32. A side of one of two similar triangles is 12 feet, and the corresponding side of the other is 8 feet. If the larger of the triangles has an area of 72 square feet, what is the area of the other?

33. The areas of two similar rectangular fields are, respectively, 9 and 6.25 acres, and the first is 120 rods long. What is the length of the second?

34. The diameters of two circles are, respectively, 100 feet and 50 feet. How much larger is the one than the other?

35. If a pipe whose diameter is 1.5 inches fills a cistern in 5 hours, in what time will another whose diameter is 35 inches fill it?

36. If it costs $ 30 to pave the bottom of a cellar whose width is 16 feet, what will it cost to pave a similar one whose width is 24 feet?

37. A gentleman has a park, in the form of a right-angled triangle, containing 105.55 square meters, the longest side of which is 15 meters. He wishes to lay out another in the same form, whose longest side shall be 3 times the length of the first. Required the area.

SIMILAR SOLIDS.

410. Similar Solids are solids having the same form without regard to size.

411. *Similar solids are to each other as the cubes of their corresponding dimensions.* Hence,

412. *The corresponding dimensions of similar solids are to each other as the cube root of their contents or volumes.*

38. If a cone 3 feet in height contains 1539 cubic feet, what are the contents of a similar cone 2 feet in height?

Solution. — $3^3 : 2^3 = 1539 : x$, or $27 : 8 = 1539 : 456$.

39. Two similar bins contain, respectively, 400 and 600 bushels. If the first is 4 feet deep, how deep is the second?

40. A sugar loaf which is 12 inches high weighs 16 pounds. How many inches may be broken from the base, that the rest may weigh 8 pounds?

41. If a sphere 6 inches in diameter weighs 16.50 kilograms, what is the weight of a sphere of the same material 12 inches in diameter?

42. Suppose the diameters of the earth and the moon to be, respectively, 8000 and 2000 miles, how many times larger is the earth than the moon?

43. If the weight of a well-proportioned man 5 ft. 6 in. in height, is 140 pounds, what should be the weight of Barnum's Chinese giant who is 8 ft. 3 in. in height?

REVIEW.

ORAL EXERCISES.

413. 1. If 6 barrels of flour cost $33, what will 11 barrels cost?

2. If 9 tons of coal cost $54, how many cords of wood at $4 a cord would cost as much as 5 tons of coal?

3. If 6 horses in a certain time consume $14\frac{3}{4}$ bushels of oats, how many bushels will 10 horses consume in the same time?

4. If $\frac{2}{3}$ of a barrel costs $$4\frac{2}{3}$, what will $\frac{5}{8}$ of a barrel cost?

5. If A can do a piece of work in 3 days, and B can do the same in 4 days, what part of the work can they together do in 1 day?

6. If A and B can in 1 day perform $\frac{2}{15}$ of a piece of work, how long will it take them to do the whole?

7. Divide 75 into two numbers that shall be to each other as 7 to 8?

8. The first three terms of a proportion are 7, 8, and 9. What is the fourth term?

9. How long will 40 bushels of oats last 3 horses, if they consume 8 bushels in 2 weeks?

10. If 7 horses consume 3 tons of hay in 6 weeks, how many tons will 6 horses consume in 4 weeks?

11. A and B run a race in which A gains 3 rods in running 11 rods. How far must he run to gain 7 rods?

12. A box of tea lasted a man and his wife 9 months. When the man was absent it would last his wife 12 months. How long would it have lasted the man alone?

13. Smith and Jones hire a pasture together. Smith pays $16 and Jones $20. Smith puts in 12 sheep. How many does Jones put in?

14. A furnished 2 loaves for a lunch, and B furnished 3, while C contributed 30 cents to be divided between A and B. How much should each receive?

Multiply the number last squared by the last root figure, and add three times the product and the square of the last root figure to the trial divisor for a complete divisor.

Multiply the complete divisor by the last root figure, subtract the product from the dividend, and to the remainder annex a new period.

Form a second trial divisor, and proceed as before until all the periods have been used.

NOTE. — Note 2, under the rule for square root, applies likewise to cube root.

Extract the cube root of the following numbers:

64. 91125.
65. 421875.
66. 571787.
67. 912.673.
68. 3796416.
69. 12977875.
70. 60236.288.
71. 101847563.
72. 258474853.
73. 6372.783864.

74. What is the cube root of 8.144865728?

8./144/865/728/ (2.012
8

120000	144865
600	
1	
120601	120601
12120300	24264728
12060	
4	
12132364	24264728

Here, as 0 occurs in the root, we annex 00 to the trial divisor, 1200, and bring down to the corresponding dividend another period.

75. Find the cube root of 64481.201.
76. Find the cube root of 37259704.
77. What is the cube root of 0.000001728?
78. What is the cube root of 1860867?
79. What is the value of $\sqrt[3]{8.144865728}$?

31. The interest of $400 is $70 for 2 y. 6 mo. What is the rate?

32. What is the cube root of the square of $\frac{1}{8}$?

33. Which is the larger, the square of $\frac{3}{4}$ or its cube, and how much?

34. Find a mean proportional between 3 and 48.

35. What is the inverse ratio of $\frac{2}{3}$ to $\frac{3}{4}$?

36. Received a discount for cash of $12\frac{1}{2}$ % on a bill of goods amounting to $128. How much did I pay?

37. Which is the better investment, 6 % stock at 120 or 8 % bonds at 144?

38. Find the missing term in the proportion $2\frac{1}{2} : x = 5 : 7\frac{1}{2}$.

39. A note for $800, which matures May 31, is discounted April 1. Required its proceeds.

40. In selling cloth at $6 per yard, I gain 20 %. What % should I gain if I sold it at $6.40?

41. A bank renewed $100000 worth of U. S. 6 per cents at $3\frac{1}{2}$ %. What is the annual loss of interest?

42. What is the interest of the present worth of a debt of $520 due in a year, money being worth 4 %?

43. When exchange on London is quoted at $4.80, what will a £200 draft cost?

44. I lent Willard Aldrich $1500 for three months. How long should he lend me $1000 to requite the favor?

45. What is the ratio of the square of 4 to the cube root of 125?

46. What is the inverse ratio of a decigram to a kilogram?

47. A party of pleasure-seekers hired a sailboat for $4, each paying as many cents as there were persons in the party. What did each pay?

48. A man with more money than learning willed $\frac{1}{2}$ his property to his wife, $\frac{1}{3}$ to his son, and $\frac{1}{4}$ to his daughter. He left $13000; how ought it to be equitably divided?

WRITTEN EXERCISES.

49. I am offered $7000 cash for my house or $7700 in a year without interest. I can get 10 % for my money. Which is the better offer?

50. I buy butter for 32 cents, and am obliged to sell it for 28 cents. What % do I lose?

51. A suit of clothes marked to sell at $40 is sold at 10% discount, and yet the seller makes 25%. What did it cost him?

52. What is 33⅓ % of the square of the cube root of $\frac{27}{64}$?

53. What is the entire surface of a 14-foot cube?

54. When Washington was inaugurated there were 13 States in the Union, and, March 4, 1881, there were 38. Required the rate per cent of increase.

55. I lost 16⅔ % of my property by fire and 20 % by failures. How much had I originally if $5700 remained?

56. I buy books at 80 cents, 20 % off, and sell for $1.00, 15 % off, and 5 % off the remainder. How much do I gain?

57. Received a consignment of 2000 barrels of flour, which I sell at $8.50, paying $74 storage and $27 cartage. My commission is 1½ %. How much do I remit?

58. My insurance at ¼ % cost me a premium of $62.40, and is ¾ of the value of my house. What do I lose in case of its total destruction by fire?

59. Received a remittance of $25375 for the purchase of cotton at 12½ ¢ per pound after deducting my commission of 1½ %. How many pounds do I purchase?

60. I buy 6 % stock at 112. What per cent do I receive on my investment?

61. What sum will cancel a note of $892, Dec. 24, 1881, dated May 30, 1878, drawing 4½ % interest?

62. Sold two pianos for $600 each, gaining 20 % on one and losing 20 % on the other. Required my gain or loss.

2. The hypothenuse is 30, and one of the sides is 18. What is the other side?

Solution. — $30^2 - 18^2 = 900 - 324 = 576$; $\sqrt{576} = 24$

3. What must be the height of a ladder to reach to the top of a house 20 feet high, the bottom of the ladder being placed 15 feet from the base of the house?

4. The hypothenuse is 157 feet, and the perpendicular 132. What is the other side?

5. Two vessels sail from the same port; one sails due south 48 miles and the other due west 36 miles. What is then their distance from each other?

6. A tree stands upon the edge of a river 100 feet wide, and a line extending from the opposite shore of the river to the top of the tree is 400 feet. What is the height of the tree, to the nearest hundredth of a foot?

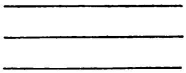
7. A park in the form of a rectangle is 40 rods long and 36 rods wide. What is the length in rods of a walk between its opposite corners?

X 8. A ladder 32 feet long was so placed in a street as to reach a window 25 feet from the ground, and when it was turned to the other side, without changing the position of its foot, it reached a window 20 feet from the ground. How wide was the street?

QUADRILATERALS.

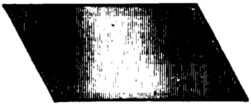
391. A **Quadrilateral** is a plane figure bounded by four straight lines.

392. **Parallel Lines** are lines in the same plane having the same direction.



Parallel Lines.

393. A **Parallelogram** is a quadrilateral having its opposite sides parallel.



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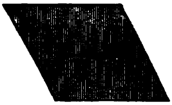
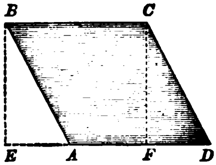


A Rhombus.

A *Rectangle* (Art. 166) is a right-angled parallelogram; a *Rhomboid* is a parallelogram having no right angles; and a *Rhombus* is a rhomboid having equal sides.



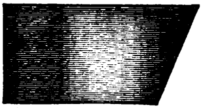
394. It will be seen by the diagram that the rhombus *A B C D* is equal to the rectangle *E B C F* of the same base and altitude (Art. 218). Hence,

The area of a parallelogram is equal to the product of the base and altitude.

9. What is the area of a parallelogram whose base is 36 feet and altitude 15 feet?

10. The base of a rhombus is 16 feet and its height 12 feet. What is its area?

11. What is the difference in the area of two floors, the one being 37 feet long and 27 feet wide and the other 40 feet long and 20 feet wide?



A Trapezoid.

395. A **Trapezoid** is a quadrilateral having only two of its sides parallel.

396. *The area of a trapezoid is equal to the product of half the sum of the parallel sides and the altitude.*

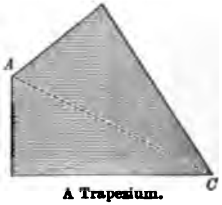
12. What is the area of a trapezoid, the longer of the two parallel sides being 120 feet, the shorter 100 feet, and the altitude 85 feet?

13. What is the area of a plank whose length is 6 meters, the width of one of the parallel ends being 60 centimeters and the other 40 centimeters?

14. The parallel sides of a field are 131 and 243 yards, and the breadth 220 yards. How many acres does it contain?

397. A **Trapezium** is a quadrilateral having no two of its sides parallel.

A *Diagonal* is a straight line joining any two angles of a plane figure not adjacent, as the line *A C*.



A Trapezium.

398. It will be seen from the above diagram that a diagonal divides a trapezium into two triangles. Hence,

The area of a trapezium is equal to the product of the diagonal and half the sum of the perpendiculars drawn to the diagonal from the vertices of opposite angles.

NOTE 1. — Any plane figure bounded by straight lines is called a *Polygon*, and may be divided into triangles; and the sum of the areas of the triangles will be the area of the figure.

NOTE 2. — For the *Circle* see Art. 221.

15. The diagonal of a trapezium is 16 feet, and the perpendiculars upon it from the opposite angles are 7 feet and 5 feet. Find the area.

Solution. — $\frac{7\text{ ft.} + 5\text{ ft.}}{2} \times 16 = 96$ sq. ft.

x **16.** What is the area of a trapezium whose diagonal is 65 feet, and the length of the perpendiculars let fall upon it from opposite angles is 14 feet and 18 feet?

x **17.** How many square yards of paving are there in a trapezium whose diagonal is found to measure 126 feet 3 inches, and the perpendiculars upon it 58 feet 6 inches and 65 feet 9 inches?

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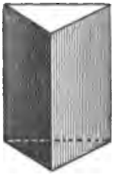

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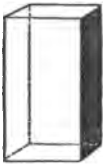

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18. What are the contents of a triangular prism whose length is 15 feet and the area of its triangular base is 6 square feet?

19. What are the contents of a quadrangular prism whose length is 6 meters, and whose base is 18 by 20 centimeters?

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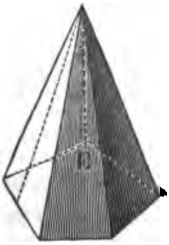
PYRAMIDS AND CONES.



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4. Two men, who are 1224 miles apart, travel towards each other; one 32 miles a day and the other 36 miles a day. In how many days will they meet?

5. The minuend being 26402 and the remainder 18725, what is the subtrahend?

6. The product is how many times the multiplicand? When is it a concrete number?

7. Multiply 40800 by 30600. Why is the product an abstract number?

8. A farmer bought 8 horses for $ 75 each, and 6 horses for $ 125 each, and sold them all for $ 120 each. How many dollars did he gain?

9. Divide 381600 by 123, and prove your work.

10. A man bought 240 acres of land at $ 26 an acre, giving in payment a house valued at $ 2820 and horses valued at $ 180 each. How many horses did he give?

416. 1. What is the difference between a figure and a number?

2. Read 40090.049. What name is given to the number at the left of the point?

3. How may you prove subtraction? Illustrate by an example.

4. Give the sum of all the numbers in the next four examples.

5. If 3008.7 is the minuend and 299.99 is the subtrahend, what is the remainder?

6. If 8467 is the remainder and 44 is the subtrahend, what is the minuend?

7. Multiply 387.5 by 6. Perform the same example by addition.

8. Divide 86784 by 87, and prove the work.

9. A man bought a cow for $ 85, a horse for $ 165, and a carriage for $ 276. How much more did he pay for the carriage than for both horse and cow?

10. A man sold 108 acres of land at $ 205 per acre, and with the money purchased horses at $ 75 each; how many did he get?

417. 1. The multiplicand is 87040, the multiplier is 6080. What is the product?

2. One cord of wood contains 128 cubic feet. How many cubic feet are there in 75 cords of wood?

3. A farmer sold to a flour merchant 45 bbl. of apples at $ 3 per bbl., 65 bbl. of potatoes at $ 2 per bbl., and received in payment 40 bbl. of flour at $ 6 per bbl., and the balance in money. How many dollars did he receive?

4. If a silk dress containing 17 yards costs $ 38.25, what is the cost a yard?

5. $84.61 \times 27 = ?$ At the right of each term write its name.

6. How many pounds of sugar at 12 cents a pound can you get for 18 dozen eggs at 16 cents a dozen?

7. Multiply 814 by 16; add 279 to the product; subtract 384 from the sum, and divide the remainder by 18.

8. Show by an example that either of the factors in multiplication may be used as multiplier without changing the value of the product.

9. A man bought 8 cords of wood at $ 6.50 per cord, 18 tons of hay at $ 21 per ton, 7 bushels of potatoes at $ 0.90 per bushel. He paid $ 75 in cash. How much does he still owe?

10. A man bought ten books; for 4 of them he paid $ 1.50 each, for 3 of them he paid $ 1.80 each, and for the rest he paid 28 cents each. How much did he pay for all?

418. 1. Divide the product of the sum and difference of 125 and 36 by 48.

2. Bought 360 acres of land for $ 32400, and sold it for $ 8400 more than cost. What was the selling price per acre?

3. ($\frac{1}{8}$ of 69543248) − ($\frac{4}{9}$ of 81369) = ?

4. I bought 1265 books, and sold $\frac{1}{5}$ of them at $ 0.50 each and the remainder for $ 0.75 each. How much did I receive for them?

5. Bought 18 lb. of steak at 24¢, 4½ doz. eggs at 36¢, 3 qt. of molasses at 18¢, and a bushel of potatoes for 75¢. Required the amount of my bill.

6. In a certain church there are 40 pews that seat 6 people, 35 that seat 5, and 18 that seat 4. The gallery will accommodate 115. At a lecture every seat is filled; the price of admission being 25¢, what were the proceeds, 62 complimentary tickets having been used?

7. A farmer bought a cow and 254 sheep for $ 1134.50. The cow cost $ 55. What did 54 sheep cost?

8. How many 5-cent pieces in $ 720?

9. Exchanged a farm worth $ 4278 for 75 sewing-machines worth $ 45 each, and $ 200 cash. Did I gain or lose, and how much?

10. $6010 \times 6020 \times 9 = 18 \times ?$

COMMON FRACTIONS.

419. 1. Subtract the sum of all the prime numbers from 1 to 37 from that of all the composite numbers from 4 to 40.

2. Name three composite numbers prime to each other.

3. Reduce $\frac{6}{25}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{7}{12}$ to fractions having the least common denominator, and find their amount.

4. Reduce to their lowest terms $\frac{96}{112}$, $\frac{126}{198}$, $\frac{182}{196}$, and $\frac{873}{1067}$, using in each case the greatest common divisor.

5. What is a *fractional unit*, and what is the *unit of a fraction?* Give an example of each.

6. Find the amount of the following mixed numbers: $4\frac{75}{325}$, $6\frac{84}{126}$, and $12\frac{25}{150}$. Reduce to lowest terms and least common denominator before you add.

7. From $175\frac{3}{4}$ take $95\frac{5}{8}$, and from $45\frac{5}{8}$ take $25\frac{7}{8}$.

8. In what two ways can fractions be multiplied by an integer? Which way is preferable? Why?

9. Multiply $175\frac{5}{7}$ by 12, 124 by $6\frac{3}{4}$, and $6\frac{2}{3}$ by $12\frac{3}{4}$.

10. A man who owned $\frac{3}{4}$ of a ship sold $\frac{2}{5}$ of his interest for $ 30000. What part of the ship did he sell, and what was the *value* of the whole ship at that rate?

420. 1. **Define** a prime number; a composite number; a fraction.

2. Find the greatest common divisor of 182 and 196.

3. Find the least common multiple of 8, 7, 10, 14.

4. Change $13\frac{1}{4}$, $61\frac{4}{9}$, $15\frac{11}{13}$, to improper fractions.

5. Reduce $\frac{225}{375}$ and $\frac{180}{468}$ to their smallest terms, using the greatest common divisors.

6. Reduce $\frac{1225}{35}$, $\frac{2170}{17}$, $\frac{1000}{73}$, to whole or mixed numbers.

7. Reduce $\frac{8}{9}$, $\frac{6}{8}$, $\frac{5}{12}$, $\frac{2}{3}$, to equivalent fractions having the least common denominator.

8. When is it necessary to reduce fractions having different denominators to equivalent fractions having a common denominator?

9. A horse traveled $48\frac{3}{10}$ miles in one day, $56\frac{3}{4}$ the next, $40\frac{12}{32}$ the third, and $45\frac{27}{40}$ the fourth. How far did he travel in all?

10. From a bin containing $25\frac{3}{8}$ bushels of grain there were taken out $5\frac{3}{4}$ bushels at one time and $6\frac{1}{2}$ bushels at another. How much remained?

421. 1. Find the greatest common divisor of 36, 108, and 420.

2. Find the least common multiple of 24, 180, 45, and 60.

3. Required the amount of $12\frac{3}{4}$, $16\frac{2}{3}$, $24\frac{4}{5}$, and $40\frac{7}{8}$.

4. Required the difference between $84\frac{1}{3}$ and $42\frac{7}{8}$.

5. Multiply $\frac{2}{3}$ of $12\frac{5}{8}$ by $36\frac{2}{5}$.

6. Divide $27\frac{3}{4}$ by $\frac{2}{3}$ of $8\frac{1}{2}$.

7. Reduce $\dfrac{\frac{7}{12}}{\frac{5}{8}}$ and $\dfrac{6\frac{3}{4}}{9\frac{2}{3}}$ to simple fractions.

8. If $\frac{2}{3}$ of a farm is worth \$7000, what is $\frac{5}{8}$ of it worth?

9. If a man travels 240 miles in $5\frac{3}{4}$ days, how far will he travel in $3\frac{1}{2}$ days?

10. A coal-dealer sold $\frac{3}{7}$ of what coal he had on hand for \$120, at the rate of \$6 a ton. How many tons had he?

422. 1. Define factoring. Name and define the terms of a fraction.

2. When are two or more numbers said to be prime to each other?

3. Reduce the following by cancellation:

$$\frac{800 \times 378 \times 44 \times 15}{160 \times 63 \times 11 \times 4}.$$

4. Reduce $\frac{4}{5}$ to a fraction having 135 for its denominator.

5. The sum of two numbers is $36\frac{3}{7}$, and one of them is $15\frac{2}{3}$. What is the other?

6. Divide $12\frac{4}{5}$ by $\frac{3}{4}$ of 17.

7. What is the value of an estate if $\frac{2}{3}$ of $\frac{5}{7}$ of it is worth $45000?

8. What is the value of $\frac{3}{4}$ of a farm if $\frac{5}{8}$ of it is worth $4000?

9. Reduce $\frac{\frac{7}{8} \times \frac{2}{3}}{4\frac{1}{2} \div \frac{1}{3}}$ to a simple fraction.

10. Is $\frac{3}{247832}$ a large or a small fraction, and why?

423. 1. Change $\frac{3003}{5005}$, $\frac{87}{145}$, $\frac{429}{715}$, to lowest terms.

2. Add $4\frac{1}{21}$, $3\frac{7}{4}$, $4\frac{8}{3}$, $\frac{6}{7}$.

3. Take $12\frac{6}{7}$ from $31\frac{4}{5}$.

4. Having $$4283\frac{13}{18}$, I gave $$1597\frac{23}{24}$ to my son. How much had I left?

5. Multiply 641 by $5\frac{3}{8}$.

6. Reduce $\frac{1}{17}$ of $3\frac{1}{7}$ of $4\frac{1}{4}$ of $63\frac{2}{3}$ of $\frac{21}{25}$ to a simple fraction.

7. $\frac{8\frac{2}{5}}{9\frac{3}{4}}$ plus $7\frac{1}{2}$ equals what?

8. If $3\frac{1}{2}$ lb. cost $$2\frac{1}{4}$, what will $\frac{3}{4}$ lb. cost?

9. $5\frac{7}{11}$ is contained how many times in $2\frac{1}{10}$?

10. If a bird can fly $10\frac{3}{4}$ miles in $\frac{1}{3}$ of an hour, how far can *it fly in* $2\frac{1}{2}$ hours?

424. 1. Find the sum and the difference of $12\frac{13}{48}$ and $20\frac{7}{12}$.

2. In what two ways can a fraction be divided by a whole number, and which is the better way?

3. If a man can travel $32\frac{4}{5}$ miles in one day, how many miles can he travel in $5\frac{4}{7}$ days?

4. A man received $\$84\frac{1}{5}$ for laboring $18\frac{3}{4}$ days. How much did he receive each day?

5 What is the value of a farm if $\frac{3}{4}$ of it is worth $500 more than $\frac{5}{7}$ of it?

6. If a merchant who owns $\frac{5}{8}$ of a store should sell $\frac{1}{3}$ of his share for $12000, what is the value of the whole store at that rate?

7. A has $12000. If $600 be added to $\frac{3}{4}$ of A's money, the sum will equal $\frac{4}{5}$ of B's money. How much money has B?

8. Reduce $\dfrac{4\frac{2}{3} + 3\frac{1}{2}}{8\frac{3}{4} \times 2\frac{5}{8}}$ to a simple fraction.

9. A man who owned 100 acres of land sold $37\frac{1}{2}$ acres to one man and $\frac{1}{5}$ of the remainder to another. How many acres had he left?

10. A man who owned $\frac{2}{3}$ of $\frac{7}{8}$ of a ship sold $\frac{1}{2}$ of his interest for $32000. What was the value of the whole ship at the same rate?

425. 1. Three cheeses weigh, respectively, $46\frac{5}{8}$, $49\frac{7}{18}$, and $57\frac{1}{4}$ lb. What is their entire weight?

2. What number is that from which if $28\frac{4}{7}$ is taken the remainder is $65\frac{22}{35}$?

3. $41\frac{1}{3} + \overline{56 - 24\frac{7}{30}} - 41\frac{11}{15} = ?$

4. Find the cost of $\frac{4}{5}$ of $156\frac{2}{3}$ acres of land at $\frac{5}{6}$ of $54 an acre.

5. What number multiplied by $33\frac{5}{8}$ will produce $297\frac{1}{2}$?

6. In $2\frac{1}{4}$ acres of land, how many building lots of $\frac{3}{8}$ of an acre each?

7. Find the value of $\frac{\frac{3}{8} \times \frac{11}{21}}{\frac{1}{18} \times 5\frac{1}{2}}$.

8. What part of 9 miles is $\frac{3}{4}$ of 8 miles?

9. $\frac{2}{3}$ of a farm is worth $9000. What is $\frac{5}{12}$ of it worth?

10. If 2 be added to both terms of the fraction $\frac{4}{5}$, will the value be increased or diminished, and how much?

426. 1. $(7\frac{1}{5} - 3\frac{7}{8}) \div 14\frac{7}{9}$.

2. At $3 a day for work, what part of a day's work can be had for $2.50?

3. Multiply 94 by $26\frac{2}{3}$, and from the product subtract the quotient of 12000 divided by $7\frac{3}{4}$.

4. Give an example in solving which it is necessary to use the least common multiple of 63 and 84.

5. The denominator of a certain fraction is $\frac{1}{2}$ of $\frac{3}{7} + \frac{7}{8} \times 3\frac{1}{5}$, and the numerator is $\frac{2}{3}$ of the denominator. What is the value of the fraction?

6. Find the amount of the sum and the difference of $6\frac{2}{3} \times 2\frac{5}{8}$ and $5\frac{1}{3} \div 3\frac{3}{4}$.

7. A boy sold a book for $$2\frac{1}{2}$, which was $\frac{3}{5}$ of the cost. What did he lose by the bargain?

8. Emma, having 2 quarts of berries, ate $\frac{1}{5}$ of them, sold $\frac{1}{3}$ of a quart, and divided the remainder equally among three friends. What did each friend receive?

9. Multiply $24\frac{3}{4}$ by $\frac{7}{8}$. Divide $24\frac{3}{4}$ by $\frac{7}{8}$. How much does the quotient exceed the product?

10. A bought $\frac{3}{8}$ of a farm for $1760. B bought $\frac{1}{2}$ the remainder at the same rate, and C took the remainder at an advance of $375. What did C's share cost?

DECIMALS.

427. 1. Change $\frac{9}{48}$ to an equivalent decimal.

2. From eight hundred thousandths take eight hundred thousandths.

3. *Write* in words: 7.008; 9090.909; 0.00042.

4. Change to an equivalent common fraction in its smallest terms 0.0025.

5. Why is it unnecessary to express the denominator of a decimal fraction?

6. From a piece of cloth containing $49\frac{7}{8}$ yd. I sold $\frac{1}{2}$ of a yd., $3\frac{3}{4}$ of a yd., and 21.125 yd. What was the length of the remnant?

7. Reduce $\frac{3}{8}$, $\frac{4}{5}$, $\frac{5}{16}$, and $\frac{9}{75}$ to decimals, and add.

8. How much will 240 men earn at $\$1.37\frac{1}{2}$ a day?

9. If 4 bushels of beans cost $12.56, what will 9 bushels cost?

10. What part of the quotient of 24 thousandths divided by 25 ten-thousandths is the product of 16 thousandths by 3 hundred?

428. 1. Change $\frac{73}{8000}$ to a decimal.

2. Change 960 hundred-thousandths to a common fraction in smallest terms.

3. Multiply the quotient of 144 ÷ 12000 by the quotient of .0144 ÷ .00012.

4. Subtract 23 ten-millionths from .02 of .006.

5. Divide 17.28 by $.083\frac{1}{3}$.

6. Multiply $.027\frac{1}{2}$ by $.36\frac{3}{4}$.

7. Change $\frac{7\frac{1}{2}}{6000}$ to a decimal.

8. Tell the shortest way of dividing a decimal by 1000. Of multiplying it by 100.

9. If .03 of a number is 90, what is .005 of it?

10. Divide 1.2 by .0025, and subtract the divisor from the quotient.

429. 1. Change $\frac{3}{40}$ to a decimal, multiply by .0008, and divide the product by .02.

2. Divide one thousand by one thousandth, and from the quotient subtract the dividend, the divisor, and their product.

3. Multiply six hundred thousandths by six hundred-thousandths, and divide the product by .02½.

4. All of the six U. S. gold coins are equal in value to how many twenty-five cent pieces?

5. Bought 15280 bricks at $ 40 per thousand. 350 were so broken that they were worthless. What was the actual cost per thousand of those used?

6. If a man receives $ 1500 a year for labor, and his expenses are $ 968, in what time can he save enough to buy a farm worth $ 3724?

7. A man lost 0.60 of his money, and then earned $ 130, when he had 0.83⅓ of the original amount. How much had he at first?

8. What cost 19375 ft. of lumber at $ 17.25 per thousand?

9. Find the amount of the following purchases: 21¾ yd. carpeting at $ 1.75; 25 yd. lining, at 12½ ¢; 2⅞ yd. silk at $ 2.25, and ¾ yd. velvet at $ 2.87½.

10. If my gas bill was $ 11 when I burned 4400 feet of gas, what will it be when gas costs ¼ more and I burn 1500 feet less?

COMPOUND NUMBERS.

430. 1. Find the difference between 25$\frac{5}{18}$ and 15.064.

2. Define a complex fraction; a mixed decimal; a denominate number; reduction.

3. Change $\frac{1}{2}\frac{6}{2}$ to a complex decimal of five places, and 0.0875 to a common fraction.

4. Reduce 65 rd. 2 yd. 1 ft. to the decimal of a mile.

5. Reduce 0.5473 of a pound troy to lower denominations.

6. At $ 5.50 a cord, what is the value of a pile of wood 80 feet long, 12 feet wide, and 4 feet high?

7. A man bought a farm containing 260 acres 45 square rods at $ 25.75 an acre. What was the cost of the farm?

8. In 16.455 metric tons how many kilograms?

9. A man paid $3.46 for sugar and coffee; he bought 6 lb. 5 oz. of coffee at 32 cents a pound, and paid 11 cents a pound for the sugar. How much sugar did he buy?

10. If an average degree of latitude is $69\frac{37}{900}$ common miles, and a meter is one ten-millionth of the distance from the equator to the pole, measured on a meridian, what is the length of a meter in inches?

431. 1. How many years, months, and days from the discovery of America, Oct. 11, 1492, to the Declaration of Independence, July 4, 1776?

2. A farmer had two farms, one of 104 A. 117 sq. rd., the other of 87 A. 78 sq. rd. He reserved 40 A. 40 sq. rd., and divided the remainder equally among his 3 sons. What was the share of each son?

3. If a car runs 16 mi. 25 rd. 12 ft. in 40 minutes, how far will it run at the same rate in 24 hours?

4. If the difference in the time of Greenwich and of St. Louis is 5 h. 55 min., what is the difference of longitude?

5. The aggregate weight of 85 hogsheads of sugar is 39 T. 1625 lb. What is the average weight per hogshead?

6. A ship sails east from Boston, longitude 71° west, 2° 30′ 20″ a day. What is her longitude at the end of 5 days?

7. Define a square; a rectangle; a cube; a solid.

8. If a man wastes 4 minutes a day, how many days, hours, and minutes will he waste in the years 1880 and 1881?

9. There is a fence enclosing a circular field 32 feet in diameter. What will be the area of a square field which the same fence will exactly surround?

10. How many cubic feet of water must be drawn from a reservoir 24 ft. 6 in. long and 20 ft. 9 in. wide, to lower the surface 8 inches?

432. 1. A room measures 14 ft. by 16 ft., and is 8 ft. high. How many rolls of paper 1½ ft. wide will cover the walls, there being 8 yd. in a roll and ⅛ allowed for openings?

2. Divide 2⅞ times ⅘ of 29⅓ by 4⅝ times $\frac{3}{11}$ of 8.

3. Change ⅜ of a great gross to integers of lower denominations.

4. How many square inches in the entire surface of 15 bricks, each 8 inches long, 4 inches wide, and 2 inches thick?

5. What date comes 275 days after May 7?

6. What is the cost of 17 gal. 3 qt. 1 pt. at $ 0.45 per gallon.

7. 3.75 × 48.34½ ÷ .5⅛ = ?

8. How many acres in ½ of a mile of street 60 ft. wide?

9. A room 10 ft. high contains 30000 cu. ft. What will it cost to carpet it at $ .75 per sq. yd.?

10. How many square yards of silk in 300 yd. of 3-inch ribbon?

433. 1. A man paid $ 660 for a piece of land 8 rods long and 10 feet wide, and sold it at 60 cents per square foot. What did he gain?

2. What will 7 bu. 3 pk. 4 qt. of nuts cost at $ 4.80 per bushel?

3. It takes 30 yards of carpeting ¾ yd. wide to carpet a room 15 ft. long. How wide is the room?

4. Bought an acre of land for $ 300, sold ½ of it at 30¢ per square foot, and the remainder at cost. What did I gain?

5. Add 0.24 lb. and 7 oz., avoirdupois.

6. What will 12750 feet of boards cost at $ 27.50 per thousand?

7. What part of an acre is a lot of land 132 ft. long and 66 ft. wide?

8. How many bags, each holding 2 bu. 1 pk. 3 qt., will it take to hold 124 bu. 0 pk. 7 qt.?

9. Find the exact number of days and hours from 9 o'clock A. M., Jan. 7, 1876, to 3 P. M. of March 1, 1881.

10. How many building lots, each 75 ft. by 125 ft., can be made out of 1 A. 46 sq. rd. 18½ sq. yd. of land?

434. 1. Mr. Howes bought a farm, 198 rods long and 150 rods wide, and paid $ 32 an acre. What did it cost him?

2. I bought a board 12 feet long, 16 inches wide at one end and 9 at the other, at $ 30 per M. What did it cost me?

3. How many cubic feet of snow will there be on an acre of land, if it is uniformly 6 inches deep?

4. In a piece of ribbon 2¼ inches wide and 167 feet long, how many square yards?

5. Of a street 50 feet wide, how many feet in length make one acre?

6. How many bricks (each 8 inches long, 4 inches wide, and 2 inches thick) will make a cubical pile 13 feet each way?

7. If a ship sails at the rate of 2 miles in 11 minutes and 11 seconds, how many days will she require to cross the Atlantic where it is 2979 miles wide?

8. I own five contiguous unfenced house-lots. Each lot is 50 ft. wide and 150 ft. deep. How many feet of boards will enclose said lots with a tight board fence 4 ft. high, and what will they cost at $ 16 per M.?

9. A grocer uses a false gallon containing 3 qt. 1½ pt. What is the actual worth of the liquor he sells for $ 240, and what does he make by the cheat?

10. What costs a pile of wood 17 ft. 8 in. long, 8 ft. wide, and 8 ft. 3 in. high, at $ 8.32 per cord?

435. 1. What part of an acre is a rectangular piece of land, 12 rods long and 110 feet wide?

2. A meter is 39.37 inches. How many meters are there in a mile?

3. Gold is 19.35 times as heavy as water. What is the weight in kilograms of a cubic meter of gold?

4. What is the cost of 25 yards 2 feet 3 inches of tubing at $ 0.25 per yard ?

5. Paid $ 5 for constructing 2½ rods of stone wall. What will a wall .875 of a mile in length cost at the same rate ?

6. Knowing the length of a meter, how can you find the length of a degree of latitude ? Write out the statement, but do not perform the work.

7. A pile of wood 8 feet wide and 4 feet high contains forty cords. How long is it ?

8. How many feet of boards will it take to cover the walls of a house 56 feet long, 25 feet wide, and 30 feet high ? How much will they cost at $ 10 per thousand feet ?

9. What will it cost to make a sidewalk 8 ft. wide and 225 ft. long, when it costs $ 30 to make one 10 ft. wide and 90 ft. long ?

10. What will be the cost of constructing a railroad 25 miles 145 rods long, at $ 700 per mile ?

436. **1.** How many yards of carpeting ¾ yd. wide will carpet a room 16 feet long and 14 feet 9 inches wide ?

2. The signal service reports 4½ inches of rain as falling in 24 hours. How many cubic yards fell on ¼ of an acre ? 3053 $\frac{225}{231}$ yds

3. Two ships on the ocean, which are 120 miles apart, are sailing towards each other, one 8¾ miles an hour, the other 10⅝ miles an hour. How far apart will they be in 4 hours and 40 minutes ?

4. What decimal of a mile is 575⅝ feet ?

5. A cubic foot of water weighs 62½ pounds avoirdupois. The specific gravity of gold is 19.25. One pound avoirdupois equals 7000 grains troy. How many grains does a cubic inch of gold weigh ?

6. A farmer sold 71200 pounds of hay at $ 22 a ton, and purchased 19625 feet of boards at $ 15 a thousand. How much money had he remaining ?

7. If a car runs 16 miles 25 rods 12 feet in 40 minutes, how far will it run at the same rate in 4 hours ?

8. The aggregate weight of 67 hhd. of sugar is 39 T. 16 cwt. 35 lb. What is the average weight per hhd.?

9. If the difference in the time of Greenwich and of Boston is 4 hours 44 minutes, what is the longitude of Boston?

10. A room is 18 ft. 8 in. long and 10 ft. 6 in. wide. One kind of carpeting three-fourths of a yard wide can be obtained for $2 per yard; another kind, a yard wide, can be obtained for $1.75 per yard. Which kind is the more expensive, and how much will it cost to carpet the room with each?

437. 1. A field containing 24 acres is 80 rods long. What is its width?

2. What is the difference between 25 rods square and 25 square rods?

3. How many tons of ice can be packed in an ice-house 50 feet long, 20 feet wide, and 12 feet high, a cubic foot of ice weighing 58.5 lb.?

4. What is the difference in time between two places whose difference in longitude is 4° 40′?

5. If 1 ounce of sugar costs 1 cent, what will be the cost of 5 T. 9 cwt. 75 lb.?

6. I have a piece of land 42 rods long and 6 rods wide. I wish to make 7 square lots of equal size. What will be the cost of boundary and cross fences at $2.37½ a rod?

7. What is the value of a pile of wood 12 feet long, 8 feet wide, and 6 feet high, at $4.50 per cord?

8. Mont Cenis Tunnel, which connects the railways of France and Italy, is 7 miles 190 rods long, and the Hoosac Tunnel is 25000 feet long. The latter is what decimal of the former?

9. What is the cost of 4 loads of coal weighing 2436, 2150, 1735, and 3462 lb., respectively, at $5.25 per ton?

x 10. My house is on a corner lot, 100 ft. on one street and 75 ft. on the other. The sidewalk is 8 ft. wide. How many cubic feet of snow do I shovel in clearing my walk after a 15-inch storm?

PERCENTAGE.

438. 1. Express the following as common fractions: $6\frac{1}{4}\%$, $12\frac{1}{2}\%$, $8\frac{1}{3}\%$, $16\frac{2}{3}\%$, $66\frac{2}{3}\%$.

2. What per cent is $\frac{7}{50}$, $\frac{11}{25}$, $\frac{2}{5}$, $\frac{7}{8}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{25}{40}$?

3. Define percentage; base of percentage; rate per cent.

4. A man owns 60 % of a ship and sells 75 % of his share. What part of the whole ship does he sell?

5. What is the difference between $87\frac{1}{2}\%$ of $ 5000, and $.87\frac{1}{2}\%$ of the same sum?

6. The difference in the time of two places is $37\frac{1}{2}\%$ of a day. How many degrees apart are the meridians of those places?

7. How much money has a merchant on deposit if $33\frac{1}{3}\%$ of one third of it is $ 700 more than 25 % of one fourth of it?

8. A man sold a house for $ 5000, which was 25 % more than it cost him. What would have been his gain per cent if he had sold it for $ 6000?

9. A broker received $ 12000 for the purchase of bank stock. The brokerage was $\frac{3}{8}\%$ on the purchase. What did he pay for the stock, and what was the brokerage?

10. What is $62\frac{1}{2}\%$ of a sum of money if $\frac{3}{4}$ of it is $ 1200 more than $\frac{2}{3}$ of it?

439. 1. A man had $ 5420 in bank. He drew out 15 % of it, and afterward $37\frac{1}{2}\%$ of the remainder. How much money had he then in bank?

2. A man sold 160 acres of land for $ 4563.20, which was 8 % less than it cost. What did it cost an acre?

3. A broker received $ 45337.50 to invest in stocks after deducting a commission of $2\frac{1}{2}\%$. What amount did he invest, and what was his commission?

4. A man owns a boat-load of corn valued at $ 1800, and insures $87\frac{1}{2}\%$ of its value at $1\frac{2}{3}\%$. What premium does he pay?

5. $\frac{3}{8}$ of 75 per cent is what per cent of $\frac{2}{3}$ of 90 per cent?

6. A man owning ¾ of a ship sold 45% of his share for $36000. What part of the ship did he still own, and what was its value at the same rate?

7. What per cent of a mile is 124 rods 2 yards 2½ feet?

8. A merchant, after paying 40% of his debts, found that $4800 would pay 75% of the remainder. What was his whole indebtedness?

9. An agent received $3675 to lay out, after deducting his commission of 2½%. What was the amount of his commission?

10. What was the value of the goods purchased, and what was the remittance, when the commission at 3⅓% amounted to $92.80?

440. **1.** What per cent of ⅝ is ⅜?

2. A bankrupt's assets are $45000, and his liabilities $67500. What per cent can he pay?

3. Bought a horse for $200. What must I ask for him in order to gain 10 per cent and still fall 10 per cent on the asking price?

4. The premium for insuring a cargo at 2½ per cent was $1000. What was the amount insured?

5. If ⅔ of a barrel of flour is sold for what ⅘ of the barrel cost, what per cent is gained?

6. A merchant paid $2500 for cotton, and sold it at 10 per cent advance, taking his pay in prints, which he sold at a loss of 10 per cent. Did he gain or lose, and how much?

7. Sold 2 horses for $450 each, thereby gaining 25 per cent on the one, and losing 25 per cent on the other. What was the per cent of gain or loss on the investment?

8. What per cent of $500 is 37½% of $1200?

9. A merchant reduced the price of a piece of cloth 18 cents per yard, and thereby reduced his profit on the cloth from 12½% to 8%. What was the cost of the cloth per yard?

10. In 1864 the greenback dollar was worth only 35$\frac{5}{7}$ cents in gold. What was the price of gold?

441. 1. A owns 35 per cent of a steamboat that is valued at $ 125000, B owns 45 per cent of it, and C owns the remainder. What is the value of each man's share?

2. A farm that cost $ 4500 was sold for $ 5400. What was the gain per cent?

3. A merchant sold $ 65000 worth of goods in a year; 40 per cent of the receipts were sales at 25 per cent profit, and the rest at 30 per cent profit. What was the cost of the goods?

4. An agent received $ 4500 with which to purchase merchandise, after deducting his commission at 2½ per cent. How much did he expend, and what was his commission?

✓ 5. A man had $ 5000 in bank. He drew out 15 per cent of it, then 20 per cent of the remainder, and afterward deposited 12½ per cent of what he had drawn. How much had he then in the bank?

6. A merchant owes $ 15000, and his assets are $ 9525. What can he pay on the dollar?

7. If by selling land at $ 80 an acre I lose 25 per cent, how must I sell it to gain 40 per cent?

8. What must be paid for insuring $ 4500 on a house, at ¾ per cent?

9. A man has secured a policy of $ 6000 on his life, at the rate of $ 26.30 a year on $ 1000. The dividend this year will reduce his payment 35 per cent. What, therefore, will his payment be?

10. A grain dealer bought corn at 55 cents a bushel, and sold it at 66 cents; and wheat at $ 1.10, and sold it at $ 1.37½. Upon which did he make the greater profit, and how much?

442. 1. How many feet are there in 45 per cent of a mile?

2. A man whose income was $ 2800 spent $ 1600. What per cent of his income remained?

3. A man invested in real estate $ 7500, which was 37½ per cent of his property. What was the value of his property?

4. A has $1600. 75 per cent of his money is equal to 62½ per cent of B's money. How much have both together?

5. Alpheus Cole bought 12500 pounds of coal, and received a discount of $16.25, or 20%. What was the price per ton?

6. What per cent of 1880 was the month of December?

7. A merchant, after paying 40% of his debts, found that $4800 would pay 25% of the remainder. What was his whole indebtedness?

8. In a town containing 2576 whites 77% of the inhabitants were colored. Required the population.

9. What was the value of the goods purchased, and what was the remittance, when the commission at 5⅓% amounted to $92.80?

10. A merchant owning 45% of a ship and cargo valued at $125000, paid 4½% for insuring his share. Required the base and the premium.

443. 1. Sold tea at 115 per cent of its cost, and thereby gained 9 cents on a pound. What was the cost per pound?

2. What sum invested at 4½ per cent will yield an annual income of $900?

3. A man owns a house worth $5000. Repairs and insurance average 1 per cent yearly. Would it be more profitable for him to rent the house at $400 per year, or to sell it and invest the money in business which will return him 6 per cent? How much?

4. A merchant received for a lot of goods $874. He had deducted 5% from the face of the bill, and yet found he had made 15% on his investment. What did he pay for the goods?

5. A merchant sold goods for $5895, and gained as much as he would have lost had he sold them for $4585. What was the gain per cent?

6. The premium for insuring a steamer at 4½ per cent was $2925. What was the value of ⅜ of the steamer?

7. A man bought a lot of apples, and sold them for 20 per cent more than they cost, by which he gained $24.80. How much did they cost him?

8. By selling flour at $6.65 per barrel I shall lose 5 per cent of its cost. For how much must I sell it to gain 5 per cent?

9. A farmer made 12% by the purchase and sale of a horse, and thus gained as much as he had lost by selling a $500 lot of land for 7⅕% less than its value. What did he pay and receive for the horse?

10. By selling goods at $1537.90, a profit of 12⅔ per cent was made. What per cent would have been gained or lost if they had sold for $1651.65?

INTEREST AND DISCOUNT.

444. 1. Find the amount of $125 for 3 years 3 months 3 days at 7%.

2. In what time, at 8%, will any principal double itself?

3. What principal, at 7%, will amount to $643.76 in 3 years 4 months 24 days?

4. What is the interest of $475 from January 1, 1880, to July 4, 1882, at 7½ per cent?

5. What is the difference between the interest and true discount of $450 for 1 year 4 months at 6 per cent?

6. The difference between the interest of $450 and the interest of $300 for a certain time is $15.30. Rate, 6 per cent. Required the time.

× 7. A note for $875, dated Jan. 1, 1879, has the following indorsements: March 10, 1880, $225; April 1, 1881, $145. What was due Dec. 31, 1881?

8. Required the avails of a note for $450, due in 6 months, discounted at a bank at 7½ per cent.

9. The proceeds of a 6 months' note discounted at a bank at 6 per cent are $800. Required the face of the note.

10. How much more is the interest of 15 cents for 15 years at 4 % than the interest of 15 dollars for 15 days at 8 % ?

445. **1.** In what time will the interest of $ 480 at 7 % equal the interest of $ 356.50 for 3 years 9 months 25 days at 8 % ?

2. What sum of money will gain $ 153.75 in 3 months 24 days at 7 % ?

3. At what rate per cent will $ 500 gain $ 84 in 2 years 4 months 24 days ?

4. What is the accurate interest of $ 1525 from March 20th to October 20th, at 4½ % ?

5. What is the compound interest of $ 1360 for 1 year 6 months at 8 %, interest compounding semi-annually ?

6. What is the present worth and the true discount of $ 1275, due in 1 year 5 months 18 days, at 6 % ?

7. What is a negotiable note ? How may a note, payable to order, be made negotiable ?

8. When is a note due, if the time for payment is not specified ? If the words "with interest" are omitted, when will interest accrue ?

9. On a note for $ 750 at 6 %, dated Jan. 15, 1878, were the following indorsements: Sept. 20, 1879, $ 250; June 12, 1880, $ 120. What was due May 25, 1881 ?

10. For what sum must a note be drawn, at 9 months 15 days, at 7 %, so that the proceeds, when discounted, may be $ 1240 ?

446. **1.** What is the difference between the simple and the compound interest on $ 700 for 2 years 6 months at 7 per cent ?

2. What is the bank discount on a note of $ 700 for 3 months at the rate of 7½ per cent ?

3. What principal, on interest at 6 per cent, will gain $ 27.47 in 1 year 3 months ?

4. At what rate per cent must $ 648 be on interest, to gain $ 81.873 in 2 years 3 months 17 days?

5. For how much must a note, payable in 30 days, be given, that, when discounted at a bank, $ 900 may be received on it, money being 6 per cent?

6. A note of $ 365 is dated July 1, 1878. Indorsements: Jan. 1, 1879, $ 85; July 1, 1879, $ 125. What was the amount due Jan. 1, 1881?

7. Find the compound interest of $ 245 for 2 years 6 months at 4½ %.

8. What must be the face of a note that, when discounted at a bank for 5 months at 8 %, the proceeds may be $ 217.35?

9. Find the bank discount and proceeds of a note of $450, payable in 90 days, discounted at 8 %.

10. A note of $ 250, dated May 16, 1880, and payable in 4 months, with interest at 6 %, was discounted July 5, 1880, at 7 %. What were the proceeds?

447. 1. What is the interest of $ 105.23 at 6 per cent, from May 6, 1879, to July 7, 1881?

2. In what time will $ 300 gain $ 47.25 at 6 per cent?

3. The interest of $ 560 for 2 yr. 4 mo. 15 d. was $ 106.40. What was the rate per cent?

4. The proceeds of a note discounted at a bank for 90 days, at 8 per cent. is $ 293.80. Required the face of the note.

5. What is the difference between the simple and the compound interest on $ 650 for 2 yr. 8 mo. at 6 per cent?

6. What principal at 8 per cent will gain $ 78.08 in 4 months 24 days?

7. If the interest on $ 500 from Jan. 6, 1880, to April 18, 1880, be $ 12.75, what will the amount of that principal be, Feb. 23, 1881?

8. Note given for $ 320, July 14, 1874, at 8 %; payment, Dec. 24, 1874, $ 180; settlement, March 30, 1875. What was due?

9. What are the avails of a note for $ 1728, due Nov. 18, 1881, and discounted July 6, 1881?

10. $860 compounds semi-annually for eighteen months at 7 per cent. What is the amount?

448. **1.** At 6 % how long will it take $175 to amount to $275?

2. A note of $500 on interest from Jan. 10, 1881, to July 10, 1881, amounted to $529.16⅔. What was the rate?

3. A note of $1250, dated July 5, 1868, was paid June 1, 1870, with interest at 8 per cent. What was paid?

4. What is the difference between the bank discount and the true discount of $450 due in 60 days, discounted at 6 per cent?

5. On a note for $425, at 8 per cent, dated March 25, 1880, were the following indorsements: June 1, 1881, $75; Dec. 30, 1881, $120. What was due Sept. 1, 1882?

6. If I pay a debt of $1410, 2 yr. 6 mo. before it is due, what discount should be made, money being worth 7 %?

7. For what sum must a note, dated May 10, for 3 months, be drawn to yield $395.80, if discounted at a bank June 10, money being worth 6 %?

8. June 15, 1880, George Page borrowed of Henry Smith $2000, and gave his note for the same, with interest, at 8 per cent. Aug. 27, 1881, a payment of $1450 was made, and a new note given for the balance. For what sum was the new note given? Write the note in its proper form.

9. A capitalist holds half a million of extended U. S. Government 5's, now paying 3½ %. What is his quarterly income?

10. Sold an 8 % R. R. stock which cost me 160, and bought Government 4's at 120. Did I increase or lessen my annual income?

STOCKS AND AVERAGE PAYMENTS.

449. **1.** A 6 % stock, bought at 120, pays what per cent on the investment?

2. What sum invested at 4½ % will yield an annual income of $1800?

3. Paid $ 9000 for stock at 10 % below par, and sold it at 112. What per cent did I gain?

4. Bought 40 shares of railroad stock, par value $ 100 per share, at 33 % below par, and immediately sold them at 20 % below par. What per cent did I gain by the transaction?

5. How many shares of B. & A. R. R. stock can be bought for $ 25000 at 164½ and brokerage at ¼, and how much money will be left?

6. If A lends B $ 300 for 4 months, how long ought B to lend A $ 800 to equal the favor?

7. If a person, owing $ 700, payable in 10 months, pay $ 300 down and $ 200 at the end of 6 months, how long after the end of 10 months may he delay payment of the balance?

8. A merchant owes in London £ 500 6 *d.* How much must he pay for a draft at $ 4.86½ a pound sterling, to remit in payment?

9. March 11, 1880, a merchant sold goods to the amount of $ 1850, on a credit of 4 months. April 7, he received $ 400; May 15, $ 270; and June 20, $ 350. When in equity should the balance be paid?

10. Mr. Adams bought goods Aug. 1, 1881, to the amount of $ 2400; for ¼ of the bill he was to pay cash, ⅓ of it he was to pay in 6 months, and the balance in 10 months. On what day may he equitably pay the whole?

PROPORTION.

450. **1.** The ratio is 2⅖, the antecedent ½ of ⅝. What is the consequent?

2. At the rate of 72 yards for £ 44 16 *s.*, how many yards of cloth can be bought for £ 5 12 *s.*?

3. If a bin 8 ft. long, 4½ ft. wide, and 2½ ft. deep, holds 67½ bu., how wide, must another bin be made, that is 18 ft. long and 3⅝ ft. deep to hold 450 bu.?

4. If 7 men build $6\frac{4}{7}$ rods of wall in 15½ days, in how many *days can 12* men do as much?

5. If 10 cents will buy a 6-ounce loaf, when flour is $11 per bbl., how large a loaf will 11 cents buy, when flour is $10 per bbl.?

6. If 9 men, by working 8 hours per day, can mow 30 acres of grass in 2½ days, how many acres can 5 men mow in 3⅔ days, by working 7½ hours per day?

7. A vessel has provisions for 50 men for 4½ months, allowing each man 1½ lb. per day. How long would the same provisions furnish 75 men, allowing 14 oz. per day to each?

8. If 36 men can dig a trench 60 rods long in 48 days, working 8 hours a day, how many men will dig a trench 80 rods long in 32 days, working 6 hours a day?

9. If 19 men build a wall 25 rods long, 4 feet thick, and 3 feet high, in 8 days, working 8 hours and 30 minutes each day, how many men will it take to build a wall 45 rods long, 7½ feet thick, and 6 feet high, in 9 days, working 9 hours and 30 minutes each day?

10. If 8 men can mow 36 acres of grass in 9 days of 9 hours each, how many men will be required to mow 48 acres in 12 days, when the days are 12 hours long?

451. 1. A and B are partners in trade, B contributing 5/9 of the capital. What is A's share of the gain, the whole gain being $4500?

2. Three men rent a pasture for $55. The first puts in 3 cows 2 months, the second 2 cows 4 months, and the third 2 horses 3 months. Each horse eats ⅓ more than a cow. What part of the rent should each man pay?

3. Ames and Howe entered into partnership the first of January, and each put in $2000. The first of May, Ames put in $1000 more. At the end of the year the profits proved to be $2800. What should each receive?

4. Four men hired a pasture containing 280 acres at $1.25 per acre. A pastured 125 sheep; B, 150; C, 200; D, 225. How much rent *ought* each to pay?

5. A, B, and C gain $ 2250. A's gain is $ 800, and B's $ 1000. C's capital is $ 3000. What is the stock of each?

6. Jan. 1, 1880, three persons began business with $ 1300 furnished by A. March 1, B put in $ 1000; August 1, C put in $ 900. The profits were, at the end of the year, $ 750. Find the gain of each partner.

7. Two partners engaged in business. One furnished $\frac{3}{5}$ of the whole capital, and the other furnished $ 4000. They gained in trade 20 per cent of their capital, but lost $ 500 from bad debts. What was each partner's share of the net gain?

8. Three persons engage in trade with a joint capital of $ 37680. A puts in $ 6 as often as B puts in $ 10, and as often as C puts in $ 14. Their annual gain is equal to C's stock. What is each partner's gain?

9. A can do a piece of work in 6 days, B can do it in 8 days, and C in 10 days. In what time can they do it by working together?

10. A man owes A $ 1800, B $ 750, and C $ 1950. His assets are: money on hand, $ 205; a horse and carriage valued at $ 260; and a stock of goods for which he is offered $ 1200. What per cent can he pay, and what will A, B, and C each receive?

ROOTS AND MENSURATION.

452. 1. Extract the square root of 925444.

2. $\sqrt{.1369} + \sqrt{1296} = ?$

3. If a line 160 feet long will reach from the top of a steeple 130 feet high to the opposite side of the street, what is the width of the street?

4. If it cost $ 312 to enclose a field 216 rods long and 24 rods wide, what will it cost to enclose a square field of equal area with the same kind of fence?

5. What is the difference between the cube and the cube root of 0.008?

6. In the center of a square garden there is a pond covering an area of 810 square feet, which is a tenth of the whole garden. How many rods of fence will enclose the garden?

7. Extract the cube root of 34012224.

8. A rectangular piece of land is 800 meters long and 600 meters wide. If you walk at the rate of a hektometer in $1\frac{1}{8}$ minutes, how long will it take you to walk from one corner to another, diagonally across the piece?

9. What are the dimensions of a cube that has the same volume as a box 12 feet 6 inches long, 10 feet wide, and 5 feet high?

10. The altitude of a pyramid having a square base is 80 feet; the length of each side of the base is 120 feet. Required its slant height, surface, and contents.

453. 1. If it costs $ 75 to paint a house 45 ft. long, what will it cost to paint a similar house 60 ft. long?

2. The pedestal of a certain monument is a cubical block of granite containing 373248 cubic inches. What is the length of one of its sides?

3. Two flagstaffs, one 80 feet high and the other 116 feet high, stand 160 feet apart on a horizontal plain. What is the distance between their tops?

4. In a square field of one acre, if a man mow a space one rod in width around it next the inner border, how many square rods does he mow?

5. A and B start from the same point, each walking 12 hours per day. A walks due north, at the rate of a mile in 15 minutes, while B walks due east, at the rate of a mile in 12 minutes. How far apart will they be at the end of the fourth day?

6. Find the entire surface and the diagonal of a cube containing 262144 cubic inches.

7. How many globes 4 inches in diameter will it take to equal in volume a globe 12 inches in diameter?

8. How much more ground will a board 20 ft. long, 2 ft. wide, protect from rain falling vertically, when it is flat on the ground, than when one end is raised 12 ft. ?

9. Two rafters, each 35 feet long, meet at the ridgepole of a roof 15 feet above the attic floor. What is the width of the house ?

10. I have a square garden, 100 feet on a side; I wish to enclose the garden by a ditch 4 feet wide. How deep must the ditch be dug that the earth thrown out may raise the whole surface of the garden one foot ?

MISCELLANEOUS.

454. **1.** Reduce to a simple fraction $\frac{\frac{3}{4} \text{ of } \frac{4}{5} \text{ of } 37\frac{1}{2}}{24\frac{1}{2} - 18\frac{3}{4}}$.

2. What decimal of $\frac{7}{8}$ of a ton is $\frac{3}{8}$ of a cwt. ?

3. A man bought a house for $2000, and sold it for 25% more than he paid for it and 12½ % less than he asked for it. What did he ask for it ?

4. On a note for $400, at 7 %, there was paid $100 annually for 3 years. How much remained due 3 years 4 months from the date of the note ?

5. Required the bank discount and the proceeds of a note for $1250, due in 90 days at 7 %.

6. Required the time of day, provided the time past noon equals $\frac{2}{3}$ of the time to midnight.

7. Find the cost of raising the surface of ¼ of an acre 9 inches, at 50 cents per cubic yard of earth.

8. If it cost $312 to enclose a field 206 rods long and 24 rods wide, what will it cost to enclose a square field of equal area with the same kind of fence ?

9. What is the cube root of 74088 ?

10. How many days will 21 men require to dig a ditch 80 feet long, 8 feet wide, and 4 feet deep, if 7 men can dig a ditch 60 feet long, 6 feet wide, and 3 feet deep in 12 days ?

455. 1. How much cloth $\frac{3}{4}$ of a yard wide will cover 27 tables 6 ft. long and 3 ft. 2 in. wide?

2. What is the amount at compound interest of $ 200, at 8 per cent, for 2 y. 6 mo. 6 d.?

3. What is the length of one side of a square farm containing 102 A. 64 sq. rd.?

4. What is the length of a cubical block of granite containing $47\frac{325}{512}$ cubic feet?

5. Divide 600 by .012, multiply the quotient by .05, and divide .005 by that product.

6. The first term of a proportion is $(4\frac{2}{7} \div .03)$, the second is $6\frac{1}{2}$, and the fourth is $8\frac{1}{3}$. Required the third term.

7. Bought a hogshead of wine for $ 420; but $10\frac{1}{2}$ gallons having leaked out, how must I sell the remainder per gallon to gain 25 per cent?

8. What is the interest of $ 1200, at $4\frac{1}{2}$ per cent, from Aug. 18, 1880, to April 30, 1882?

9. If 25 men, in 6 days of 10 hours each, build 200 rods of wall, how many rods will be built by 12 men in 5 days of 8 hours each?

10. My friend lends me $ 7000 for 15 days. How long must I lend him $ 7500 to requite the favor?

456. 1. In the number 78.342, the value of the 7 is how many times as great as the value of the 2?

2. From $\frac{3\frac{1}{3}}{7\frac{1}{2}}$ of $12\frac{1}{2}$ take $4\frac{5}{6}$.

3. A merchant who owns .36 of a ship sells 85% of his share for $ 22950. What is the value of the whole ship at that rate?

4. The capital stock of a company is $ 80000; its gross earnings annually are $ 8000, and its expenses $ 3200. What per cent on his investment does a stockholder receive who purchased at 20% above par?

5. How much more is the compound than the simple interest on *$ 625 at 8% for 2* years 6 months?

6. Find the date when due, the bank discount, and the proceeds of a note of $ 1250, dated Feb. 12, 1881, payable in 90 days.

7. If 6 men can reap 14 acres of wheat in 2 days of 10 hours each, in how many days of 8 hours each can 5 men reap 25 acres ?

8. The length of a rectangular field containing an acre is twice its width. What is the length of its diagonal ?

9. Find the solid contents of a cone whose height and diameter are each 12 inches.

10. The length of a meter being known, how can the length of the earth's diameter be ascertained ?

457. 1. I hire $ 200 for 1 y. 8 mo. 18 d. at 8 %, at simple interest, and loan it at compound interest. Required my gain.

2. If 8 men in 4 days of 12 hours each can mow 18 acres of grass, how many acres can 6 men mow in 10 days of 9 hours each ?

3. Bought a house for $ 10000 gold, and sold it for $ 15000 currency. If gold at time of selling is worth 125, what is my gain per cent ?

4. How many gross of tacks can be bought for $ 12, if each tack cost .02 of a mill ?

5. What will be the cost of a close fence, 5½ ft. high, around a field 14 rd. long, 12½ rd. wide, the boards costing $ 42 per thousand ?

6. Bought a piece of cloth containing 72 yd. for $ 280 less 10 %, and sold it at $ 4.50 per yd., receiving in payment a note on 90 days, which I immediately have discounted at a bank at 6 %. Do I gain or lose, and how much ?

7. If a bullet ½ inch in diameter weighs 1¾ oz., what will be the weight of a cannon-ball 7 inches in diameter ?

8. What is the area of one side of a cubical block containing the same number of cubic feet as one that is 27 ft. by 8 ft. by 125 ft. ?

9. Sold 20 loads of hay, each weighing 16 cwt. 40 lb., at $ 35 per ton. How long must what I receive be on interest, at 8 % simple interest, to amount to as much as would have been received for 20 tons ?

10. I paid $ 4.50 for a line that would just reach from the top of a spire to the ground, at a distance of 56 ft. from the foot of the spire; price being $ 0.50 per lb., allowing 18 ft. to a pound, what was the height of the spire ?

458. 1. After 4 per cent of a flock of sheep had been killed by dogs, and 68 had been sold to a butcher, $\frac{4}{7}$ of the original flock were left. Required the number of sheep in the flock.

2. A pile of 4-foot wood 256 ft. long and 5 ft. high was sold for $ 152.56. What did a cord cost ?

3. The floor of a public house, 56 by 85 feet, is of boards 14 ft. long, 6 in. wide. There are 8 nails in each board. Allowing 68 nails to a pound, how many pounds are in the floor ?

4. Divide $ 1000 between two children aged, respectively, 6 years and 8 years, in proportion to the square of their ages.

5. If you hire $ 1200 for 1 y. 8 mo. 19 d., at 8 per cent simple interest, and lend it for the same rate and time at compound interest, payable semi-annually, how much do you gain ?

6. The square root of a number is 2304. What is its cube root ?

7. A ¾-in. faucet fills a cistern in 3 hours. How long will a 1½-in. faucet require ?

8. Exchange is 2 per cent below par. How large a 60-day draft can you buy for $ 1939, the rate of discount being 6 per cent ?

9. At what rate will $ 508.50 earn $ 89.609 in 2 y. 2 mo. 13 d. ?

10. Sent my agent $ 7315 with which to buy apples, after deducting 4½ per cent commission. What per cent is his commission of $ 6300 ?

459. 1. What would be my annual income on $10000, invested in 5 per cent bonds at 5 per cent premium ?

2. What would be the cost of a section of U. S. land containing 640 acres at $1.50 per centare ?

3. Calling the earth's diameter 7912.5 miles, what is the length in miles of a minute on the equator ?

4. Give two ways of changing the form of a fraction without changing the value, and explain why the value is not changed.

5. What will it cost to carpet a room 18 by 15 ft. with carpeting 27 in. wide at $2.50 per yard ?

6. Two men starting at different points travel till they meet; one finds his watch 70 minutes fast, the other 120 minutes slow. How far, and in what direction, does each go ?

7. Sold two farms for $3000 each; on one I make 16⅔ per cent, and on the other I lose 12 per cent. Find the net gain or loss.

8. How much must be invested in U. S. 4's at 120 to produce a semi-annual income of $960 ?

9. A man having a field 40 rods square sold to A 100 square rods, to B 4 acres, and to C 20 rods square. How much remained unsold ?

10. What per cent of the square root of 21316 is the cube root of 150568768 ?

460. 1. What will be the cost of fencing a section of land at 12½ cents per foot ?

2. If 40 yards of tapestry will carpet a room 18 feet long, what is its width, tapestry being ¾ yd. wide ?

3. Reduce to a simple fraction $\frac{\frac{3}{4} \text{ of } \frac{5}{7} \text{ of } 12\frac{4}{9}}{18\frac{1}{2} - 12\frac{8}{9}}$.

4. What is the length of a pile of wood containing 12 cords if its width is 9 feet and its height 6 feet ?

5. A merchant bought 45 pieces of cloth, each piece containing 30 yards, at $ 3.75 per yard, on 9 months' credit, and sold the same immediately at $ 4 per yard, on 4 months' credit. What was his gain at date of purchase?

6. Required the proceeds of a note for $ 4500, dated June 15, running 4 months, and discounted August 18 at 7½%.

7. What sum of money will amount to $ 520 in 3 years 9 months, at 4%?

8. If 6 men in 16 days of 12 hours each build a wall 15 feet long, 8 feet high, and 3 feet thick, how many men will be required to build a wall 45 feet long, 9 feet high, and 6 feet thick, in 24 days of 9 hours each?

9. What is the distance, in rods, from the center to each corner of a section of land?

10. How many cubic feet of water must be drawn from a reservoir 24 feet 6 inches long, and 20 feet 9 inches wide, to lower the surface 8 inches?

461. 1. Write a note due in 90 days, dated March 30, 1881, and signed by John Smith, on which Wm. White can obtain $ 850 at a bank, the bank discounting at 8 per cent.

2. Find the difference between 3⅞ divided by .31, and .31 divided by 3⅞.

3. For what must hay be sold per ton to gain 18¾ per cent, if by selling at $ 46 per ton 25 per cent be gained?

4. At $ 50 an acre, what will be the value of a field which can be divided into 12 lots, each 80 feet square?

5. How many reams of paper will be required to make 32 octavo books, each containing 348 pages?

6. A commission merchant receives $ 720 to be expended in butter, reserving his commission of 2½ per cent on the amount expended. He pays 37 cents per pound. How many pounds can he buy?

7. A sells cloth at a profit of 9 per cent; B sells at a profit of 7 per cent, but sells 5 yards while A is selling 3. Which will make a greater profit on a capital of $ 1000? Give the *reason for your answer.*

8. There is a fence enclosing a circular field 48 feet in diameter. What will be the area of a square field which the same fence will exactly surround ?

9. A loans $ 325.50 for 3 y. 4 mo. 24 d., and receives for interest $ 77.469. B loans $ 1.80, and his money amounts in the same time to $ 2.259. Which receives the greater per cent ?

10. Find the difference between the square root and the cube root of .0064

462. 1. What will 37⅞ bushels of salt cost, if 13⅔ bushels cost $ 11.75 ?

2. How many stones 10 inches long, 9 inches wide, and 4 inches thick, will it take to build a wall 80 feet long, 20 feet high, and 2¼ feet thick ?

3. If you buy eggs for 12½ cents, and sell them for 15 cents per dozen, for how much must wood be sold, that cost you $ 4.20 per cord, to gain in the same ratio ?

4. What must I pay for 25 shares of bank stock, at 7⅝ per cent advance, the par value being $ 250 per share ?

5. For how much less than its face should a note of $ 840 be sold, if payable in 4 months ?

6. What is the value of a lot of land in the form of a right-angled triangle, the base being 15 yards and the hypothenuse 25 yards, at 8½ cents per square foot ?

7. In a school-room 35 feet long, 30 feet wide, and 12 feet high, there are 40 pupils, each breathing 10 cubic feet of air per minute. In what time will the air be unfit for respiration, if no pure air be admitted ?

8. A man bought 3 loads of hay. The first load weighed 4832 lb., tare 1124 lb.; the second weighed 4628 lb., tare 1136 lb.; the third weighed 4976 lb., tare 1142 lb. What did it cost him at $ 33.37½ per ton ?

9. A cargo of 4000 bushels of wheat, worth $ 1.20 per bushel, is insured at ¾ of 1½ per cent on ⅔ of its value. If the cargo be lost, how much will the owner of the wheat lose ?

10. Mr. B. mortgaged his farm for $ 6000, Oct. 1, 1879, to be paid in 6 years, with interest at 8 per cent. Three months from date he paid $ 500; Sept. 10, 1880, $ 1126; March 31, 1881, $ 2000; and Aug. 10, 1881, $ 876.50. How much was due at the expiration of the time?

463. **1.** What cost a piece of land 70 rods 5½ feet long, 52 rods 8¼ feet wide, at $ 256 per acre?

2. What is the interest of $ 725 for 9 months 4 days, at 5 per cent?

3. Bought a farm for $ 4200, agreeing to pay $ 600 down and the rest in six equal semi-annual installments. When could I justly make one payment for the whole?

4. A chimney 5 ft. square and 50 ft. high has two flues, each 1 ft. square. How many bricks, 8 in. by 4 in. by 2 in., were used in its construction?

5. Having used my carriage three years, I am willing to sell it at a loss of 20 per cent. If I receive $ 250 for it, what was the cost?

6. What is the circumference of the largest wheel that can be got through a door 5 ft. wide and 12 ft. high?

7. A, B, C, and D hired a pasture for $ 75. A put in 7 cows, 6 weeks; B 3 cows, 13 weeks; C 4 cows and 9 sheep, 7 weeks; D 42 sheep, 5 weeks. A cow would eat as much as 3 sheep. What should each pay?

8. Divide $ 3000 among A, B, and C, giving A $ 50 more than B, and B $ 250 more than C.

9. If a cistern 6 ft. in diameter hold 80 barrels of water, what must be the diameter of a cistern of the same depth to hold 1280 barrels?

10. A can do a piece of work in ⅚ of an hour; B can do ⅚ of it in one hour. In what time can both do it?

APPENDIX.

ROMAN NOTATION.

464. Roman Notation uses seven capital letters in expressing numbers:

I,	V,	X,	L,	C,	D,	M.
1,	5,	10,	50,	100,	500,	1000.

All other numbers are expressed by repeating or combining these letters, according to the following

465. Principles.

1. *Repeating a letter repeats its value.*

Thus, I stands for one; II, two; III, three; X, ten; XX, twenty; XXX, thirty, etc.

2. *When a letter is placed before another of greater value, the difference of their values is expressed.*

Thus, IV stands for four; IX, nine; XL, forty; XC., ninety.

3. *When a letter is placed after another of greater value, the sum of their values is expressed.*

Thus, VI stands for six; XI, eleven; XV, fifteen; XXV, twenty-five, etc.

4. *A dash (—) over a letter increases the value of the letter a thousand fold.*

Thus, $\overline{V}$ stands for five thousand; $\overline{X}$, ten thousand; $\overline{L}$, fifty thousand.

466. TABLE.

I	1	XIV	14	XC	90
II	2	XV	15	C	100
III	3	XVI	16	CC	200
IV	4	XVII	17	CCC	300
V	5	XVIII	18	CD	400
VI	6	XIX	19	D	500
VII	7	XX	20	DC	600
VIII	8	XXX	30	M	1000
IX	9	XL	40	MD	1500
X	10	L	50	$\overline{C}$	100000
XI	11	LX	60	$\overline{M}$	1000000
XII	12	LXX	70	MDCXX	1620
XIII	13	LXXX	80	MDCCCLXXXI	1881

Express in figures:

1. XXXIX.
2. LXXXIII.
3. CDXIX.
4. $\overline{X}$MCCXXII.
5. MMDXLIV.
6. MDCCCXCVIII.

Express in letters:

7. Twenty-nine.
8. Seventy-three.
9. Ninety-eight.
10. One hundred sixty-one.
11. Fifteen hundred eighty.
12. Two thousand eight hundred.
13. One thousand seven hundred seventy-six.

FUNDAMENTAL PROCESSES.

467. The **Fundamental Processes** of arithmetic, or those upon which all others depend, are addition, subtraction, multiplication, and division.

These are also sometimes called the ground rules of arithmetic.

PRIME NUMBERS.

468. No direct method of detecting prime numbers has been discovered.

The prime numbers to 1009 are included in the following

TABLE.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009

CIRCULATING DECIMALS.

469. A **Circulating Decimal** is a decimal in which a figure or a set of figures continually repeats. Thus,

$$\tfrac{2}{3} = 0.666\ldots \text{ and } \tfrac{8}{11} = 0.727272\ldots$$

470. A **Repetend** is the part of the decimal that continually repeats.

It may be marked, when a single figure repeats, by a dot (.) over it, and when a set of figures, by a dot over the first *and the last* of the set. Thus,

$0.666\ldots = 0.\dot{6}$, read repetend 6; and $0.727272\ldots = 0.\dot{7}\dot{2}$, read repetend 72.

471. A **Pure Circulate** has in the decimal no figure but the repetend. Thus,

$0.\dot{9}\dot{3}$ is a pure circulate.

472. A **Mixed Circulate** has in the decimal a part before the circulate. Thus,

$0.9\dot{3}\dot{6}$ is a mixed circulate.

473. A *Circulating Decimal* arises from the reduction to a decimal of a common fraction whose denominator contains other prime factors than 2 or 5.

14. Express the circulate $0.\dot{9}\dot{3}$ as a common fraction in its lowest terms.

$$\begin{array}{r@{\;\times\;}r@{\;=\;}l}
0.\dot{9}\dot{3} & 100 & 93.93\ldots \\
0.\dot{9}\dot{3} & 1 & \underline{0.93\ldots} \\
\hline
0.\dot{9}\dot{3} & 99 & 93.
\end{array}$$

$$0.\dot{9}\dot{3} = \frac{93}{99} = \frac{31}{33}.$$

Solution. — As the repetend has two places of figures, we multiply it by 100, and have 93.93. Subtracting from this once $0.\dot{9}\dot{3}$ gives a number with the same figures as the circulate, but which do not repeat. Thus, 99 times the circulate = 93, and once the circulate must be $\frac{93}{99}$, or $\frac{31}{33}$. That is,

474. *A repetend is equal to a common fraction whose numerator is the figures of the repetend, and the denominator as many 9's as there are figures in the repetend.*

15. Express the circulate $0.09\dot{3}$ as a common fraction in its simple form.

$$\textit{Solution.} — 0.09\dot{3} = 0.09\tfrac{3}{9} = \frac{9\frac{3}{9}}{100} = \frac{7}{75}.$$

Reduce to common fractions in their simplest form:

16. $0.\dot{7}5\dot{3}$.

17. $0.\dot{5}9\dot{4}$.

18. $0.4\dot{2}\dot{5}$.

19. $7.34\dot{5}$.

20. $0.\dot{1}3\dot{5}$.

21. $53.00\dot{2}4\dot{3}$.

22. What decimal will express the difference between $29.25\dot{9}$ and $25.\dot{0}4\dot{7}$?

23. What decimal will express the product of 5.9 by $0.\dot{0}\dot{8}$?

24. What decimal will express the quotient of $4.2\dot{7}$ divided by $0.\dot{4}\dot{2}$?

TABLES.

475. Surveyors' Measures.

7.92 inches are 1 link, l.
25 links " 1 rod, rd.
4 rods " 1 chain, ch.
80 chains " 1 mile, mi.

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10 square chains are 1 acre.

NOTE. — A *Gunter's Chain* is the unit of measure, and is 4 rods, or 66 feet long, and consists of 100 links.

476. Mariners' Measures.

6 feet are 1 fathom.
120 fathoms " 1 cable-length.
7⅓ cable-lengths " 1 mile.

1.15 common miles are 1 nautical mile.
3 nautical miles " 1 marine league.

477. Apothecaries' Measures.

Weight.	*Liquid.*
20 grains are 1 scruple, ℈.	60 minims (♏) are 1 fluid drachm, f. ʒ.
3 scruples " 1 dram, ʒ.	8 fluid drachms " 1 fluid ounce, f. ℥.
8 drams " 1 ounce, ℥.	16 fluid ounces " 1 pint, O.
12 ounces " 1 pound, ℔.	8 pints " 1 gallon, Cong.

478. Miscellaneous.

Blue grass seed	14 lb. = 1 bu.	Oats	32 lb. = 1 bu.
Timothy seed	45 lb. = 1 bu.	Barley	48 lb. = 1 bu.
Clover seed	60 lb. = 1 bu.	Wheat	60 lb. = 1 bu.
Wheat bran	20 lb. = 1 bu.	Beans	60 lb. = 1 bu.
Corn or rye meal	50 lb. = 1 bu.	Castor beans	46 lb. = 1 bu.
Corn or rye	56 lb. = 1 bu.	Potatoes	60 lb. = 1 bu

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100 pounds dry fish are 1 quintal.
196 pounds flour " 1 barrel.
200 pounds beef or pork " 1 barrel.

25. The distance between two places, as measured by a surveyor, is 120 chains 75 links. How far are they apart in miles?

26. A rectangular field is 30 chains 25 links long, and 25 chains 40 links wide. How many acres are there in the field?

27. A piece of land is in the form of a right-angled triangle. The sides forming the right angle measure 14 chains 50 links and 24 chains 20 links. How many acres in the piece?

28. When a vessel at sea has made 500 nautical miles, how many common miles has she gone over?

29. How many prescriptions of 20 grains each can be put up from 1 ℔ 8 ℥ 2 ʒ?

30. How many minims are there in 12 fluid ounces?

31. What is the value of a car-load of corn weighing 20000 pounds at 63 cents a bushel?

32. How much is made by buying 4 barrels of clear pork at $19.50 a barrel, and retailing it at 14 cents a pound?

GOVERNMENT LANDS.

479. Government Lands are divided by parallels and meridians into *Townships*, 6 miles square, each containing 36 *Sections*, or square miles. Each section is subdivided into *half-sections* and *quarter-sections*.

480. The *Townships* are numbered with reference to two special lines, the one running east and west, called the *base* line, and the other running north and south, called the *principal meridian.* Thus,

A township in the fourth tier of townships north of the base line, and the second in that tier west of the principal meridian, would be designated *Township* 4 *N., Range* 2 *W.*

481. The *Sections* into which a township is divided are designated by numbers beginning with the northeast corner section, and running westward with the north tier of sections, eastward with the second tier, and so on. Thus,

A section in the third tier from the north in a township and the fourth toward the west in the tier would be designated section 16.

33. A section is in the fourth tier from the north in a township and the third toward the west in the tier. What is its number?

34. Draw a plan of a township laid off into 36 sections, and designate by number each section?

LONGITUDE AND TIME.

482. **Longitude** is the distance east or west from a given meridian.

All places within 180° east of the given meridian are in *east longitude,* and all places within 180° west are in *west longitude.*

483. To find the difference in longitude between two places,

If both places are on the same side of the prime meridian, take the difference of their longitudes; but if one is east and the other west of the prime meridian, take the sum of their longitudes.

NOTE. — If the sum of the longitudes exceeds 180°, it must be subtracted from 360° to give the true difference in longitude, it being impossible for two places to be more than 180° apart.

484. The earth by turning from west to east causes 360° of longitude to pass under the vertical rays of the sun once in 24 hours, and makes the time shown by the clock later in all places east of any given meridian, and earlier in all places west of it.

485. TABLE.

15° of longitude	correspond to	1 hour in time.
15′ of longitude	"	1 minute in time.
15″ of longitude	"	1 second in time.
1° of longitude	"	4 minutes in time.
1′ of longitude	"	4 seconds in time.

486. From the table is derived the

Rule.

Divide the difference of longitude, in degrees, minutes, and seconds, by 15, and the quotient will give the difference of time in hours, minutes, and seconds.

Multiply the difference in time, in hours, minutes, and seconds, by 15, and the product will give the difference in degrees, minutes, and seconds.

35. When it is noon at New York, 74° 0′ 3″ W., what time is it at San Francisco 122° 26′ 15″ W.?

	122°	26′	15″
	74°	0′	3″
15)	48°	26′	12″
	3 h.	13 min.	45 sec.
	12 h.	0 min.	0 sec.
	8 h.	46 min.	15 sec.

Solution. — Both cities being west of Greenwich, their difference of longitude is found by subtraction. As 15°, 15′, 15″ make a difference of 1 h., 1 min., 1 sec. of time, respectively, $\frac{1}{15}$ of the difference in longitude considered as hours, minutes, and seconds, or 3 h. 13 min. 45 sec., will be the difference in time. San Francisco, being the more westerly, has earlier time than New York. Hence, when it is 12 M. at the latter city, it is 3 h. 13 min. 45 sec. before 12 M. at San Francisco, or 8 o'clock 46 min. 15 sec. A. M.

36. What is the difference of time between Greenwich and Washington, 77° 2′ 48″ west?

37. Paris is in 2° 20′ 15″ east longitude, and Boston 71° 4′ 9″ west. When it is 6 o'clock in the morning in Boston, what time is it in Paris?

38. How many degrees east of Greenwich is Rome, whose time is 49 min. $48\frac{4}{5}$ sec. later than that of Greenwich?

39. The difference of time between Baltimore and New Orleans is 53 min. 30 sec. What is the difference in longitude?

40. A gentleman traveling west from Augusta, Maine, in longitude 69° 50′ west, found, on arriving at Des Moines, that his watch, which was right when he left Augusta, was 1 h. 35 min. 20 sec. faster than the time at Des Moines. What is the longitude of Des Moines?

LEGAL INTEREST.

487. The **Legal Rate** of interest is the rate established by law.

488. When no rate is mentioned the legal rate is that given in the left-hand column; and if specified in writing, any rate not exceeding that in the right-hand column is legal.

TABLE.

State.	Rate.		State.	Rate.		State.	Rate.	
Alabama...	8	8	Louisiana	5	8	N. Y.	6	6
Arkansas	6	10	Maine......	6	Any	N. C.	6	8
California	7	Any	Maryland	6	6	Ohio	6	8
Colorado...	10	Any	Mass.	6	Any	Penn.	6	6
Conn.	6	6	Michigan	7	10	R. I.	6	Any
Dakota ...	7	12	Minnesota	7	10	S. C.	7	Any
Delaware	6	6	Mississippi	6	10	Tennessee	6	6
Florida ...	8	Any	Missouri...	6	10	Texas	8	12
Georgia ...	7	Any	Montana	10	Any	Utah	10	Any
Illinois ...	6	8	Nebraska	7	10	Vermont	6	6
Indiana ...	6	8	Nevada ...	10	Any	Virginia...	6	8
Iowa	6	10	N. H.	6	6	W. Ter. ...	10	Any
Kansas ...	7	12	N. J.	6	6	W. Va. ...	6	6
Kentucky	6	8	N. Mexico	6	12	Wisconsin	7	10

TWELVE PER CENT INTEREST.

489. The **Twelve per cent method** of computing interest is often the most convenient.

At 12 % 1 year's interest = .12 of the principal.
" " $\frac{1}{12}$ y., or 1 month's, interest = .01 " "
" " $\frac{1}{10}$ m., or 3 days', interest = .001 " "

490. Hence, to find interest at 12 %,

Multiply .01 *of the principal by the time in months.*

Or,

Multiply .001 *of the principal by* $\frac{1}{3}$ *of the time in days.*

41. What is the interest of $ 825 for 2 y. 7 mo. 7 d. at 4 %?

$ 825 = Principal.
$ 8.25 = 1 mo's interest.
31.2$\frac{1}{3}$ = Time in months.
275
1650
825
2475
3) $ 257.675 = 12 % interest.
$ 85.89 = 4 % interest.

Solution. — 1 month's interest at 12 % is $\frac{1}{100}$ of the principal, or $ 8.25; 31.2$\frac{1}{3}$ months' interest is 31.2$\frac{1}{3}$ × $ 8.25, or $ 257.675. The interest at 4 % = $\frac{1}{3}$ of 12 % interest, or $ 85.89.

42. Find the interest of $ 840 for 73 days at 10 %.

$ 840 = Principal.
$ 0.84 = 3 days' interest.
24$\frac{1}{3}$ = $\frac{1}{3}$ time in days.
28
336
168
12) $ 20.44 = 12 % interest.
$ 1.703 = 1 % interest.
$ 17.03 = 10 % interest.

491. Connecticut Rule for Partial Payments.

When at least a year's interest has accrued at the time of a payment, and in the case of the last payment, follow the United States Rule.

When less than a year's interest has accrued at the time of a payment, except the last, find the difference between the amount of the principal for an entire year, and the amount of the payment for the remainder of the year after it is made, for a new principal.

When the interest which has accrued at the time of a payment exceeds the payment, find the interest upon the principal only.

Note.—At the option of the teacher, the exercises under the United States Rule (Art. 282), may be performed by this rule.

ANNUAL INTEREST.

492. Annual Interest is interest payable annually.

The annual interest not paid when due draws simple interest.

43. What is the amount due on a note of $500, interest payable annually, on which no payments have been made, at the end of 3 years 6 months and 12 days?

Solution.

Principal	$500.00		
1st annual interest		$30.00	
Int. on 1st annual int. 2 y. 6 mo. 12 d. .			$4.56
2d annual interest		30.00	
Int. on 2d annual int. 1 y. 6 mo. 12 d. .			2.76
3d annual interest		30.00	
Int. on 3d annual int. 6 mo. 12 d. . .			.96
4th annual interest		16.00	
	$500.00	$106.00	$8.26
Total interest, $106 + $8.26 =	114.28		
Amount due	$614.28		

493. Rule for Annual Interest.

Compute the interest annually, and simple interest on each annual interest for the time it shall remain unpaid.

44. A debt of $600 was contracted Jan. 1, 1880; allowing interest annually at 6%, and if no payments be made, what will be due April 1, 1884?

45. A note was given May 16, 1881, for $1250, interest annually at 5%; if no payment is made, what will be due March 16, 1884?

46. A note was given March 14, 1880, for $576, interest annually at 6%. What will be due Sept. 25, 1883, no payments having been made?

494. When **Partial Payments** have been made on a note, or other obligation drawing annual interest, the following is

The New Hampshire Rule.

If in any year, reckoning from the time the annual interest began to accrue, payments have been made, compute interest upon them to the end of the year in which they are made.

The amount of payments is to be then applied, first, to cancel interest upon annual interest; second, to cancel annual interest; and thirdly, to the extinguishment of the principal.

If, however, at the date of any payment there is no interest except the accruing annual interest, and the payment, or payments, do not exceed the annual interest at the end of the year, deduct the payment, or payments, without interest on the same.

Omit the last paragraph of this rule and it is the **Vermont Rule.**

47. What was due, July 1, 1881, on a note dated July 1, 1878, for $1000, with 6% annual interest, and on which was paid, Nov. 1, 1879, $400?

Solution.

Principal		$1000		
1st annual interest			$60.00	
Int. on 1st annual int. 2 y.				$3.60
2d annual interest			60.00	
		$1000	$120.00	$3.60
Payment Nov. 1, 1879	$400.00			
Int. on paym't, July 1, 1880	14.00			
Am't of paym't July 1, 1880	$414.00 =	290.40 +	120.00 +	3.60
Principal, July 1, 1880,		$709.60		
3d annual interest			$42.58	
Due July 1, 1881		$709.60 +	$42.58 =	$752.18

48.

$2000. MANCHESTER, N. H., April 1, 1873.

On demand, I promise to pay Charles West & Son, two thousand dollars, value received, with interest annually.

JAMES GODDARD.

Payments: Sept. 19, 1875, $500; Dec. 3, 1879, $600; and Aug. 9, 1880, $775. What will be due May 19, 1883?

49.

$5000. PLYMOUTH, N. H., Jan. 13, 1874.

On demand, we promise to pay to the order of John M. Monroe & Co., five thousand dollars, value received, with interest annually.

PRESBY & LORD.

Payments: Sept. 23, 1878, $2000; Feb. 19, 1880, $1500; May 29, 1881, $125; and June 11, 1883, $20. What will be due at settlement, Aug. 30, 1885?

NOTE. — At the option of the teacher, each of the above exercises may be performed by the Vermont Rule.

AVERAGE OF ACCOUNTS.

495. The **Balance** of an account is the difference between its debtor and creditor sides.

496. The **Average** of an account is the equitable time of the balance becoming due, or being entitled to interest.

50. What is the balance of the following account, and at what date should the balance begin to draw interest?

Dr. JOHN L. MARTIN. **Cr.**

1880.			1880.		
May 16	Mdse. on 60 d.	$300	May 20	Mdse. on 30 d.	$200
June 3	" " 60 d.	50	July 19	" " 60 d.	200
July 1	" " 30 d.	150			

Solution.

	Debits.			*Credits.*	
July 15,	300 × 26 d. =	7800 d.	June 19,	200 × 0 d. =	0 d
Aug. 2,	50 × 44 d. =	2200 d.	Sept. 17,	200 × 90 d. =	18000 d.
July 31,	150 × 42 d. =	6300 d.		$400	18000 d.
	$500	16300 d.			16300 d.
	400		1700 ÷ 100 = 17 d.		1700 d.
	$100 balance.				

June 19 — 17 days = June 2, average time.

We select for convenience June 19, the earliest date at which any of the items of account become due, as the point of reckoning, and find the aggregate of the terms of credit of the credit items, with reference to the selected date, to be equal to the credit of $ 1 for 18000 days, and the aggregate of the terms of credit of the debit items to be equal to the credit of $ 1 for 16300 days.

Striking the balance, it appears, at the selected date, $ 100 subject to a credit equal to the credit of $ 1 for 1700 days is in favor of John L.

Martin. But the credit of $1 for 1700 days is equal to that of $100 for $\frac{1}{100}$ of 1700 days, or 17 days; hence the $100 was due in equity 17 days before June 19, or June 2.

If, however, the balance of items and terms of credit had been both on the same side of the account, the balance would have been due after, instead of before, the selected date.

497. Rule for Averaging Accounts.

Select the earliest date at which any of the items of account becomes due, and therefrom reckon the terms of credit.

Multiply each term of credit by the number denoting the corresponding item, and divide the balance of the sums of the products by the balance of the sums of the items of the account, and the quotient will be the average term of credit.

When the balances are both on the same side of the account, the time must be added to the selected date, but subtracted from that date when the balances are on different sides.

The same result may be reached by what is called the *Interest Method.* Thus,

Compute interest upon each item for the days intervening, between its becoming due and the earliest date at which any item becomes due. Divide then the balance of interest by the interest of the balance of items for one day, and the quotient will be the average term of credit.

NOTE 1. — A convenient rate of interest in averaging accounts is 12 %, which is .01 of the principal for 30 days and .001 for 3 days.

NOTE 2. — The note under the rule Art. 334 applies in averaging accounts.

51. Find when the balance of the following account averages due.

Dr. WILLIAM HOLT. Cr.

1881.			1881.		
June 20	Mdse. on 30 d.	$600.60	July 15	Cash	$650
July 10	" net	149.40			

52. Find the balance of the following account, and, allowing each item to be on 30 days, the time the balance becomes due.

Dr. BRYANT AND ROBERTS. Cr.

1881.			1881.		
Sept. 30	Mdse.	$550	Oct. 1	Mdse.	$400
Oct. 15	"	850	" 5	"	30

✓**53.** Find the time when a note for the balance of the following equated account should begin to draw interest.

Dr.		Cr.	
Due March 2	$600	Due March 14	$400

54. Find the face of a note which must be given for the balance of the following account, and the date at which it should begin to draw interest.

Dr. JAMES GRIMSHAW. Cr.

1881.			1881.		
Aug. 10	Mdse. on 4 mo.	$200	Sept. 11	Cash.	$60
" 28	" " 6 mo.	200	Dec. 10	"	140
			1882.		
Sept. 1	" net	150	Jan. 29	"	100

BUSINESS FORMS.

RECEIPTS.

498. A **Receipt** is a written acknowledgment that money or other property of value has been received.

Receipt for Payment on Account.

$250. BOSTON, Oct. 19, 1881.

Received from Henry F. Jordan Two hundred fifty dollars on account.

Smith & Watson.

Receipt for Rent.

Received, Portland, May 5, 1882, *from James Johnson, Thirty-one dollars for rent of house, No.* 14 *Austin Street, for month ending April* 30, 1882.

$ 31. *Catharine A. Pettes.*

Receipt in Full.

$ $26\frac{84}{100}$. PHILADELPHIA, Aug. 1, 1882.

Received from John Randall Twenty-six $\frac{84}{100}$ *Dollars in full of all demands to date.*

S. A. Caswell & Co.

ORDERS.

499. **An Order** is a written request to deliver money or goods to some person mentioned, or to his order, or to the bearer, on account of the person signing the request.

Order for Money.

BURLINGTON, Dec. 12, 1881.

Messrs. Simmons & Son,

Gentlemen: Please pay S. P. Wright, or order, Seventy-five Dollars, and charge to our account.

Reed, Pratt, & Co.

Order for Goods.

MANCHESTER, Nov. 4, 1881.

Mr. W. N. Goddard,

Please pay to James Brewer, or order, Sixty Dollars in Goods from your store, and charge to the account of

Charles Dole.

DUE-BILLS.

500. A **Due-Bill** is a simple acknowledgment of a debt *in writing.*

Due-Bill for Goods.

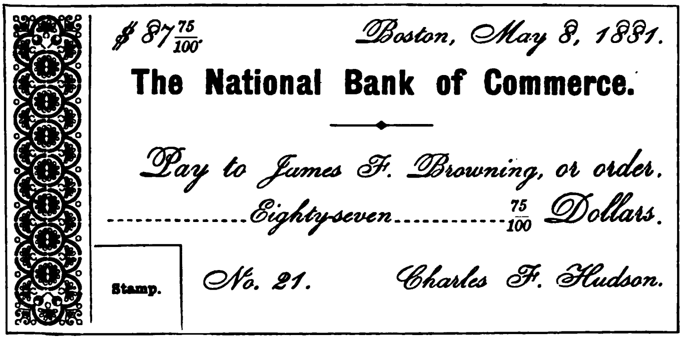
Due, New York, Aug. 9, 1882, *to H. L. Chase, Twenty-three* $\frac{65}{100}$ *Dollars in goods from my store.*

$ 23$\frac{65}{100}$. *Henry C. Carter.*

CHECKS.

501. A **Check** is a written order addressed to a bank by a person having money deposited, requesting the payment on presentation of a certain sum of money to a person named therein, or to his order.

Bank Check.



87\frac{75}{100}$ Boston, May 8, 1881.

The National Bank of Commerce.

Pay to James F. Browning, or order.

..............Eighty-seven............$\frac{75}{100}$ Dollars.

Stamp. No. 21. Charles F. Hudson.

TAXES.

502. A **Tax** is a sum of money assessed upon the person, property, income, or business of individuals for public use.

503. A **Poll Tax** is a tax upon the person, a **Property Tax** is a tax on property, and an **Income Tax** is a tax on income.

504. **Assessors** are officers appointed to take an inventory of taxables, and to apportion the tax to be raised among the tax-payers.

55. A town is to be taxed $14562. The taxable property is $1146000. There are 540 polls, each assessed $1.50. What will be A's tax, whose property is assessed at $8500, and who pays one poll-tax?

Solution.

$1.50 × 540 = $810, sum to be assessed on the polls.
$14562 — $810 = $13752, sum to be assessed on the property.
$13752 ÷ $1146000 = 0.012, or 12 mills on $1 of valuation.
$8500 × 0.012 = $102, A's tax on property.
$102 + $1.50 = $103.50, A's entire tax.

505. Rule for Assessment of Taxes.

Deduct the amount of the poll-taxes, if any, from the entire tax to be raised, and the remainder, divided by the value of the taxable property, will give the rate.

Multiply the value of each individual's taxable property by the rate, and to the product add the poll-tax, if any, and the sum will be the individual's entire tax.

NOTE. — In Massachusetts, the assessors "in each year assess upon the polls the state and county taxes authorized or required by law; provided, however, that in case either of said taxes shall exceed in amount the sum of one dollar upon each poll, the excess above said amount and in every case the whole amount assessed for other purposes shall be apportioned upon property. . . . The state tax assessed upon poll and property and the county tax assessed upon poll and property shall each constitute an entire and indivisible tax." — *Chap.* 299 *of the Acts of* 1879.

Computation of taxes may be facilitated by the construction of a table. Thus, if the rate on $1 is 12 mills, we can have the following

TABLE.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$0.012	$8	$0.096	$60	$0.72	$400	$4.80	$2000	$24.00
2	0.024	9	0.108	70	0.84	500	6.00	3000	36.00
3	0.036	10	0.12	80	0.96	600	7.20	4000	48.00
4	0.048	20	0.24	90	1.08	700	8.40	5000	60.00
5	0.060	30	0.36	100	1.20	800	9.60	6000	72.00
6	0.072	40	0.48	200	2.40	900	10.80	7000	84.00
7	0.084	50	0.60	300	3.60	1000	12.00	8000	96.00

56. What is B's tax, by the table, his valuation being $ 2545, and he paying one poll-tax of $ 1.75?

57. Find, by the table, C's tax on $ 9565, D's on $ 1764, and E's on $ 5630, and each paying a poll-tax of $ 1.50.

58. The taxable property of a certain town is $ 1000000. The number of polls, 600. The tax to be raised is, State $ 348, County $ 1500, and Town $ 12100. The state and county tax are each to be assessed upon the polls, but so much as either of these taxes shall exceed $ 1 on a poll, is, with the town tax, to be assessed upon the property. What will be each poll-tax? What will be the rate of county tax? What will be the rate of town tax? What will be A's county tax, his valuation being $ 5000, and he paying for one poll? What will be his entire tax?

DUTIES, OR CUSTOMS.

506. **Duties**, or **Customs**, are taxes levied on imported goods and the tonnage of vessels.

They are collected at custom-houses by the government officers in charge.

507. A **Specific Duty** is a fixed tax upon an article without regard to its value.

508. An **Ad Valorem Duty** is a tax at a certain rate on the cost of the goods in the country from which they are imported.

509. **Tare** is an allowance made for the weight of the box, cask, etc., containing the goods.

It may be estimated by actual weighing, or by a schedule furnished by the government.

Leakage is an allowance for waste of liquors in casks, and *Breakage* is an allowance on liquors in bottles.

Gross Weight is the weight before any allowances are made, and *Net Weight* is the weight after the allowances are made.

510. **The values of Foreign Moneys, to be taken in estimating the values of foreign merchandise at custom-houses, as declared by the Secretary of the Treasury, Jan. 2, 1882, are shown in the following**

TABLE.

Country.	Monetary Unit.	Standard.	Value in U. S. Money.
Austria	Florin	Silver	$0.406
Belgium	Franc	Gold and silver	0.193
Bolivia	Boliviano	Silver	0.823
Brazil	Milreis of 1000 reis	Gold	0.546
British N. A.	Dollar	Gold	1.00
Chili	Peso	Gold and silver	0.912
Cuba	Peso	Gold and silver	0.932
Denmark	Crown	Gold	0.268
Ecuador	Peso	Silver	0.823
Egypt	Piaster	Gold	0.049
France	Franc	Gold and silver	0.193
Great Britain	Pound Sterling	Gold	4.866½
Greece	Drachma	Gold and silver	0.193
German Empire	Mark	Gold	0.238
India	Rupee of 16 annas	Silver	0.39
Italy	Lira	Gold and silver	0.193
Japan	Yen	Silver	0.887
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	0.894
Netherlands	Florin	Gold and silver	0.402
Norway	Crown	Gold	0.268
Peru	Sol	Silver	0.823
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks	Silver	0.658
Sandwich Islands	Dollar	Gold	1.00
Spain	Peseta of 100 centimes	Gold and silver	0.193
Sweden	Crown	Gold	0.268
Switzerland	Franc	Gold and silver	0.193
Tripoli	Mahbub of 20 piasters	Silver	0.743
Turkey	Piaster	Gold	0.044
U. S. of Columbia	Peso	Silver	0.823
Venezuela	Bolivar	Gold and silver	0.193

59. The Bay State Iron Co. imported from England 200 tons pig-iron, invoiced at £ 725 3 *s.* 4 *d.*, and paid a duty of $7 per ton. *Find the duty and the cost of the importation.*

60. Jordan, Marsh, & Co. import from Paris 3 cases silk goods containing 2664.5 meters, invoiced at 6 francs per meter. They pay a duty of 60%. What does the government receive?

61. What is the duty on an importation from Russia of 516 bales of flax, weighing 7880 poods, at $20 per ton, a pood being a Russian weight of about 36 lb.?

62. Spaulding, Nash, & Co. receive from Cuba an invoice of 50381 gallons of molasses, valued at 11102.7 pesos. They pay a duty of 5¢ per gallon and 25% *ad valorem.* How many dollars do they pay the collector of customs?

63. R. H. White & Co. import from Germany a case of dress-goods, containing 640.4 meters, costing in Hamburg 2.05 marks per meter less a discount of 8%. The width of the goods is 43½ inches. The duty paid is 8¢ per square yard and 40% *ad valorem.* Find it.

MEASUREMENT OF ROUND TIMBER.

511. **Round Timber** (or logs) is usually estimated in cubic feet.

512. **Spars** from 10 to 4 inches in diameter, inclusive, are estimated by the inch diameter, taken clear of bark, at one third of their length from the larger end.

Spars above 7 inches should have 4 feet of length, and below 7 inches should have 5 feet of length, to every inch of diameter.

513. The **Mean Girt** of a tapering piece of round timber is the girt, clear of bark, at one third its length from the larger end.

514. To find the contents of round timber in cubic feet,

Multiply the length in feet by the square of one fourth of the mean girt in inches, and divide the product by 144.

NOTE. — The rule gives about a fifth less than the exact quantity, so much being allowed for crooks and waste.

64. How many cubic feet of timber in a log whose length is 30 feet, and whose mean girt is 42 inches?

65. The mean girt of a piece of round timber is 60 inches, and its length 24 feet. Required its contents in cubic feet.

515. To find the side of squared timber that may be hewn or sawed from a log,

Multiply the diameter of the smaller end of the log by 0.707.

66. The smaller end of a log is 21 inches in diameter. What is the side of the squared beam that may be sawed from it?

67. How many board feet in a log when squared, the length of the log being 18 feet, and its diameter at the smaller end 24 inches?

68. A log is in the form of a cylinder, 6 feet in circumference and 20 feet long. What is the value of the largest squared timber that can be hewn from it, at $30 a thousand feet, board measure?

GAUGING.

516. Gauging is finding the capacity of casks.

517. The **Mean Diameter** of a cask is very nearly equal to the head diameter increased by 0.55 to 0.70 of the difference between the bung and head diameters, according as the staves are curved little or much.

518. To find the capacity of casks,

Multiply the product of the square of the mean diameter and the length, expressed in inches, by 0.0034 *for gallons, or by* 0.0129 *for liters.*

As a cubic foot is about 7½ gallons, in finding the capacity of a cistern it is sufficiently accurate to estimate 7½ gallons to a cubic foot.

69. A cask whose mean diameter is 22 inches and length 30 inches will contain how many gallons?

70. What is the capacity of a cask in gallons whose mean diameter is 30 inches and length 38 inches?

71. What is the capacity in liters of a cask, staves much curved, whose head diameter is 24, bung diameter 30, and length 36 inches?

72. How many gallons in capacity is a rectangular cistern whose inside dimensions are 4 feet 3 inches, 3 feet 6 inches, and 4 feet?

TONNAGE OF VESSELS.

519. The **Tonnage** of a vessel is the number of tons' burden it will carry.

520. Shipwrights generally make their estimates of tonnage by the following

Rule.

For a single-deck vessel, take the length in feet above the deck from the forepart of the mainstem to the after-part of the sternpost, the breadth at the widest part above the main wales on the outside, and the depth from the under side of the deck plank to the ceiling of the hold. From the length subtract three fifths of the breadth, multiply the remainder, breadth and depth, together, and the product divided by 95 *will give the tonnage.*

For a double-deck vessel, take the length above the upper deck, for the depth half the breadth, and proceed as before.

73. What is the tonnage of a single-decked vessel whose length is 75 feet, breadth 20 feet, and depth 9 feet?

74. What is the tonnage of a double-decked vessel whose length is 160 feet and breadth 30 feet?

FARMERS' ESTIMATES.

521. **Grain** in a bin or granary occupies nearly $1\frac{1}{4}$ as many cubic feet as there are bushels.

522. **Corn** on the ear will yield about half its bulk in shelled corn.

523. **Wheat,** according to quality, less a sixth for toll, will yield from 26 to 33 pounds of flour per bushel.

524. **Mixed Hay,** in large mow, is estimated at 500 cubic feet, and **Clover,** at 550 cubic feet to a ton of 2000 pounds.

525. **Horses,** young cattle, and sheep are estimated to consume daily, for each 100 pounds of weight, about 3 pounds of hay; and oxen and cows, about $2\frac{1}{2}$ pounds.

As food for stock, 100 pounds of average meadow hay is equal to about 56 pounds of corn, 56 pounds of wheat middlings, 60 pounds of oats, or 32 pounds of cotton seed meal.

526. **Net Weight** of fat beeves is about $\frac{3}{5}$ of the live weight; of fat swine, $\frac{4}{5}$; of fat sheep, $\frac{1}{2}$; and of fat fowl, $\frac{7}{10}$.

Average beeves, net weight, will cut up: rump and sirloin, $\frac{1}{5}$; thigh and round, $\frac{1}{5}$; forequarter and rattlerand, $\frac{2}{5}$; hide, $\frac{1}{10}$; and tallow, $\frac{1}{10}$.

An average swine, net weight, will cut up: hams and shoulders, $\frac{1}{4}$; and sides and clear pork, $\frac{1}{3}$.

75. I have a bin 8 feet long, 4 feet wide, and 3 feet deep. How many bushels will it contain?

76. A wagon 8 feet long, $3\frac{1}{2}$ feet wide, and 2 feet deep, is filled with corn in the ear. How many bushels of shelled corn will it yield?

77. How much average meadow hay will suffice to keep 3 horses 120 days, whose weight is 1200 pounds each, provided grain is allowed to replace one third of the hay?

78. How many bushels of best wheat must be carried to mill to get back, after allowing a sixth for toll, a barrel of flour?

79. When corn is 75 cents a bushel, what is the corresponding value of average meadow hay as food for stock?

80. The live weight of a fat ox is 1550 pounds. If slaughtered, how many pounds of his net weight will cut up into rump and sirloin, and how many into round?

81. The live weight of 5 fat swine is 2250 pounds. How many pounds of the net weight will cut up into hams and shoulders, and how many into sides, or clear pork?

STONE AND BRICK WORK.

527. A **Perch** of stone or masonry is 16½ feet long, 1½ feet thick, and 1 foot high, or 24¾ cubic feet.

528. In **Rubble** masonry, a cubic yard laid in the wall requires 1⅕ cubic yards of undressed stone, and ⅕ of a cubic yard of mortar.

In **Ashlar** work, about ⅛ of the volume of the stone is allowed for mortar.

A *mason*, with a helper, can in a day lay in courses 4 cubic yards of rubble stone dry, or 3 cubic yards in mortar.

529. **Bricks** when laid will average for each square foot of surface on the face of the wall about twice as many bricks in number as the wall is inches thick.

530. A **Cask of Lime** is about 2½ bushels, or 240 pounds, and absorbs about 2½ times its bulk, or 2¼ times its weight, of water in slacking.

A cask of lime, with about 10 bushels of sharp sand, will make mortar for laying about 1000 bricks, or, with the addition of 5 pounds of hair, mortar for 35 square yards of plastering, one coat work, or 30 square yards, two coat work slipped.

531. **A Cask of Cement** of 300 pounds, with twice its bulk of sharp sand, will make mortar for laying 650 bricks; or, with four times its bulk, or about 12 bushels, of clean gravel, concrete for 9 square yards of flooring surface.

532. In **Paving**, about 40 bricks laid flatwise, or 75 bricks laid edgewise, are allowed for one square yard.

A *mason*, with a helper, in a day can lay in mortar, 8-inch work, 1400 bricks, or 12-inch work, 2000 bricks; and bricks flat in sand 20 square yards, or in cement, 12 square yards.

82. What will be the cost of the material for mortar for plastering 900 square feet, the price of lime being $0.90 per cask, sand 8 cents per bushel, and hair 6 cents per pound?

83. I have a walk 4 feet wide and 224 feet long. What will it cost to pave it with brick, laid flat in sand, brick $7.50 a thousand, wages of the mason $2.75 per day, and of a helper $1.50 per day?

84. What will it cost to concrete the bottom of a cellar 40 feet long and 24 feet wide, cement being $2 a barrel and gravel 8 cents a bushel?

85. What will it cost to build a 12-inch brick wall, 6 feet high and 100 feet long, laid in mortar, brick being $8 per thousand, lime $1.10 per cask, sand 10 cents per bushel, wages of the mason $3 per day, and of the helper $1.75 per day?

86. A cellar is 34 feet long, 27 feet wide, and 9 feet deep. Its walls are 1½ feet thick, made of rubble stones laid in mortar. The stone undressed cost $2.50 per perch, the lime cost $1 per cask, the sand used with it for mortar 10 cents a bushel, and a cask of lime and 10 bushels of sand made 15 cubic feet of mortar. The mason who built the walls was paid $3 a day, and his helper $2. What did the material and on work of the cellar cost?

BUILDERS' ESTIMATES.

533. Shingles are usually 16 inches long, and on an average 4 inches wide, and are put up 4 bundles to the 1000.

1000 shingles, laid 4 inches to the weather, will cover 107 square feet; laid 4½ inches to the weather, 120 square feet; and laid 5 inches to the weather, 133 square feet.

534. Clapboards are usually 4 feet long, and put up 10 bundles to the 1000.

100 of 4-foot clapboards, laid 4 inches to the weather, will cover 130 square feet; laid 4½ inches to the weather, 150 square feet; and laid 5 inches to the weather, 165 feet.

535. Laths are usually 4 feet in length, and are put up 10 bundles to a 1000.

100 laths, set ¼ of an inch apart, will cover 5½ square yards.

A workman in a day will set of laths about 100 square yards, lay of shingles on a roof about 2000, or put on of outside boards about 1000 feet.

536. Nails are put up 100 pounds to a keg.

6 pounds of 4-penny, or 5 pounds of 3-penny, nails are allowed for laying 1000 shingles; 3½ to 4½ pounds of 5-penny nails for laying 1000 clapboards; 7 pounds of 3-penny nails for setting 1000 laths.

537. Paint for outside work may have for first coat 16½ pounds of *white lead*, ground in oil, to a gallon of *linseed oil;* and for second or third coat, 20 pounds of white lead to a gallon of linseed oil.

For inside work, the *spirits of turpentine* may replace from one third to two thirds of the oil.

538. A Gallon of linseed oil weighs about 7½ pounds, and a gallon of spirits of turpentine about 7 pounds.

One pound of prepared white lead paint will cover of first coat about 4 square yards, and of subsequent coats from 4½ to 5½ square yards.

A day's work for a painter is, of plain outside work, from 80 to 100 square yards, and of inside work, from 40 to 65 square yards.

1679 [illegible]

87. Each of the two sides of the roof of a certain building is 34 feet long and 25 feet wide. How many shingles will be required for it if laid 4 inches to weather, and how many 4-penny nails must be allowed? What will be the cost of laying the shingles, at $2.50 per day for labor?

88. How many clapboards, laid 4 inches to the weather, will be required to cover the side of a building 30 feet high and 63 feet long, no allowance being made for openings?

89. A close board fence, 4 feet high and 64 feet long, is to be painted both sides with two coats of white lead paint. When the lead, ground in oil, is 9 cents a pound, and linseed oil 72 cents a gallon, what will the paint required cost? How much must be paid a painter for putting it on, if he covers 80 square yards a day at $2.50?

90. A room is 20 feet long, 18 feet wide, and 10 feet high. Allowing 108 square feet for openings and mopboards, how many laths will be required for its ceiling and walls? What will be the cost of nails for setting them at 4½ cents a pound?

91. A hipped-roof barn is 64 feet long, 40 feet wide, and 20 feet high to the roof. Allowing 360 square feet for openings, how much will rough boards for the sides cost at $20 a thousand feet, and how much should each of two men, at $2.25 a day, be paid for putting on the boards?

PROGRESSIONS.

539. A **Series** of numbers is a succession of numbers, increasing or decreasing according to some fixed law.

540. The **Terms** of a series are the numbers forming the eries.

The first and last terms are called the *extremes*, and the intervening terms the *means*. Thus,

3, 6, 9, 12, is a series in which 3 and 12 are the extremes, and 6 and 9 the means.

541. A series is *ascending* or *descending*, according as the series increases or decreases from the first term.

ARITHMETICAL PROGRESSION.

542. An **Arithmetical Progression** is a series of numbers which increase or decrease by a common difference. Thus,

2, 4, 6, 8, 10, 12, is an ascending series,
12, 10, 8, 6, 4, 2, is a descending series,

in each of which 2 is the common difference.

To find either Extreme.

92. The first term of an arithmetical progression is 3, and the common difference 2. What is the fifth term?

Solution.

1st term = 3. 3d term = 3 + (2 × 2).
2d term = 3 + 2. 4th term = 3 + (2 × 3).
5th term = 3 + (2 × 4) = 11.

93. The fifth term of an arithmetical progression is 11, and the common difference 2. What is the first term?

Solution.

5th term = 11. 3d term = 11 — (2 × 2).
4th term = 11 — 2. 2d term = 11 — (2 × 3).
1st term = 11 — (2 × 4) = 3.

Hence,

543. To find an extreme,

Multiply the common difference by the number of terms less one, and the product plus the smaller extreme will be the larger; or, the larger extreme minus the product will be the smaller.

> **94.** The number of terms of an arithmetical progression is 100, the common difference 3, and the first term 5. What is the last term?

95. A man bought 34 yards of cloth, and agreed to give 12 cents for the first yard, 12⅓ cents for the second yard, 12⅔ cents for the third yard, and so on. What did the last yard cost him?

> **96.** A man travels 10 days, increasing each day's travel by ⅜ of a mile. If he goes the last day 17 miles, how many miles did he start with?

97. If 16 persons give in charity, and the first gives 5 cents, the second 9 cents, and so on in arithmetical progression, how much does the last person give?

To find the Sum of the Terms.

98. The first term of an arithmetical progression is 2, the last term 12, and the number of terms 6. What is the sum of all the terms?

Solution.

$$\begin{array}{lcccccl} \text{Let} & 2, & 4, & 6, & 8, & 10, & 12, \text{ be an arithmetical series,} \\ \text{and} & 12, & 10, & 8, & 6, & 4, & 2, \text{ be the series reversed.} \\ \hline \end{array}$$

$14 + 14 + 14 + 14 + 14 + 14 = 84$, twice the sum of the series.

But $84 = (2 + 12) \times 6$, or the sum of the extremes multiplied by 6; and half of 84, or $\frac{(2 + 12) \times 6}{2} =$ the sum of the series.

Hence,

544. To find the sum of the terms,

[illegible] the sum of the extremes by the number of ter

99. The first term of a series is 2, the last term 478, and the number of terms 86. What is the sum of the series?

100. A man agreed to labor 12 months. For the first month he was to be paid $ 7, and for the last $ 51. If he was to receive the same addition to his wages each successive month, what sum would he receive for his year's labor?

GEOMETRICAL PROGRESSION.

545. A **Geometrical Progression** is a series of numbers which increase, or decrease, by a common rate or ratio. Thus,

3, 9, 27, 81, 243, is an ascending series;
243, 81, 27, 9, 3, is a descending series.

In the first series the rate, or ratio, is 3, and in the last $\frac{1}{3}$.

To find any Term.

101. The first term of a geometrical progression is 4, the rate 2, and the number of terms 5. What is the last term?

Solution.

1st term $= 4$. 3d term $= 4 \times 2^2$.
2d term $= 4 \times 2$. 4th term $= 4 \times 2^3$.
5th term $= 4 \times 2^4 = 64$.

102. The first term of a geometrical progression is 1458, and the rate $\frac{1}{3}$. What is the seventh term?

Solution.

$$\left(\frac{1}{3}\right)^6 = \frac{1}{729};\ 1458 \times \frac{1}{729} = \frac{1458}{729} = 2.$$

Hence,

546. To find any term,

Multiply the first term by that power of the ratio whose exponent is equal to the number of terms less one.

103. The first term of a series is 10, the rate 20, and the number of terms 5. What is the last term?

> **104.** When the first term is $120, the ratio 1.06, and the number of terms 4, what is the last term?

> **105.** What will $50 amount to in 4 years at 6 % compound interest?

To find the Sum of the Series.

106. A geometrical progression consists of 2, 6, 18, 54, the ratio being 3. What is the sum of all the terms?

Solution.

$$\begin{array}{rl} 6 + 18 + 54 + 162 & = 3 \text{ times the series.} \\ 2 + 6 + 18 + 54 & = \text{once the series.} \\ \hline 162 - 2 & = 2 \text{ times the series.} \\ \dfrac{162 - 2}{2} & = 80 = \text{the series.} \end{array}$$

Subtracting like terms of once the series from three times the series, there remains 162 — 2, as two times the series, or 80 as the sum of the series.

Hence,

547. To find the sum of the series,

Multiply the last term by the ratio, subtract the first term, and the remainder divided by the ratio less one will give the sum.

107. What is the sum of a geometrical progression whose extremes are 6 and 768, and ratio 2?

108. The first term of a geometrical progression is 10, the ratio $\frac{3}{4}$, and the number of terms 5. What is the sum of the series to the nearest hundredth?

109. The first term of a geometrical series is $100, the rate 1.06, and the number of terms 4. What is the sum of the series?

110. A lady, wishing to purchase 10 yards of velvet, thought $4 a yard too high a price. She, however, agreed to give 1 cent for the first yard, 4 cents for the second, 16 cents for the *third,* and so on. What was the cost of the velvet?

COLLEGE ENTRANCE-EXAMINATION PAPERS.

548. *Brown University*, 1880.

1. Divide fifty millionths by six hundred twenty-five ten-thousandths, and express the quotient in words.

2. A merchant owned $\frac{11}{16}$ of a stock of goods; $\frac{4}{5}$ of the whole stock were destroyed by fire, and $\frac{7}{12}$ of the remainder damaged by water. How much did the merchant lose, provided the uninjured goods were sold at cost for $ 4200, and the damaged at half the cost?

3. How many hektoliters of oats can be put into a bin that is 2^{m} long, 1.3^{m} wide, and 1.5^{m} deep?

4. Sold a village lot for $ 230, which was 8 per cent less than cost. Had it been sold for $ 300, what would have been the gain per cent?

549. *Dartmouth College*, 1880.

1. $$\frac{2\frac{5}{8} \div \dfrac{4}{\frac{5}{10}} \times 2}{2 - \dfrac{\frac{1}{2}}{4} \div 5} = ?$$

2. Find the sum and product of $\frac{7}{5}$, $\frac{1}{3}$, $\frac{5}{8}$.

3. Find the cube root of 3845672000.

4. Find the square root of 3534400.5.

5. A platform bears a weight of 100 lb. per square foot. What is the weight in kilograms per square meter?

6. A horse that cost $6\frac{1}{4}$ per cent of $ 25000 was sold for $ 1000. What was the loss per cent?

550. *Trinity College*, 1880.

1. Subtract thirty million twenty-six thousand three from 45007021. Find what number must be added to the difference to make one hundred million, and write the answer in words.

2. The sum of $\frac{2}{3}$ and $\frac{4}{15}$ is diminished by $\frac{1}{10}$. How many times does the difference contain $\frac{3}{11}$ of the sum of $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{1}{10}$?

3. Divide 375 by .75, and .75 by 375, and find the sum and the difference of the quotients.

4. A loaded wagon weighs 2 T. 3 cwt. 48 lb.; the wagon itself weighs 18 cwt. 75 lb. The load consists of 215 packages, each of the same weight. Find the weight of each, and reduce it to grams and kilograms.

5. Define interest, and give and explain the rule for computing the interest on any sum of money for any time and at any rate per cent.

6. Extract the square root of 184.2 to three places of decimals.

551. *Harvard University*, 1880.

1. Find the greatest common divisor of 315, 504, and 441.

2. Express as a decimal $\frac{2}{3} \times \frac{\frac{3}{4} - \frac{1}{5} \times \frac{10}{6}}{3\frac{1}{3} - 2\frac{5}{6}}$.

3. How many hektoliters are there in 57 gallons $3\frac{1}{2}$ pints?

1 liter = 0.2642 gal.; 1 gallon = 8 pt.

4. How much paper, $1\frac{1}{8}$ yd. wide, will be needed to paper the walls of a room 10 feet high, 18 feet long, 12 feet broad?

5. I sold a lot of sugar for $230, and thereby lost 8 per cent of the cost. What per cent should I have gained if I had sold the sugar for $300?

552. *Yale College*, 1879.

1. Find the value of $\left(\frac{4\frac{6}{7}}{8\frac{9}{10}} \text{ of } \frac{3}{7\frac{1}{8}}\right)$ divided by $\frac{6}{11}$, and extract the square root of the quotient to two decimal places.

2. Find a fourth proportional to .37, 8.9, 4.3, and extract the cube root of it to two decimal places.

3. Reduce 16 rods 2 feet and 8 inches to the decimal of a mile.

4. What is the length in meters and decimeters of a side of a square which contains .1335 are?

553. *Dartmouth College,* 1879.

1. $\dfrac{4\frac{1}{2} + 2\frac{1}{3} \div \frac{2}{3}}{6\frac{1}{2} - 1\frac{2}{3} \times \frac{3}{2}} = ?$

2. Find the least common multiple and the greatest common divisor of 6, 8, 20, and 36.

3. How many meters in 25 feet?

4. Find the square root of 3530641.

5. Gold was quoted at $1.12½. What was a one-dollar greenback worth?

6. $1200 includes a sum to be invested, and a commission of five per cent of the sum to be invested. What is the sum to be invested?

554. *Cornell University,* 1880.

1. Define a composite number, a factor, an abstract number, the cube root of a number, equation of payments.

2. What is the value of 50 lb. 8 oz. of gold at $20.59¼ per ounce?

3. Given the meter equal to 39.37 inches, reduce one mile to kilometers. Give the metric table of weights.

4. Divide $\frac{2}{3}$ of $7\frac{3}{4}$ by $\frac{4}{5}$ of $12\frac{1}{1}\frac{1}{2}$. Prove the result by reducing the fractions to decimals, and working the example anew.

5. A man said, "I will spend half my income, save a third of it, and devote a fourth to business." His income was $780 a year. Point out his blunder, and divide his income rightly in the proportion intended by him.

6. How long must $125 be on interest at 7½ per cent to gain *$15?*

7. Received 6 per cent dividend on stock bought at 25 per cent below par. What rate of interest did the investment pay?

8. Find the cube root of .726572699.

555. *Yale College,* 1880.

1. Reduce to a common denominator, and add:

$$\left(\frac{3}{8}\times\frac{4}{5}\times\frac{5}{9}\right),\ \frac{7}{15},\ \frac{3}{4},\ \text{and}\ \frac{9}{10}.$$

2. Divide $\left(\frac{4}{6\frac{4}{7}}-\frac{1}{7}\right)$ by $\frac{8}{11}$.

3. Find, to three decimal places, the value of $\frac{1}{\sqrt{3}}$.

4. Find the fourth term of a proportion of which the first, second, and third terms are, respectively, 3.81, 0.056, 1.67.

5. Reduce 3 R. 13 sq. rd. 8 sq. ft. to the decimal of an acre.

6. (*a*) In a board 4^{m} long and 0.4^{m} wide, how many square decimeters?

(*b*) Divide 2700^{Hl} by 90^{cl}.

556. *Harvard University,* 1879.

1. Simplify $\dfrac{1+\frac{8}{3}+\left(\frac{4}{3}\right)^2}{\left(\frac{4}{3}\right)^2-1}$.

2. Find the length in dekameters of the side of a square, the area of which equals the area of a rectangle which is 1 kilometer 8 meters long, and $4\frac{12}{25}$ hektometers wide.

3. Find the least common multiple of the *even* numbers *from 10 to 20* inclusive.

4. The interest on $2500 for 2 months 12 days is $45. Find the rate.

5. If 25 men, working 8 hours a day, do $\frac{3}{8}$ of a piece of work in 24 days, in how many days of 10 hours each will 30 men finish the piece of work?

557. *University of the State of New York*, 1881.

1. Copy and add: 20570, 6206, 98.007, 63000, 426.000626, 4287, 63.961, 102030, 405.0607, 8090, 543.21, 1028848.414995.

2. Express by Arabic notation, $\overline{\text{MDXCV}}\text{DCCCLXIV}$.

3. Express by Roman notation, 84796.

4. Numerate 20567189.004321098.

5. Divide 31984875832 by 96813.

6. Find the value of

$$\overline{(28-7)\times 6}+\overline{(92+7)\div 9}-\overline{(86+10)\div 12}.$$

7. Divide, using cancellation,

$$15 \times 80 \times 27 \times 28 \text{ by } 7 \times 20 \times 8.$$

8. Change $\frac{2}{21}$, $\frac{12}{87}$, $\frac{26}{169}$, and $\frac{1}{4}$, to similar fractions having their least common denominator, and

9. Reduce their sum to decimal form.

10. Find the greatest common divisor of 7955, 8769, 6401.

11. How much must be paid for making 52 rd. 14 ft. 8 in. of fence, at $0.75 per foot?

12. A traveler, on reaching a certain place, found that his watch, which kept correct time for the place he left, was 2 h. 22 min. slower than the local time. Had he traveled eastward or westward, and how far, in circular measure, had he come?

13. What per cent (expressed in words) of 30000 bushels are 50 bushels?

14. What number diminished by 36% of itself = 336?

15. What is the value of a lot 70 rd. long and 20 rd. wide, at $ 47.25 per acre?

16. A cistern has 3 pipes. The first will fill it in 12 hours, the second in 16, and the third in 18 hours. If all run together, in what time will they fill it? (State this example as a proportion, if you can.)

17. What is the difference between the simple interest on $ 328 for 2 y. 7 mo. at 7%, and the compound interest on the same sum for the same time at 6%?

18. Find the balance due March 4, 1881, on a note dated Jan. 1, 1879, for $ 580, at 5%, on which a payment of $ 85 has been made every 6 months, using the United States rule.

19. How much should be discounted on a bill of $ 3725.87, due in 8 mo. 10 d., if paid immediately, money being worth 5%?

20. Bought bonds at 115, and sold at 110, losing $ 300. How many bonds of $ 1000 each did I buy?

21. If A puts in $ 4000 capital for 8 months, B $ 6000 for 7 months, and C $ 3500 for 1 year, and they gain $ 2320, what is each partner's share of the gain?

22. If 5 horses eat as much as 6 oxen, and 8 horses and 12 oxen eat 12 tons of hay in 40 days, how much hay will 7 horses and 15 oxen eat in 65 days?

23. Find the value of $\sqrt[3]{0.000238328}$.

24. A steamer goes due north at the rate of 15 miles an hour, and another due west 18 miles an hour. How far apart will they be in 6 hours?

25. Find the cost, at 30 cents per sq. yd., of cementing the bottom and sides of a cubical cistern that will hold 300 barrels.

26. What is the area of a circle 5 ft. in diameter?

27. What is the difference between 5 square feet and 5 feet square? Illustrate by a diagram.

ANSWERS.

Art. 37.

009
69.
98.
59.
989.
499.
1465.
484.
149.
31.355.
39.928.
61.665.
317.40.

Art. 39.

89.
978.
396.
5485.
052.
788.
018.7.
43.91.
131.31.
100.66.
393.30.
230.05.
018.
443.
736.
023.
026.
6986.

103. 176.40.
104. 153.89.
105. 281.72.
106. 5233.97.
107. $125.65.
108. $511.69.
109. $168.08.
110. $532.40.
111. 998.
112. 4391.
113. $9665.68.
114. 5619.
115. $75.13.
116. 318381.
117. $145.17.
118. $360.
119. $14170.70.
120. 2815.
121. 24445.
122. 313.54.
123. 150.390.
124. 29002 ft.
125. 3578392.
126. $1046.87.
127. $175430
128. 62611.
129. $39320.
130. 1323925.63.
131. 1863189.
132. 563972744718.
133. 509006545503.418.
134. 323497.
135. 340522022.
136. 1380855.262.
137. $32545.24.
138. $24005.79.

Art. 46.

45. 332.
46. 223.
47. 205.
48. 222.
49. 1114.
50. 3212.
51. 2213.
52. 1118.
53. 1221.
54. $325.
76. 309.
77. 192.
78. 192.
79. 46.65.
80. 803.153.
81. $391.05.
82. $55.14.
83. $338.80.

Art. 48.

84. 5196.
85. 4969.
86. 1859.
87. 1056.
88. 29962.
89. 3541.
90. 56.39.
91. 14.251.
92. $6.27.
93. $83.96.
94. $95.81.
95. $29.99.
97. 34456.
98. 97820.
99. 22968.

100. 9903.
101. 9154.
102. 1.
103. 6.552.
104. 811.95.
105. 9615.5
106. 78.44.
107. $486.57.
108. $1836.75.
109. 5541.
110. 26983.
111. 11001.
112. 107.91.
113. 389.
114. $740.75.
115. 163864.
116. $4066.94.
117. 269535.
118. 267369.
119. 1785837.

120. $291.25.
121. $3527.82.
122. 891; 460; 280.
123. $173.32.
124. 1382.
125. 1330 miles.
126. $5555.75.
127. 380247.
128. $1031.32.

Art. 58.

38. 3024.
39. 13701.
40. 5545.
41. 49608.
42. 20496.
43. 5216.
44. 86415.
45. 218709.
47. 280.56.
48. 30.675.
49. 3271.8.
50. 49899.71.
51. $497.40.
52. $1155.282.
53. $6588.00.
54. $76838.76.
81. *23231.*
82. 60525.
83. 10150.
84. 55114.
85. 115842.
86. 501030.
87. 10003.121.
88. 18102.25.

Art. 60.

89. 266843.
90. 200718.
91. 7684.
92. $2923.83.
93. 31055.6.
94. 4824.342.
95. 9309.65.
96. 4378905.
97. $159438.30.
98. 41105.24.
99. 2951129.6.
100. $46471.68.
101. 8210.90.
102. 66909.44.
103. 27835.83.
104. $292636.36.
105. 1054.875.
106. 59073353.
107. 14518308.
108. $675729.95.
109. 4193735.
110. 26502.93.
111. 325631.74.
112. 53794576.
113. $11545.45.
114 40444770.
115. 299023.58.
116. 68697853.
117. $68479.40.
118. $86214.193.
119. 48085.
120. 127716.
121. 4143204.
122. 50684.13.
123. 58501.12.
124. $2062.50.
125. 205732.57.
126. 639446.5.
127. 886864.86.
128. 349732.16
129. $5642.455.
130. $9810.944.
131. $219187.65.
132. 264798 bush.
133. 3575806.65.
135. 473970.
136. 7854000;138000

Art. 61.

138. 814000.
139. 274470.
140. 493020.
141. 603900.
142. $67244.10.
143. 2760720.
144. $3208100.
145. 2051636.8.
146. 27306000.
147. $23364000.
148. 16320000 miles.
149. 4992000 acres.

150. $110.75.
151. 9984.
152. $1061.40; could have saved $1129.00.
153. 732 miles.
154. 163200 lb.
155. 4598 lb.
156. $86.45.
157. 150696.
158. $1353.50.
159. $100.
160. $268.95.
161. $4409.
162. $1326.

Art. 71.

36. 383.
37. 821.
38. 971.
39. 1142.
40. 58.6.
41. 12.23.
42. 11.90.
43. 4.467.
45. 715¼.

46. $678\frac{7}{12}$.
47. $612\frac{3}{8}$.
48. $8423\frac{4}{7}$.
49. $673\frac{1}{4}$.
50. 13.34.
51. $60.71\frac{1}{6}$.
52. $4.348\frac{5}{9}$.
53. 405.
54. $205\frac{3}{7}$ miles.
55. $5.45.
56. $2313\frac{1}{5}$.

Art. 72.

69. $470\frac{6}{21}$.
70. 234.
71. $1359\frac{21}{23}$.
72. 5.63.
73. 1.063.
74. 50.5.

Art. 74.

76. $3.18.
77. $32.02.
78. $990\frac{202}{361}$.
79. $10.67\frac{700}{793}$.
80. $965\frac{54}{982}$.
81. $4.47\frac{219}{785}$.
82. $5.509\frac{285}{1234}$.
83. $\$8.503\frac{15}{987}$.
84. $505\frac{4050}{6942}$.
85. $1487\frac{612}{873}$.
86. $75\frac{875}{841}$.
87. $2.056\frac{706}{2007}$.
88. $8.75\frac{696}{987}$.
89. $1208\frac{208}{456}$.
90. $2453\frac{53}{359}$.
91. $6888\frac{12}{95}$.
92. $125.
93. 24.
94. $\$6848.76\frac{42}{98}$.
95. $26\frac{10}{115}$.
96. $3.44\frac{240}{512}$.
97. 3007.
98. $114\frac{52}{432}$.
99. 342.
100. $52.88.
101. $\$136\frac{860}{865}$.
103. $61.87\frac{22}{44}$.
104. $106.558\frac{12}{86}$.
105. $31.25.
106. 137.125.
107. $30303\frac{1}{33}$.

Art. 75.

110. $19.65.
111. 39.62; 3.962.
112. 45.54.
113. $137\frac{56}{90}$.
114. $1090.18\frac{1800}{6400}$.
115. $28\frac{9272}{51000}$.
116. $494\frac{116000}{186000}$ sec.
117. $1024\frac{6469}{8291}$.
118. $768\frac{3078}{3782}$.
119. $58881\frac{148}{1234}$.
120. $11956\frac{3955}{5678}$.
121. $1.542\frac{2028}{2961}$.
122. $.529\frac{216}{349}$.
123. $977\frac{1908}{6384}$.
124. $1.788\frac{123}{945}$.
125. $585\frac{267}{1252}$.
126. $169\frac{659}{1189}$.
127. $100\frac{484}{609}$.
128. 1016400.
129. $30515\frac{290}{2090}$.
130. $\$1.11\frac{215}{850}$.
131. $3\frac{75}{203}$.
132. $.871\frac{400}{975}$.

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133. 2555.
134 101.
135 3.
136. $\$26.77\frac{80}{160}$.
137. 14.
138 640.
139. 106.
140. 64.
141. 35; and 1400 sq. mi. over.
142. 750.
143. 108.
144 71.
145. 217.

Art. 76.

33. 3207.
34. 39817.
35. 1492.
36. $2063.38.
37. 12616.
38. $371.14.
39. $15.30.
40. 6.
41. $35405.
42. 28618.
43. $0.14.
44. $\$0.24\frac{270}{310}$.
45. $\$4.71\frac{108}{252}$.
46. 27.
47. $8.
48. $1820.85.
49. 873.
50. $3632.25.
51. $137\frac{58}{67}$ hhds.
52. $13\frac{880}{2240}$.
53. $123.
54. 192.
55. 1808.
56. $7137.
57. 17.
58. 759.
59. 550.
60. 17.
61. $269880.
62. 1160.
63. 13.
64. $5817.
65. $\$4.53\frac{3}{8}$.
66. 215.
67. $4064, gain.

Art. 82.

8. $2^2, 3, 7;\ 2^4, 3^2;\ 2^5, 5$.
9. $2, 3, 7, 11;\ 2^6, 3^2;\ 2^4, 3^2, 7$.

Art. 83.

10. 2, 3, 5, 7.
11. $2^2, 3^2, 7, 11$.
12. 2, 3, 71.
13. 3, 5, 11, 37.
14 3, 5, 7, 11.
15. $2^4, 5^2, 7$.

16. $2^2, 3^2, 5, 19$.
17. $2^3, 3, 5^2, 13$.
18. $2^2, 4007$.
19. $3^3, 7^2, 13$.
20. $3^2, 31, 37$.
21. $2^4, 11, 71$.
22. 37.
23. 3, 7, 11, 19.
24. $2^2, 3^2, 5^2, 7$.
25. 7, 11, 17, 31.

Art. 85.

32. $3\frac{1}{4}$.
33. $4\frac{1}{2}$.

Art. 86.

34. $2\frac{1}{4}$.
35. $12\frac{1}{2}$.
36. $3\frac{8}{9}$.
37. 109.
38. 24.
39. $9\frac{1}{3}$.
40. 7.
41. 19.
42. 8.
43. 4.
44. $0.30.

Art. 90.

50. 9.
51. 6.

Art. 91.

52. 45.
53. 15.
54. 3.
55. 12.
56. 2.
57. 16.
58. 9.
59. 3.
60. 3 ft.

Art. 92.

62. 48.
63. 25.
64. 22.
65. 86.
66. 55.
67. 39.
68. 2.
69. 87.
70. 56.
71. 28.
72. 6.
73. 8.
74. 12.
75. 31.
76. 14.
77. 696.

Art. 96.

83. 210.
84. 1260.

Art. 97.

85. 1848.
86. 504.
87. 1320.
88. 2835.
89. 72.
90. 86100.
91. 7000.
92. 7560.
93. 7200.
94. 7920.
95. 350.

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96. $2^3, 3^2, 5, 7$.
97. $2^7, 3^2$.
98. 13.
99. 492.
100. 80.
101. 12.
102. 1777.
103. 54528.
104. 6.
105. 105.

Art. 112.

35. $\frac{86}{84}$.
36. $\frac{24}{32}$.
37. $\frac{24}{64}$.
38. $\frac{182}{196}$.
39. $\frac{120}{168}$.
40. $\frac{171}{189}$.
41. $\frac{36}{48}$.
42. $\frac{360}{576}$.

Art. 114.

53. $\frac{3}{4}$.
54. $\frac{5}{6}$.

Art. 115.

55. $\frac{9}{10}$.
56. $\frac{7}{19}$.
57. $\frac{8}{85}$.
58. $\frac{425}{272}$.
59. $\frac{374}{361}$.
60. $\frac{29}{37}$.
61. $\frac{8}{9}$.
62. $\frac{4}{5}$.
63. $\frac{107}{117}$.
64. $\frac{11}{12}$.
65. $\frac{22}{125}$.
66. $\frac{81}{11}$.
67. $\frac{19}{67}$.
68. $\frac{13}{17}$.
69. $\frac{81}{422}$.
70. $\frac{19}{67}$.
80. $\frac{197}{1}$.
82. $\frac{356}{13}$.
83. $\frac{410}{11}$.

Art. 116.

84. $\frac{1895}{15}$; $\frac{2247}{21}$.
85. $\frac{1495}{13}$.
86. $\frac{147}{16}$.
87. $\frac{1067}{19}$.
88. $\frac{738}{7}$.
89. $\frac{616}{158}$.
90. $\frac{194}{13}$.
91. $\frac{9554}{97}$.
92. $\frac{75}{19}$.
93. $\frac{1000}{7}$.
94. $\frac{4861}{60}$.
95. $\frac{6901}{274}$.
96. $\frac{2147}{14}$.
97. $\frac{178010}{1000}$.
105. 43.
106. $63\frac{48}{11}$.

Art. 117.

107. 9.
108. $27\frac{2}{9}$.
109. $169\frac{5}{19}$.
110. 1.
111. $1\frac{108}{858}$.
112. $1\frac{5}{12}$.
113. $7\frac{12}{17}$.
114. 128.
115. $119\frac{8}{81}$.
116. $4\frac{12}{459}$.
117. $9\frac{4}{29}$.
118. $90\frac{10}{11}$.
119. $91\frac{28}{45}$.
120. $100\frac{10}{111}$.
121. $95\frac{285}{907}$.

Art. 120.

131. $\frac{48}{56}$, $\frac{49}{56}$.
133. $\frac{96}{120}$, $\frac{100}{120}$, $\frac{75}{120}$.
134. $\frac{27}{86}$, $\frac{28}{86}$, $\frac{88}{86}$.

Art. 121.

135. $\frac{10}{45}$, $\frac{21}{45}$.
136. $\frac{39}{91}$, $\frac{28}{91}$.
137. $\frac{63}{108}$, $\frac{124}{108}$.
138. $\frac{125}{1200}$, $\frac{144}{1200}$.
139. $\frac{18}{84}$, $\frac{32}{84}$, $\frac{27}{84}$.
140. $\frac{105}{126}$, $\frac{70}{126}$, $\frac{72}{126}$.
141. $\frac{49}{105}$, $\frac{50}{105}$, $\frac{51}{105}$.
142. $\frac{15}{8}$, $\frac{4}{8}$, $\frac{96}{8}$.
143. $\frac{297}{2475}$, $\frac{935}{2475}$, $\frac{675}{2475}$, $\frac{66}{2475}$.
144. $\frac{700}{1000}$, $\frac{70}{1000}$, $\frac{7}{1000}$, $\frac{7000}{1000}$.
145. $\frac{186}{24}$, $\frac{9}{24}$, $\frac{20}{24}$, $\frac{14}{24}$.
146. $\frac{35}{40}$, $\frac{36}{40}$, $\frac{30}{40}$.
147. $\frac{20}{45}$, $\frac{27}{45}$, $\frac{24}{45}$.

Art. 123.

156. $2\frac{17}{48}$.
157. $1\frac{527}{1260}$.

Art. 124.

158. $\frac{58}{63}$.
159. $2\frac{1}{21}$.
160. $1\frac{19}{32}$.
161. $2\frac{81}{77}$.
162. $\frac{115}{204}$.
163. $8\frac{35}{72}$.
164. $2\frac{239}{360}$.
165. $2\frac{269}{300}$.
166. $9\frac{167}{180}$.

Art. 125.

167. $40\frac{3}{8}$.
168. $488\frac{1}{2}$.
169. $27\frac{1}{8}$.
170. $313\frac{33}{80}$.

Art. 126.

182. $\frac{23}{168}$.
183. $\frac{1}{5}$.
184. $\frac{4}{21}$.

Art. 127.

185. $\frac{48}{120}$.
186. $\frac{19}{85}$.
187. $\frac{48}{144}$.
188. $\frac{148}{711}$.
189. $\frac{1}{51}$.
190. $\frac{4}{5}$.
191. $\frac{81}{1000}$.
192. $\frac{7}{84}$.
193. $\frac{1}{864}$.
194. $\frac{1}{62}$.
195. $17\frac{24}{35}$.
196. $21\frac{23}{36}$.
197. $8\frac{5}{12}$.
198. $6\frac{17}{20}$.
199. $13\frac{67}{171}$.
200. $280\frac{62}{81}$.
201. $4\frac{49}{288}$.
202. $\$5\frac{19}{24}$.
203. $2\frac{23}{40}$.
204. $2\frac{19}{40}$.

Art. 128.

212. $1\frac{5}{9}$.
213. $15\frac{1}{5}$.
214. $37\frac{18}{31}$.
215. 49.
216. $10\frac{11}{15}$.
217. $2\frac{86}{101}$.
218. $43\frac{1}{2}$.
219. 144.
221. $1090\frac{1}{2}$.
222. 5131.
223. $2927\frac{1}{7}$.
224. 10025.
225. $621.

Art. 129.

232. 84.
233. $24\frac{13}{18}$.
234. 325.
235. $97\frac{6}{7}$.
236. 88.
237. $165\frac{6}{13}$.
238. $17\frac{5}{71}$.
239. $243\frac{5}{17}$.
241. $1072\frac{1}{3}$.
242. $1119\frac{1}{6}$.
243. 791.
244. $1719\frac{3}{8}$.
245. $1281\frac{1}{4}$.

Art. 131.

257. $13\frac{1}{8}$.
258. $\frac{16}{55}$.
259. $1\frac{128}{500}$.
260. $\frac{28}{99}$.
261. $2\frac{4}{5}$.
262. $\frac{8}{14}$.
263. $2\frac{2}{9}$.
264. $\frac{3}{7}$.
265. $\$89\frac{5}{48}$.
266. $82\frac{2}{15}$.
267. $1\frac{67}{77}$.
268. $25\frac{17}{245}$.
269. $\$122\frac{29}{32}$.
270. $13\frac{27}{160}$.

Art. 132.

282. $\frac{7}{228}$.
283. $\frac{4}{843}$.
284. $\frac{8}{517}$.
285. $\frac{5}{316}$.
286. $\frac{67}{5460}$.
287. $\frac{4}{97}$.
288. $\frac{1}{665}$.

289. $\frac{5}{588}$.
291. $\frac{3}{40}$.
292. $15\frac{27}{70}$.
293. $3\frac{11}{45}$.
294. \$$6\frac{3}{8}$.
304. $88\frac{13}{16}$.
305. $29\frac{5}{11}$.
306. $87\frac{1}{2}$.
307. $52\frac{1}{3}$.
308. 170.
309. $151\frac{5}{11}$.
310. 40.
311. 162.
312. $111\frac{13}{17}$.
313. $21\frac{41}{48}$.
314. 64.
315. 16.
316. 7.
327. $1\frac{13}{86}$.
328. $\frac{8}{15}$.

Art. 133.

329. $\frac{5}{6}$.
330. $\frac{4}{7}$.
331. $1\frac{13}{141}$.
332. $1\frac{5}{12}$.
333. $\frac{2}{5}$.
334. $\frac{41}{189}$.
335. $2\frac{1}{12}$.
336. $5\frac{221}{975}$.
337. $2\frac{48}{49}$.
338. $11\frac{47}{78}$.
339. \$$\frac{16}{25}$.
340. \$18.
341. $4\frac{1}{4}$ hours.
343. $1\frac{19}{26}$.
344. $19\frac{1}{5}$.
345. $16\frac{16}{21}$.
346. $\frac{1}{2}$.
347. $8\frac{1}{26}$.
348. $\frac{40}{53}$.
349. $\frac{100}{777}$.
350. 45 days.
351. 4 hours.

Art. 134.

359. $\frac{3}{5}$.
360. $\frac{6}{7}$.
361. $\frac{5}{72}$.
362. $\frac{1}{3}$.
363. $\frac{1}{6}$.
364. $\frac{5}{72}$.
365. $\frac{6}{7}$.
366. $\frac{15}{16}$.
367. $\frac{13}{14}$.
368. $\frac{13}{22}$.
369. $\frac{6}{17}$.
370. 112.
371. \$52.50.
372. $\frac{53}{78}$.
373. $\frac{2}{3}$.
374. \$$4\frac{97}{100}$.

Art. 135.

389. 1672.
390. 1863.
391. 6400.
392. 520.
393. \$1400.
394. 330.
395. $\frac{3}{8}$.
396. $25\frac{1}{2}$.
397. \$$808\frac{1}{2}$.
398. \$6550.
399. \$2625.
400. \$8919.

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401. $52\frac{1}{5}$.
402. \$$\frac{3}{8}$.
403. $11\frac{1}{4}$.
404. \$6.40.
405. $103\frac{1}{8}$.
406. $2\frac{42}{59}$.
407. $73\frac{1}{3}$ min.
408. \$$13\frac{25}{72}$.
409. $\frac{9}{14}$.
410. 7 pieces and 2 ft.
411. \$$6\frac{2}{3}$.
412. 1 to 1.
413. 48.
414. $1\frac{4}{5}$.
415. $1\frac{1}{2}$.
416. \$30.
417. $8\frac{13}{108}$.
418. \$414.
419. \$561.

Art. 136.

34. $\frac{719}{1247}$.
35. 576.
36. 15552.
38. $1\frac{83}{150}$.
39. $18\frac{39}{40}$.
40. $15\frac{40}{79}$.
41. 34.
42. $2\frac{37}{160}$.
43. $\frac{2}{21}$.
44. 27.
45. $\frac{81}{301}$.
46. 76.
47. $2\frac{13}{72}$.
48. $\frac{53}{91}$.
49. $13\frac{3}{10}$.
50. $\frac{3}{4}$.
51. $\frac{1}{4}$.
52. $\frac{77}{144}$.
53. 560.
54. $\frac{11}{16}$.
55. \$141.
56. \$$107\frac{71}{80}$.
57. \$$1\frac{7}{8}$.
58. $19\frac{3}{4}$.
59. \$5.68; \$59.64.
60. 70 cents.
61. 56.
62. $11\frac{1}{4}$.
63. $1\frac{1}{4}$.
64. A's \$300. B's \$560. C's \$100.
65. $\frac{7}{11}$.
66. \$1485.
67. $31\frac{1}{2}$.
68. \$$6\frac{29}{64}$.
69. \$$\frac{3}{4}$.
70. \$500.
71. A 560 gal. B 400 gal.
72. $2\frac{68}{189}$.

Art. 142.

54. 71.5.

19.000.
43.600.
0.640.
53.000.
15.60.
4.70.
13.00.
18.0156.
401.6000.
176.4700.
$\frac{9}{40}$.
$\frac{7}{8}$.

Art. 143.

$\frac{1}{40}$.
$\frac{561}{1000}$.
$\frac{27}{500}$.
$\frac{2}{5}$.
$\frac{5}{8}$.
$\frac{3}{8}$.
$\frac{46}{125}$.
$11\frac{3}{4}$.
$4\frac{7}{16}$.
$\frac{8}{32}$.
$9\frac{3}{8}$.
$115\frac{7}{8}$.
$\frac{11}{800}$.
$200\frac{24}{25}$.
$\frac{1}{64}$.
.6.
$.416\frac{2}{3}$.
$.3636\frac{4}{11}$.

Art. 144.

.275.
.4375.
.03125.
.7.
.32.
4.096.
4.015625.
2.09375.
5.0078125.
$.0769\frac{3}{13}$.
.5833.
.9474.
$.0933\frac{1}{3}$.

102. 1.232.
103. 51.9383.
104. 11.933877.

Art. 145.

111. 204.655.
112. 132.912.

Art. 146.

115. 253.80.
116. 9650.
117. 0.014582.
118. 84.5688.
119. 21.723.
120. 26.461.
121. 143.3282
122. 0.00038665.
123. .0000505.
124. .0001292.
125. 1.000000.
126. $160.875.
127. $61.7925.
128. 0.01020201.
139. 0.023.

Art. 147.

142. 125.36.
143. 0.756.
144. 10.01.
145. 790.
146. 0.081.
147. 0.893+.
148. 0.001365.
149. 4000.
150. 150000.
151. 1.12.
152. 360.984.
153. 15.004+.
154. 210.

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155. { 0.0403.
400.0003.
157. { 0.0087.
0.00162.
0.000009.
158. 5.325.
159. 8.99927.

160. 0.855.
161. { 6345.3654.
4.1958.
0.0000495.
162. 500.
163. $\frac{1}{32}$.
164. 23.9268 sum.
165. 14.54368 days.
166. 612.
167. { $ $2999.45\frac{1}{3}$ A's.
$ $2249.62\frac{1}{3}$ B's.
$ $2249.62\frac{1}{3}$ C's.
168. 18817.92728.
169. 58.872.
170. $16444.9602.
171. 4.9835.
172. 96.
173. 165.135.
174. $0.18.
175. $447.70.
176. $ $33\frac{8 27}{625}$.
177. 1.
178. $\frac{1}{70}$.
179. $ $219.54\frac{6}{11}$.
180. $5941.06.
181. $76\frac{52}{113}$.
182. $68\frac{587}{896}$ tons.
183. 420.
184. 4000.
185. $3\frac{8}{9}$.

Art. 154.

23. $27.26.
24. $40.93.
25. $5143.15.
26. $19.36.
27. 250.
28. $41.19.
29. $94.86+.
30. $48.
31. $44\frac{22}{67}$.
32. 760.
33. $ $163.12\frac{1}{2}$.
34. $50.95\frac{10}{41}$.

Art. 156.

44. $198.
45. $56.
46. $232.50.

47. 98.
48. 18.
49. $ 49.
50. $ 679.50.
51. 2117.
53. $ 5.10.
54. $ 81.25.
56. $ 248.60.
57. $ 904.042.
59. $ 12.
60. $ 12.50.
61. $ 1.50.

Art. 159.

62. $ 90.47.
63. $ 222.71.
64. $ 58.55.
65. $ 56.23.
66. $ 104.78.
67. $ 226.50.
68. $ 1156.25.
69. $ 84.
70. $ 443.35.
71. $ 126.64.

Art. 187.

13. 1279 pt.
14. 1311 cu. ft.
15. 35790 lb.

Art. 188.

16. 562068 sq. ft.
17. 273749 cu. in.
18. 487 pt.
19. 879 pt.
20. 248160 ft.
21. 419887 gr.
22. 87320 lb.
23. 31556930 sec.
24. 391256 cu. in.
25. 163734 sec.
26. 3200 cu. ft.
27. 1306 gi.
28. 684592 min.
29. 4320 sheets.
30. *$ 3267.*
31. *$ 6352.50.*
33. 88 yd.
34. $\frac{2}{3}$ gr.
35. $\frac{27}{28}$ sec.
36. $\frac{7}{9}$ gi.
37. $4\frac{4}{15}$ qt.
39. 48 lb.
40. 326.7 sq. ft.
41. 568.08 min.
42. 6601.76 sq. yd.

Art. 189.

51. 19 bu. 3 pk. 7 qt. 1 pt.
52. 48 cu. yd. 15 cu. ft.
53. 17 T. 17 cwt. 90 lb.

Art. 190.

54. 12 A. 144 sq. rd. 144 sq. ft.
55. 5 cu. yd. 23 cu. ft. 725 cu. in.
56. 60 gal. 3 qt. 1 pt.
57. 13 bu. 2 pk. 7 qt. 1 pt.
58. 47 mi.
59. 72 lb. 10 oz. 15 pwt. 7 gr.
60. 43 T. 13 cwt. 20 lb.
61. 365 d. 5 h. 48 m. 50 sec.
62. 8 cu. yd. 10 cu. ft. 728 cu. in.
63. 45° 28′ 54″.
64. 25 cd.
65. 40 gal. 3 qt. 0 pt. 2 gi.
66. 67 wk. 6 d. 9 h. 52 min.
67. 4 bundles 1 ream.
68. 1 A. 80 sq. rd.
69. 3 mi. 195 rd.
71. $\frac{1}{20}$ mi.
72. $\frac{25}{48}$ lb.
73. $\frac{7}{48}$ day.
74. $\frac{3}{4}$ gal.
75. $\frac{2}{15}$ bu.
77. 0.024 T.
78. 0.0075 A.
79. 0.3945 day.
80. 1.364 A.

Art. 191.

88. 71 rd. 1 ft. 10 in.
90. 2 yd. 2 ft. 8.94 in.

Art. 192.

91. 68 sq. rd. 155 sq. ft. $82\frac{2}{7}$ sq. in.
92. 10 oz. 13 pwt. 8 gr.
93. 232 d. 6 h. 32 min. $43\frac{7}{11}$ sec.
94. 248 rd. 4 yd. 2 ft. 8 in.
95. 67 lb. 4 oz.
96. 5 cwt. 64 lb.
97. 4 yd. 2 ft. 5.25 in.
98. 18 h. 15 m. 50.4 s.
107. $\frac{2}{9}$ mi.
108. 0.53 rd.

Art. 193.

110. $\frac{4667}{10890}$ A.
111. $\frac{8}{9}$ lb.
112. $\frac{37189}{58400}$ y.
113. $\frac{7}{9}$ mi.
114. 0.6725 ctl.
115. 0.282 T.
116. 0.875 rd.
117. 0.761 d.

Art. 194.

118. $\frac{1}{3}$.
119. $\frac{4183}{5944}$.
120. $\frac{44}{133}$.
121. 0.125.
122. $11.17\frac{57}{79}$.

Art. 195.

126. 63 cu. yd. 11 cu. ft. 842 cu. in.
127. 199 gal. 1 qt.
128. 77 d. 8 h. 26 m. 56 sec.
129. 64° 28′ 32″.

131. 1 T. 15 cwt. 53 lb.
132. 167 mi. 240 rd.
133. 2 A. $49\frac{1}{21}$ sq. rd.
135. 13 bu. 2 pk. 6 qt.
136. 14 mi. 231 rd. 5 yd. 0 ft. 2 in.
137. 28 A. 76 sq. rd.
138. 16° 30′ 51″.
139. 5 oz. 5 pwt. 20.8 gr.
140. 12 gal. 2 qt. 0 pt. $3\frac{1}{5}$ gi.
141. 133 d. 4 h. 39 min. 21.6 sec.
142. 181 rd. 2 yd. 2 ft. 8.72 in.

Art. 196.

144. 5 y. 9 mo. 24 d.
145. 1 y. 7 mo. 20 d.
146. 33 y. 2 mo. 20 d.
147. 86 y. 5 mo. 28 d.
149. 60 days.
150. 282 d. 11 h.
152. 141 wk. 4 d. 22 h. 16 min.
153. 37 mi. 170 rd. 1 yd.
154. 10 T. 8 cwt. 53 lb.
155. 355 A. 49 sq. rd. $21\frac{3}{4}$ sq. yd.
156. 263° 51′ 40″.
158. 17 wk. 3 d. 10 h. 17 min.
159. 3 mi. 124 rd. 1 yd. $1\frac{1}{6}$ ft.
160. 3 oz. 17 pwt. 14 gr.
161. 13° 10′ 35″.
162. 61 gal. 1 qt. 1 pt.

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163. 3779 in.
164. $ 351.75.
165. 6 T. 12 cwt. 6 lb.
166. $\frac{8}{98555}$.
167. 98 T. 3 ctl. 10 lb.
168. 19 rd. 6 ft. 6 in.
169. 19 A. 136 sq. rd. 68 sq. ft. 9 sq. in.
170. 786 days.
171 2 T. $726\frac{1}{3}$ lb.
172. 196 d. 0 h. 49 m.
173. 0.39625.
174. $16\frac{17}{32}$ cd. ft.
175. 55.
176. $ 15.25.
177. 78 y. 2 mo. 24 d.
178. Lose $ $10.78\frac{7}{8}$.
180. None.
181. 547 d. 20 h.

Art. 213.

25. 151.845^{m}.
26. 1663.70^{a}.
27. 17.4^{st}.
28. 2040^{l}.
29. $527.
30. 356.2075^{Ha}.
32. 121.253 lb.
33. 365.976 sq. yd.
34. 1111.95 acres.
35. $88.9+^{\text{K}}$.
36. $74+^{\text{Hl}}$.
38. $ 1.84+.
39. $ 4637.60+.
40. 2586.145+.
41. 93.57^{m}.
42. $ 6.68+.
43. $ 922.36.

Art. 222.

6. 820.9375 sq. ft.
7. 7854 sq. ft.
8. 1256.64 ft.
9. 400 ft.
10. 31 yd.
11. 0.78125 yd.
12. $13\frac{1}{2}$ A.
13. 154.56+ ft.
14. 232 sq. ft.
15. 14.6770^{Ha}.
16. $ 240.
17. 41 sq. rd. 147+ sq. ft.
18. 66.25 sq. ft.
19. 5 A. 159 sq. rd. $260\frac{1}{4}$ sq. ft.
20. Area, $128\frac{29}{32}$ sq. yd.; Cost $232.03+

Art. 226.

25. 15 cu. ft.
26. $1157\frac{5}{8}$ cu. ft.
27. $3\frac{2}{3}$ ft.
28. 3 cu. m.
29. $793\frac{1}{3}$ cu. yd.
30. 8.700 cu. m.
31. 8.3776 cu. yd.
32. 1026 sq. ft.

Art. 227.

33. 8 cd.
34. 8 cd.
35. $5\frac{1}{16}$.
36. $5\frac{5}{7}$ ft.
37. $\frac{63}{64}$ cd.
38. $ 236.25.

Art. 230.

40. 24 bd. ft.
41. $66\frac{2}{3}$ bd. ft.
42. $283\frac{1}{3}$ bd. ft.
43. $ 40.986.
44. $ 52.92.

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45. 72 ft.
46. $ 5.67+.
47. 1220 bd. ft.
48. $ 7.68.
49. 24.75^{Hl}.
50. 9 rolls.
51. 38.4 bu.
52. $65\frac{5}{8}$ gal.
53. 8500 bricks.
54. 4 T. and $3\frac{5}{9}$ T.
55. $7002\frac{17}{54}$ lb.
56. 1782 cu. ft.

Art. 231.

50. 99.
51. 0.003.
52. 0.01875.

53. $0.42\frac{6}{7}$.
54. 6400.
55. $\$132.37\frac{1}{2}$.
56. { $10000 land.
{ $4000 house.
57. 124 acres.
58. $2195.
59. $42\frac{18}{81}$.
60. 15 sq. yd. 3 sq. ft. 128 sq. in.
61. $62\frac{2}{9}$ yd.
62. $1613\frac{1}{3}$ cu. yd.
63. 47520 bricks.
64. 95040 bricks.
65. 85.84 sq. rd.
66. $316\frac{157}{320}$ tons.
67. $37\frac{1}{2}$ sq. yd.
68. $\$32.08\frac{1}{2}$.
69. $11.50.
70. $8739\frac{59}{119}$.
71. $32\frac{13}{16}$ cd.
72. $303\frac{3}{16}$ sq. ft.
73. $74\frac{31}{36}$ yd.
74. $5\frac{5}{11}$ acres.
75. $\$20.14\frac{22}{27}$.
76. $15.40.
77. 0.5184.
78. 0.625.
79. 80 yards.
80. July 26, 11 h. 45 min. P.M.
81. $8\frac{1}{3}$ sq. yd.
82. $\$8.49\frac{3}{8}$.
83. Ans. $\frac{27}{80}$.
84. 88 sq. rd.
85. $23\frac{8}{11}$ ft.
86. 18 ft.
87. 1926 ft.
88. $16\frac{4}{5}$ yd.
89. 5832.
90. 475 ft.
91. $12015
92. $\frac{97}{160}$.
93. $351.
94. $\$53.33\frac{1}{3}$.
95. 95 d. 5 h. 10 min.
96. 13500 yd.
97. yd. wide; $7.50.
98. 2.1 T.
99. $364.
100. $2.25.
101. $\$173.45\frac{5}{16}$.
102. $13\frac{17}{64}$.
103. $\$0.88\frac{4}{7}$.
104. $\$0.52\frac{1}{2}$.

Art. 241.

35. $3605.
36. $315.

Art. 242.

37. 2380 tons.
38. 205.20.
39. 22.61 mi.
40. 1211 men.
41. $487.50.
42. $51.64.
43. $0.8136.
44. $511.05.
45. $4122.50
46. $11223.87.
47. $9.34.
48. 14748.
49. $12825.
50. $5700.
51. $337.97.
64. $9\frac{1}{2}$ %.

Art. 243.

65. $8\frac{1}{2}$ %.
66. $6\frac{2}{13}$ %.
67. $25\frac{3}{5}$ %.
68. $9\frac{4}{5}$ %.
69. $83\frac{1}{3}$ %.
70. 14 %.
71. 5 %.
72. $7\frac{1}{2}$ %.
73. 15 %.
74. $12\frac{1}{2}$ %.
82. $324.40.
83. 295 sheep.

Art. 244.

84. $1562.50.
85. 14.4 tons.
86. $66\frac{2}{3}$ bu.
87. 1040 yd.
88. 3240 rd.
89. $3500.
90. 7000 lb.
91. $\$166\frac{2}{3}$.
92. $2400.
93. $9000.
94. 13655.

Art. 246.

102. $1.55.
103. $12160.
104. $50 loss.
105. $10.44.
106. $\$22.78\frac{1}{4}$.
107. $3780.
108. $250.
109. $6\frac{1}{4}$ %.
110. $41\frac{3}{17}$ %.
111. $3140.
112. $490.
113. $\$8.34\frac{3}{4}$.
114. $1460.
115. 4 % loss.
116. $\$0.57\frac{7}{24}$.
117. $\$19.16\frac{2}{3}$.
118. $\$66.66\frac{2}{3}$.
119. Neither.

Art. 248.

123. $141.95.
124. $8.75.
125. $157.93.
126. $59.80.
127. $5012.50.
128. $12876.
130. $18.94.
131. $214\frac{2}{13}$ bbl.
132. $1426.80.
133. 3.83+%.
134. $38.59.
135. $4170.
136. $200.

Art. 252.

139. $73.
140. $124.76.
141. $1581.

142. $53625.
143. $2400.
144. 2 %.
145. $3075.

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146. $71\frac{4117}{6178}$.
147. $92\frac{1}{2}$ %; $7\frac{1}{2}$ %.
148. $36\frac{6}{19}$ %.
149. Lose $0.05.
150. 110 %.
151. $1097.25.
152. 115 bbl.
153. $115.
154. $26163.26.
155. $5.31.
156. $\$5\frac{5}{9}$.
157. $361.80.
158. $12375.
159. $6\frac{5898}{16520}$ %.
160. $195.
161. $3033.75.
162. $2\frac{1}{2}$ %.
163. $4500.
164. $10872.75.
165. 20 %.
166. $1008.77\frac{11}{57}$.
167. $42\frac{2}{3}$ yd.
168. $\$2.80\frac{80}{89}$ per bbl.
169. 5 %.
170. $22790.
171. 195.
172. $33\frac{1}{3}$ %.
173. $2.50.
174. $99\frac{1}{4}$ %.
175. $4.
176. $7\frac{407}{499}$ %.
177. 298000.

Art. 259.

12. $2116.80.
13. $12.
16. $1728.75.
17. $\$343.93\frac{1}{3}$.
18. $584.6256.
19. $158.0298.
20. $8.2586.
21. $19.3909.
22. *$125.71.*
23. $299.33.
24. $\$6380.55\frac{5}{9}$.

Art. 263.

28. $\$5.8409\frac{1}{8}$.
29. $1500.
30. $\$1.9958\frac{2}{5}$.
31. $\$90.686\frac{3}{4}$.
32. $\$35.7857\frac{1}{2}$.
33. $\$18.4255\frac{1}{2}$.
34. $\$30.87987\frac{1}{2}$.
35. $4339.40.
36. $1610.
37. $9205.
38. $\$15.986\frac{2}{3}$.
39. $\$1.3410\frac{5}{12}$.
40. $\$10.9450\frac{5}{6}$.
41. $63.54.
42. $\$2358.90\frac{5}{18}$.
43. $140.5872.
44. $175.
45. $110.1012.
46. $818.545.
47. $112.425.
48. $565.935.
49. $508.0765.

Art. 264.

52. $1.2177.
53. $\$144.66\frac{2}{3}$.
54. $\$1.30\frac{1}{2}$.
55. $101.25.
56. $17.10.
57. $167.96.
58. $962.6446.
59. $201.
60. $779.3264.
61. $4535.7355.
62. $4.774.
63. $446.25.
64. $26.9458.
65. $173.20.
66. $426.449.
67. $637.19.
68. $22.575.
69. $279.77505.
70. $320.28.
71. $\$178.87\frac{1}{2}$.
72. $\$131.15\frac{2}{3}$.
73. $125.306.
74. $7.06446.
75. $10.54928.
76. $0.4216.
77. $17.7818.
78. $569.201.
79. $900.2532.
80. $1261.936.
81. $1025.814.
82. $\$860.57\frac{1}{3}$.
83. $656.19776.
84. $498.1983.
85. $952.899.
86. $1968.
87. $968.66304.
88. $\$204.83\frac{1}{3}$.
89. $901.6677.
90. $1237.88254.
91. $633.63554.

Art. 265.

93. $45.62.
94. $16.52.
95. $236.73.
97. 9 %.

Art. 266.

98. 6 %.
99. 6 %.
100. $6\frac{1779}{1781}$ %.
101. $15\frac{3}{4}$ %.
102. 7 %.
103. $4\frac{1}{2}$ %.
104. 9 %.
105. 10 %.
106. $5\frac{1}{2}$ %.
107. 4 %.
109. 5 years.

Art. 267.

110. 2 y. 11 mo. $28\frac{8}{49}$ d.
111. 1 y. 4 mo. 20 d.
112. 2 mo. 6 d.
113. 1 mo. 18 d. or 48 d.
114. 2 y. 6 mo.

115. 16 y. 8 mo.
116. 7 y. 4 mo. $26\frac{2}{3}$ d.
117. 2 y. 1 mo. $4\frac{158}{228}$ d.
118. 4 y. 7 mo. $16\frac{3}{8}$ d.
119. 9 y. 6 mo. $8\frac{4}{7}$ d.
122. $ 75.

Art. 268.

123. $ 3600.
124. $ 300.
125. $ 718.23$\frac{21}{28}$.
126. $ 14000.
127. $ 24000.
128. $ 675.60.
129. $ 14215.38$\frac{6}{13}$.
130. $ 14000.
131. $ 14400.
132. $ 36956250.

Art. 282.

134. $ 394.57.
135. $ 448.35.
136. $ 492.06.
137. $ 342.40.
138. $ 1447.08.
139. $ 722.17.
140. $ 5947.63.
141. $ 413.43.
142. $ 1828.69.
143. $ 1280.20.

Art. 283.

145. $ 349.21.

Art. 285.

151. $ 161.63.

Art. 286.

152. $ 111.94.
153. $ 65.60.
154. $ 93.70.
155. $ 1125.51.
156. $ 1273.82.
157. $ 212.24.
158. $ 41.85 gain.
159. $ 133.82.

Art. 287.

161. $ 1934.84.
162. $ 232.46.

Art. 291.

7. $ 42.

Art. 292.

8. $ 3122.17.
9. $ 28.81.
10. $ 356.44.
11. $ 130.32.
12. $ 300.

Art. 294.

13. $ 6.25.
14. $ 329.
15. $ 26.89.
16. $ 393.73.
17. $ 460.64.
18. $ 20.
19. $ 213.82.
20. $ 734.25.

Art. 300.

23. Bank discount $ 17.94. Proceeds $ 857.06.
24. Bank discount $ 3.05. Proceeds $ 82.55.
25. Bank discount $ 6.30. Proceeds $ 593.70.
26. $ 4880.
27. $ 10635.40.
28. $ 479.56.
29. $ 953.23.
30. $ 1013.33.
31. $ 284.39.
32. $ 640.12.
33. $ 957.93.
34. $ 323.77.
35. $ 693.74.
36. $ 839.88.
37. $ 494.92.
38. $ 8567.37.
39. $ 235.50.
40. $ 795.07.
41. $ 555.27.
42. $ 568.96.
43. $ 888.85.
45. $ 1518.60.
46. $ 300.

Art. 301.

47. $ 450.
48. $ 1600.
49. $ 842.00+.
50. $ 509.34.

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51. $ 24 discount.
52. $ 40.
53. April 15, 1878.
54. $ 6000.
55. Dec. 24.
56. $ 385.25.
57. $ 12.43.
58. $ 3.41.
59. $ 362.30.
60. April 4, 1882.
61. $ 175 loss.
62. $ 502.77.
63. $ 773.63.
64. $ 1722.64.

Art. 314.

8. $ 2050.
9. $ 34087.50.
10. $ 1268.75.
12. $ 128.12$\frac{1}{2}$.
13. $ 124.57$\frac{1}{407}$.
14. $ 130.
16. $ 125.
17. $ 360.
18. Neither.
20. $ 21100.
21. $ 16350.
22. $ 54500.
24. $4\frac{6}{11}$%.
25. 6's, .24% greater.
26. $6\frac{2}{5}$%.

28. $\$85\frac{5}{7}$.
29. At $133\frac{1}{3}$.

Art. 323.

5. $1175.64.
6. $3900.
7. $\$2520.84\frac{57}{80}$.
9. $2517.69+.
10. $4000.
11. $1164.
12. $450.
14. $\$1474.12\frac{1}{2}$.
15. $2998.50.
17. $4000.
18. $500.
19. $2985.067+.

Art. 330.

21. £160 8 s.
22. $\$6437.43\frac{3}{4}$.
23. $\$462.13\frac{61}{103}$.
24. $\$289.01\frac{127}{173}$.
25. 16614.
26. $309.70.
27. $5948\frac{92}{191}$ marks.
28. $6117.03.
29. $5434.03\frac{1}{8}$ francs.

Art. 334.

7. March 18.
8. Sept. 13.
9. 3 mo. 3 d.
10. July 17, 1881.
11. May 4.
12. 46 days.

Art. 335.

13. Sept. 18, 1881.
14. Feb. 12, 1881.
15. July 1, 1881.
16. Feb. 23, 1882.

Art. 336.

17. $66\frac{2}{3}$ %.
18. $6740.
19. $1053.
20. $585.
21. 85 %.
22. $45\frac{115}{183}$ %.
23. $12.25.
24. $\$120.25\frac{23}{40}$.
25. $35.21+.
26. 1 y. 2 mo.
27. $600.
28. 6 %.
29. 8 y. 4 mo.
30. $69.79.
31. $\$167.21\frac{1}{4}$.
32. $40.04.
33. $1700.40.
34. $41745.
35. $1050.
36. $80.
37. 92 days.
38. $2\frac{3}{10}$ % discount.
39. $265.95.
40. 24 days.

Art. 348.

27. 9.
28. 15.
29. 40.
30. 9.
31. $\frac{1}{3}$.
32. 8.
33. 30.
34. 75.

Art. 349.

36. $127.50.
37. $91.
38. 90 men.
39. 18 days.

Art. 350.

40. $632.
41. $22.68.
42. 1363 miles.
43. 9 days.
44. $\$31\frac{1}{2}$.
45. $11\frac{1}{4}$ days.
46. 7 mo.
47. $\$11\frac{1}{2}$.
48. $41\frac{1}{5}$ days.
49. 7722.
50. $\$33.91\frac{2}{3}$.
51. 96 men.
52. $\$39.40\frac{5}{8}$.

Art. 352.

54. 216.
55. $7\frac{1}{5}$ years.
56. 24 men.
57. 16 acres.
58. $4\frac{4}{7}$.
59. $\$10\frac{1}{2}$.
60. 208 miles.
61. 1800 tons.
62. $17\frac{7}{25}$ days.
63. $50\frac{50}{63}$ weeks.
64. 72 acres.
65. 600 bricks.
66. $169\frac{7}{17}$ cd.
67. 240.
68. $2.88.
69. $8\frac{16}{23}$ ft.
70. $\$189\frac{7}{12}$.

Art. 356.

7. A's gain $504.
   B's " $756.
   C's " $420.

Art. 357.

8. A's gain $720.
   B's " $990.
   C's " $1080.
9. A's share $1137.50.
   B's share $1365.
   C's share $1722.50.
10. Hall's $312.
    Bishop's $416.
11. B's loss $102\frac{6}{7}$ bbl.
    C's " $154\frac{2}{7}$ bbl.
    D's " $102\frac{6}{7}$ bbl.
12. A's share $100.
    B's " $75.
    C's " $125.

13. A's gain \$84.
B's " \$36.
Flour per bbl. \$8.

Art. 358.

15. A's loss $\$98.96\frac{88}{97}$.
B's loss $\$224.32\frac{26}{97}$.
C's loss $\$316.70\frac{19}{97}$.

16. A pays \$400.
B " \$480.
C " \$450.

17. A pays \$24.75.
B " \$22.05.

18. A's loss $\$71\frac{3}{7}$.
B's " $\$133\frac{1}{3}$.
C's " $\$95\frac{5}{21}$.

19. A's gain \$960.
B's " \$1200.

20 A's gain $\$1244\frac{4}{9}$.
B's gain $\$1155\frac{5}{9}$.

21. Davis's profits \$720.
Wood's profits \$1440.
Furbush's profits \$1000.

22. A's gain \$2200.
B's " \$1650.
C's " \$2000.

Art. 363.

8. 529.
9. 4096.
10. 28561.
11. $\frac{9}{16}$.
12. $15\frac{5}{8}$.
13. 12.96.
14. 15625.
15. 161051.
16. 0.003375.
17. 12.25.
18. $203\frac{1}{16}$.
19. 0.000343.

Art. 375.

26. 96.
27. 165.
28. 427.
29. 847.
30. 974.
31. 2.59.
32. 2.05.
33. 43.2.
34. 0.097.
35. 0.237+.
36. 0.97.
37. 279.
38. 4.03.
39. 3.86.
40. 6.25.
41. 23.19+.
42. 8.426.
43. 0.0447.

Art. 376.

45. $\frac{61}{87}$.
46. $\frac{3}{7}$.
47. $7\frac{3}{4}$.
48. $6\frac{1}{7}$.
49. 7.031+.
50. $\frac{15}{32}$.
51. $8\frac{1}{2}$.
52. 0.96+.
53. 1.5.
54. $31\frac{1}{3}$.
55. 12.108+.
56. 9.019+.
57. 0.935+.
58. 0.829.
59. 476 men.
60. 49 rd.

Art. 384.

64. 45.
65. 75.
66. 83.
67. 9.7.
68. 156.
69. 235.
70. 39.2.
71. 467.
72. 637.
73. 18.54.
75. 40.1.
76. 334.
77. 0.012.
78. 123.
79. 2.012.
80. 0.423.
81. 0.205+.
82. 0.019+.
83. 2.92+.

Art. 385.

85. $1\frac{2}{39}$.
86. $3\frac{2}{3}$.
87. 0.84+.
88. 0.57.
89. 4.33+.
90. 5.503+.
91. 25.81+ inches.

Art. 390.

3. 25 ft.
4. 85 ft.
5. 60 miles.
6. 387.30 ft.
7. 53.81+ rd.
8. 44.94+ ft.

Art. 394.

9. 540 sq. ft.
10. 192 sq. ft.
11. 199 sq. ft.

Art. 396.

12. 9350 sq. ft.
13. 3 sq. m.
14. $8\frac{1}{2}$ a.

Art. 398.

16. 1040 sq. ft.
17. $871\frac{187}{288}$ sq. yd.

Art. 400.

18. 90 cu. ft.
19. $216000^{\text{cu cm}}$.
20. $153\frac{233}{2880}$ cu. ft.

Art. 403.

21. 69.825 cu. ft.
22. 62.42 cu. m.
23. 80041500 cu. ft.

Art. 405.

25. 169.74+ cu. ft.
26. $61\frac{37}{162}$ cu. ft.

Art. 407.

27. 1963.5 sq. in.
28. 0.101787+ sq. m.
29. 65450 cu. ft.
30. 4849059600 cu. mi.

Art. 409.

32. 32 sq. ft.
33. 100 rd.
34. 5890.5 sq. ft.
35. $\frac{9}{980}$ hours.
36. $67.50.
37. 949.95 sq. m.

Art. 412.

39. 4.57+ ft.
40. 2.5 in.
41. 132^{K}.
42. 64 times.
43. $472\frac{1}{2}$ lb.

Art. 413.

49. Neither.
50. $12\frac{1}{2}\%$.
51. $28.80.
52. $\frac{3}{16}$.
53. 1176 sq. ft.
54. $192\frac{4}{13}\%$.
55. $9000.
56. $16\frac{3}{4}$ cents.
57. $16644.
58. $8382.40.
59. 200000 lb.
60. $5\frac{5}{14}\%$.
61. $1035.1657+.
62. Lose $50.
63. $88.73+.
64. $84.906.
65. $7\frac{1}{7}$ years.
66. $1251.99.
67. $100.00+.
68. $1539.75.
69. $811.08+.
70. $17968.75.
71. $1279.30.
72. $1653.69+.
73. 2.5.
74. 282.0638.
75. $4\frac{1}{6}$ cents.
76. $12\frac{3}{4}$ oz.
77. $\$3.35\frac{15}{31}$.
78. $1484.84\frac{28}{33}$.
79. Ames
$5165.91+.
Stevens
$3874.43+.
Conant
$2152.46+.
Hubbell
$807.17+.
80. $313.532.
81. 127.279+ ft.
82. 452.54+ rd.
83. 29.6.
84. 2 m. 66.274 rd.
85. 16 ft.
86. 74.83+ in.
87. 208.71+ ft.
88. $240 or $330.
89. 38.1575 ft.
90. 79.1959.
91. 373248.
92. 23.32+ ft.
93. 1.
94. 42336 sq. in.
95. 24.
96. $91\frac{1}{5}$.
97. 2 : 5.
98. $3.64.
99. 72.
100. $\$2666.66\frac{2}{3}$.
101. 20 days at the distance of 500 miles.
102. 55 min. $23\frac{1}{13}$ sec.
103. 36 men.
104. $67\frac{1}{2}$ days.
105. $375 A's.
$625 B's.
$1000 C's.
106. 407.29+ in.
107. 1.04.
108. 42.5 ft.
109. $22\frac{2}{49}$ gal.
110. 5 mo. 21 d.
111. First 123.
Second 37.
Third 22.
112. 300 sq. yd.
113. 27.

Art. 414.

2. 1020.
3. 8.
4. 1125.
5. 4.
6. $\$4166\frac{2}{3}$.
7. 365.
8. 4.
9. 24.
10. 744 hours.

Art. 415.

1. 18.
2. 365.
3. 1776.
4. 18 days.
5. 7677.
7. 1248480000.
8. $330.
9. $3102\frac{54}{128}$.
10. 19.

Art. 416.

4. 99084.19.
5. 2708.71.
6. 8511.
7. 2325.
8. $997\frac{45}{87}$.
9. $26.
10. 295 and $15 over.

Art. 417.

1. 529203200.
2. 9600 cu. ft.

3. $25.
4. $2.25.
5. 2284.47.
6. $2\frac{1}{4}$ lb.
7. $717\frac{13}{18}$.
9. $361.30.
10. $12.24.

Art. 418.

1. $293\frac{25}{48}$.
2. $\$113\frac{1}{8}$.
3. 8656742.
4. $885.50.
5. $7.23.
6. $135.
7. $229.50.
8. 14400.
9. Lose $703.
10. 18090100.

Art. 419.

1. 424.
3. $2\frac{359}{600}$.
4. $\frac{6}{7}$; $\frac{7}{11}$; $\frac{13}{14}$; $\frac{9}{11}$.
6. $23\frac{5}{78}$.
7. $79\frac{11}{12}$; $19\frac{49}{72}$.
9. $2108\frac{4}{7}$; 837; 85.
10. $\frac{3}{10}$; $100000.

Art. 420.

2. 14.
3. 280.
4. $\left\{\begin{array}{l}\frac{53}{4}.\\ \frac{553}{9}.\\ \frac{206}{13}.\end{array}\right.$
5. $\left\{\begin{array}{l}\frac{3}{5}.\\ \frac{5}{18}.\end{array}\right.$
6. $\left\{\begin{array}{l}37.\\ 127\frac{11}{17}.\\ 13\frac{51}{73}.\end{array}\right.$
7. $\frac{32}{36}$, $\frac{27}{36}$, $\frac{15}{36}$, $\frac{24}{36}$.
9. $191\frac{1}{10}$ miles.
10. $13\frac{1}{8}$ bushels.

Art. 421.

1. *12.*
2. *360.*
3. $95\frac{11}{120}$.
4. $41\frac{11}{24}$.
5. $306\frac{11}{80}$.
6. $4\frac{61}{68}$.
7. $\frac{14}{15}$; $\frac{81}{116}$.
8. $\$6562\frac{1}{3}$.
9. $146\frac{2}{23}$ miles.
10. $46\frac{2}{3}$ tons.

Art. 422.

3. 450.
4. $\frac{60}{133}$.
5. $20\frac{16}{21}$.
6. $\frac{448}{459}$.
7. $94500.
8. $4800.
9. $\frac{7}{162}$.

Art. 423.

1. $\frac{3}{5}$; $\frac{3}{5}$; $\frac{3}{5}$.
2. $16\frac{9}{28}$.
3. $18\frac{33}{85}$.
4. $2685\frac{55}{72}$.
5. $3445\frac{3}{8}$.
6. $\frac{2101}{56}$.
7. $8\frac{47}{130}$.
8. $\$\frac{27}{56}$.
9. $\frac{231}{620}$.
10. $80\frac{5}{8}$ miles.

Art. 424.

1. $32\frac{109}{120}$; $8\frac{81}{120}$.
3. $187\frac{3}{7}$ miles.
4. $4\frac{352}{675}$.
5. $14000.
6. $57600.
7. $12000.
8. $\frac{196}{595}$.
9. 50 acres.
10. $\$109714\frac{2}{7}$.

Art. 425.

1. $153\frac{5}{16}$.
2. $94\frac{12}{35}$.
3. $31\frac{11}{30}$.
4. $5640.
5. $8\frac{23}{26}$.
6. 6 lots.
7. $\frac{9}{14}$.
8. $\frac{2}{3}$.
9. $5625.
10. Diminished $\frac{4}{15}$.

Art. 426.

1. $\frac{9}{40}$.
2. $\frac{5}{6}$.
3. $958\frac{26}{33}$.
5. $\frac{2}{3}$.
6. 35.
7. $\$1\frac{2}{3}$.
8. $\frac{19}{45}$.
9. $21\frac{21}{32}$; $28\frac{4}{7}$; $6\frac{141}{224}$.
10. $\$1841\frac{2}{3}$.

Art. 427.

1. 0.1875.
2. 0.79992.
4. $\frac{1}{400}$.
6. 24.5 yd.
7. 1.6075.
8. $330.
9. $28.26.
10. 0.5.

Art. 428.

1. 0.009125.
2. $\frac{6}{625}$.
3. 1.44.
4. 0.0001177.
5. 207.36.
6. 0.01010625.
7. 0.00125.
9. 15.
10. 479.9975.

Art. 429.

1. 0.003.
2. 998998.999.
3. 0.00144.
4. 166.
5. $40.937.
6. 7 years.
7. $300.
8. $334.21875.

9. $\$49.81\frac{1}{4}$.
10. $\$9.06\frac{1}{4}$.

Art. 430.

1. 10.2485.
3. $0.72727\frac{3}{11}$; $\frac{7}{80}$.
4. 0.20445+.
5. 6 oz. 11 pwt. 8.448 gr.
6. \$165.
7. $\$6702.24\frac{7}{32}$.
8. 16455^{k}.
9. 13 lb. $1\frac{5}{11}$ oz.
10. 39.37+ in.

Art. 431.

1. 283 y. 8 mo. 23 d.
2. 50 A. 105 sq. rd.
3. 578 mi. 286 rd. 3 ft.
4. 88° 45′.
5. $936\frac{13}{17}$ lb.
6. 58° 28′ 20″ W.
8. 2 d. 0 h. 44 min.
9. 2.32+ sq. rd.
10. 338 cu. ft. 1584 cu. in.

Art. 432.

1. $11\frac{2}{3}$ rolls.
2. $6\frac{178}{485}$.
3. 7 gro. $2\frac{2}{5}$ doz.
4. 1680 sq. in.
5. Feb. 6.
6. $\$8.04\frac{3}{8}$.
7. $353.7\frac{18}{41}$.
8. $3\frac{7}{11}$ A.
9. \$250.
10. 25 sq. yd.

Art. 433.

1. \$132.
2. \$37.80.
3. $13\frac{1}{2}$ ft.
4. \$6384.
5. 10.84 oz.
6. $\$350.62\frac{1}{2}$.
7. $\frac{1}{5}$.
8. 53 bags.
9. 1880 d. 6 h.
10. 6 lots.

Art. 434.

1. \$5940.
2. $37\frac{1}{2}$ cents.
3. 21780 cu. ft.
4. $3\frac{23}{48}$ cu. yd.
5. $871\frac{1}{5}$ ft.
6. 59319 bricks.
7. $11\frac{10201}{19200}$.
8. $\left\{\begin{array}{l}\text{5600 bd. ft.}\\ \$89.60.\end{array}\right.$
9. \$15.
10. \$75.79.

Art. 435.

1. $\frac{1}{2}$ A.
2. 1609.3+.
3. 19350^{K}.
4. $\$6.43\frac{3}{4}$.
5. \$560.
7. 160 ft.
8. $\left\{\begin{array}{l}\text{4860 bd. ft.}\\ \$48.60.\end{array}\right.$
9. \$60.
10. $\$17817.18\frac{3}{4}$.

Art. 436.

1. $34\frac{26}{27}$ yd.
2. $151\frac{1}{4}$ cu. yd.
3. $29\frac{7}{12}$ mi.
4. 0.1090+ mi.
5. $4873\frac{1331}{1728}$.
6. $\$488.82\frac{1}{2}$.
7. 96 mi. 154 rd. 6 ft.
8. $1188\frac{39}{67}$ lb.
9. 71° 0′ W.
10. $\$58.07\frac{11}{27}$ cost of the \$2 kind.
    $\$38.11\frac{1}{9}$ cost of the \$1.75 kind.
    The \$2.00 kind the more expensive.

Art. 437.

1. 48 rd.
2. 600 sq. rd.
3. 351 T.
4. 18 min. 40 sec.
5. \$1756.
6. \$313.50.
7. \$20.25.
8. 0.6235+.
9. \$25.68+.
10. 1830 cu. ft.

Art. 438.

1. $\frac{1}{16}$; $\frac{1}{8}$; $\frac{1}{12}$; $\frac{1}{6}$; $\frac{2}{3}$.
2. 14%; 44%; 40%; $87\frac{1}{2}$%; 60%; $71\frac{3}{7}$%; $62\frac{1}{2}$%.
4. 45%.
5. \$4331.25.
6. 135°.
7. \$14400.
8. 50%.
9. $\left\{\begin{array}{l}\$11955.17.\\ \$44.83.\end{array}\right.$
10. \$9000.

Art. 439.

1. $\$2879.37\frac{1}{2}$.
2. \$31.
3. $\left\{\begin{array}{l}\$44231.71.\\ \$1105.79.\end{array}\right.$
4. \$26.25.
5. 75%.
6. $\left\{\begin{array}{l}\frac{88}{80}.\\ \$44000.\end{array}\right.$
7. $38\frac{481}{528}$%.
8. $\$10666.66\frac{2}{3}$.
9. \$89.63+.
10. $\left\{\begin{array}{l}\$2784.\\ \$2876.80.\end{array}\right.$

Art. 440.

1. $67\frac{1}{2}$%.
2. $66\frac{2}{3}$%.
3. \$244.44+.
4. \$40000.
5. 20%.

6. **Loss $25.**
7. $6\frac{1}{4}$ % loss.
8. 90 %.
9. $4.
10. $2.80.

Art. 441.

1. A's $43750.
   B's $56250.
   C's $25000.
2. 20 %.
3. $50800.
4. $4390.24.
   $109.76.
5. $3600.
6. $63\frac{1}{2}$ cents.
7. $\$149.33\frac{1}{3}$.
8. $33.75.
9. $102.57.
10. On wheat.
    5 % greater.

Art. 442.

1. 2376 ft.
2. $42\frac{6}{7}$ %.
3. $20000.
4. $3520.
5. $13.
6. $8\frac{86}{188}$ %.
7. $32000.
8. 11200.
9. $1740.
   $1832.80.
10. $56250.
    $2531.25.

Art. 443.

1. 60 cents.
2. $20000.
3. Renting, by $50.
4. $760.
5. $12\frac{1}{2}$ %.
6. $24375.
7. $124.
8. $7.35.
9. Paid $300.
   Received $336.
10. *Gained 21 %.*

Art. 444.

1. $153.51+.
2. 12 y. 6 mo.
3. $520.
4. $89.359+.
5. $\$2.66\frac{2}{3}$.
6. 1 y. 8 mo. 12 d.
7. $640.51.
8. $432.84.
9. $825.17.
10. $0.04.

Art. 445.

1. 3 y. 2 mo. 27+ d.
2. $6936.09.
3. 7 %.
4. $40.23.
5. $169.82.
6. $\$1171.87\frac{1}{2}$ present worth.
   $\$103.12\frac{1}{2}$ discount.
9. $508.35.
10. $1313.56.

Art. 446.

1. $6.98.
2. $13.56.
3. $\$366.26\frac{2}{3}$.
4. $5\frac{1}{2}$ %.
5. $904.97.
6. $190.40.
7. $28.57.
8. $225.
9. $9.30 bank discount.
   $440.70 proceeds.
10. $251.36.

Art. 447.

1. $13.697+.
2. 2 y. 7 mo. 15 d.
3. 8 %.
4. $300.
5. $5.55.
6. $2440.
7. $\$550.87\frac{1}{2}$.
8. $154.61.
9. $1688.26.
10. $953.49.

Art. 448.

1. 9 y. 6 mo. 9 d.
2. $11\frac{2}{3}$ %.
3. $\$1440.83\frac{1}{3}$.
4. $0.269.
5. $303.97.
6. $210.
7. $400.07.
8. $742.
9. $4375.
10. Lessen $1\frac{2}{3}$ %.

Art. 449.

1. 5 %.
2. $40000.
3. $24\frac{4}{9}$ %.
4. $19\frac{27}{67}$ % gain.
5. 151 shares.
   $\$122\frac{3}{4}$ left.
6. $1\frac{1}{2}$ mo.
7. 19 mo.
8. $2432.62+.
9. Sept. 22, 1880.
10. Feb. 6, 1882.

Art. 450.

1. $\frac{25}{192}$.
2. 9 yds.
3. $8\frac{16}{23}$ ft.
4. $9\frac{1}{24}$ days.
5. $7\frac{18}{50}$ oz.
6. $22\frac{11}{12}$ acres.
7. $5\frac{1}{7}$ mo.
8. 96 men.
9. 102 men.
10. 6 men.

Art. 451.

1. $2000.
2. 1st man $15.
   2d man $20.
   3d man $20.

3. Ames \$1600.
Howe \$1200.
4. A \$62.50.
B \$75.
C \$100.
D \$112.50.
5. A's $\$5333\frac{1}{3}$.
B's $\$6666\frac{2}{3}$.
6. A's $\$388\frac{212}{301}$.
B's $\$249\frac{51}{301}$.
C's $\$112\frac{38}{301}$.
7. 1st man \$900.
2d man \$600.
8. A's \$3516.80.
B's $\$5861.33\frac{1}{3}$.
C's $\$8205.86\frac{2}{3}$.
9. $2\frac{26}{47}$ days.
10. 37 %.
A \$666.
B \$277.50.
C \$721.50.

Art. 452.

1. 962.
2. 36.37.
3. 93.27 ft.
4 $\$187\frac{1}{5}$.
5. 0.199999488.
6. $21\frac{9}{11}$ rd.
7. 324.
8. 12 minutes.
9. 8.55—.
10. Slant h't 100 ft.
Surface 24000 sq. ft.
Contents 384000 cu. ft.

Art. 453.

1. $\$133\frac{1}{3}$.
2. 72 in.
3. 164 ft.
4. 46.596+ rd.
5. 307.3+ miles.
6. 24576 sq. in. entire surface.
110.8 in. diagonal.
7. 27.
8. 8 sq. ft.
9. 63.24 ft.
10. $6\frac{1}{104}$ ft.

Art. 454.

1. $3\frac{21}{23}$.
2. $0.0214\frac{2}{7}$.
3. $\$2857\frac{1}{7}$.
4. \$172.46.
5. \$22.60 bank discount.
\$1227.40 proceeds.
6. 4 h. 48 min. P.M.
7. \$151.25.
8. \$187.20.
9. 42.
10. $9\frac{13}{27}$ d.

Art. 455.

1. 76 yd.
2. \$242.92.
3. 128 rd.
4. $3\frac{5}{8}$ ft.
5. 0.000002.
6. $183\frac{41}{278}$.
7. \$10.
8. \$91.80.
9. 64 rd.
10. 14 days.

Art. 456.

1. 35000 times.
2. $\frac{13}{18}$.
3. \$75000.
4. 5 %.
5. \$8.16.
6. Due May 16, 1881.
Bank discount $\$19.37\frac{1}{2}$.
Proceeds $\$1230.62\frac{1}{2}$.
7. $5\frac{5}{14}$ days.
8. 20 rd.
9. 452.39 cu. in.

Art. 457.

1. \$0.91.
2. $25\frac{5}{16}$ acres.
3. 20 % gain.
4. $4166\frac{2}{3}$.
5. \$202.0095.
6. Gain \$66.98.
7. $274\frac{2}{5}$ lb.
8. 900 sq. ft.
9. 2y. 8mo. 28d.
10. 152+ ft.

Art. 458.

1. 175 sheep.
2. $\$3.81\frac{2}{5}$.
3. 80 lb.
4. \$360 share of 1st.
\$640 share of 2d.
5. \$8.46.
6. 174.4+.
7. 45 min.
8. \$2000.
9. 8 %.
10. 5 %.

Art. 459.

1. $\$476.19\frac{1}{21}$.
2. \$3885120.
3. $1.15\frac{199}{2400}$ mi.
5. \$100.
6. One 17½° west.
The other 30° east.
7. $\$19\frac{37}{77}$ gain.
8. \$57600.
9. 460 sq. rd.
10. $364\frac{28}{73}$ %.

Art. 460.

1. \$2640.
2. 15 ft.
3. $1\frac{19}{101}$.
4. $28\frac{4}{9}$ ft.
5. $\$449\frac{2208}{3553}$ gain.
6. $\$4442.81\frac{1}{4}$.
7. $\$452.17\frac{9}{28}$.
8. 36 men.
9. 226.27 rd.
10. $338\frac{1}{12}$ cu. ft.

Art. 461.

1. $867.94.
2. $12\frac{21}{50}$.
3. $43.516.
4. $88\frac{56}{368}$.
5. $1\frac{9}{20}$.
6. $1898\frac{784}{1517}$ lb.
7. B.
8. 1421.2296 sq. ft.
9. B $\frac{1}{2}$% greater.
10. 0.105+.

Art. 462.

1. 32.56\frac{127}{328}$.
2. 17280.
3. $5.04.
4. 6726.56\frac{1}{4}$.
5. $17.22.
6. $112.50.
7. $31\frac{1}{2}$.
8. 184.129\frac{7}{8}$.
9. $1636.
10. $2969.30.

Art. 463.

1. $5908.
2. $27.59.
3. At 18 months.
4. 31050.
5. $312.50
6. 40.84+ ft.
7. $15.75 A's share.
   14.62\frac{1}{2}$ B's share.
   18.37\frac{1}{2}$ C's share.
   $26.25 D's share.
8. 816\frac{2}{3}$ C's share.
   1066\frac{2}{3}$ B's share.
   1116\frac{2}{3}$ A's share.
9. 24 ft.
10. $29\frac{31}{81}$ minutes.

Art. 474.

16. $\frac{251}{888}$.
17. $\frac{22}{37}$.
18. $\frac{421}{990}$.
19. $\frac{6611}{900}$.
20. $\frac{5}{37}$.
21. $\frac{196102}{3700}$.
22. $4.21\dot{2}9\dot{5}$.
23. $0.\dot{4}7\dot{6}$.
24. $10.08\dot{3}$.

Art. 478.

25. 1.509375 mile.
26. 76.835 A.
27. 17.545 A.
28. 575 miles.
29. 486.
30. 5760.
31. $225.
32. $34.

Art. 486.

36. 5 h. 8 min. $11\frac{1}{5}$ sec.
37. 10 h. 53 min. $37\frac{3}{5}$ sec.
38. 12° 27′ 14″.
39. 13° 22′ 30″.
40. 93° 40′ W.

Art. 493.

44. $768.12.
45. 1435.41.
46. $707.54.

Art. 494.

48. $1177.84.
49. $4584.33

Art. 497.

51. Aug. 7, 1881.
52. Nov. 12, 1881.
53. Feb. 6
54. Dec. 16, 1881.

Art. 505.

56. $32.29.
57. C's $116.28.
   D's $22.69.
   E's $69.06.
58. Each poll $1.58.
   County rate 0.0009.
   Town rate 0.0121.
   A's county tax $5.50.
   A's entire tax $66.58.

Art. 510.

59. $1400 duty.
   $4929.02+ cost.
60. $1851.29+.
61. 2532\frac{6}{7}$.
62. $5105.979+.
63. $182.68+.

Art. 514.

64. 22.96+ cu. ft.
65. $37\frac{1}{2}$ cu. ft.

Art. 515.

66. 14.847 in.
67. 431.869+ bd. ft
68. $13.12+.

Art. 518.

69. $49.36\frac{4}{5}$ gal.
70. 116.28 gal.
71. 369.309+ l.
72. 446.25 gal.

Art. 520.

73. $119\frac{7}{19}$ tons.
74. $672\frac{12}{19}$ tons.

Art. 526.

75. $76\frac{4}{5}$ bu.
76. $22\frac{2}{5}$ bu.
77. 8640 lb.
78. $5\frac{31}{33}$ bu.
79. $15.

80. 186 lb. rump and sirloin.
186 lb. round.

81. 450 lb. hams & shoulders.
600 lb. sides.

Art. 532.

82. $ 5.71$\frac{3}{7}$.
83. $ 51.02$\frac{2}{9}$.
84. $ 35.08$\frac{4}{27}$.
85. $ 179.64.
86. $ 362.20+.

Art. 538.

87. 15.888 M. shingles.
95.328 lb. nails.
$ 19.86 cost of labor.

88. 14.53 hundred.

89. $ 2.47 cost of paint.
$ 1.78 paid painter.

90. 20$\frac{4}{5}$ hundred laths.
$ 0.64+ cost of nails.

91. $ 76 cost of boards.
$ 4.27$\frac{1}{2}$ to each man.

Art. 543.

94. 302.
95. 23.
96. 9 miles.
97. 65 cents.

Art. 544.

99. 20640.
100. $ 348.

Art. 546.

103. 1600000.
104. $ 142.92+.
105. $ 63.12+.

Art. 547.

107. 1530.
108. 30$\frac{65}{128}$.
109. 437.46+.
110. $ 3495.25.

Art. 548.

1. Eight ten-thousandths.
2. $ 29741.25.
3. 39^{Hl}.
4. 20 %.

Art. 549.

1. $\frac{105}{816}$.
2. 2$\frac{43}{120}$ sum.
$\frac{7}{24}$ product.
3. 1566.7+.
4. 1880.0001+.
5. $488.25+^{K}$.
6. 36$\frac{16}{19}$ %.

Art. 550.

1. Eighty-five millions eighteen thousands nine hundred eighty-two.
2. 9$\frac{1}{11}$.
3. 500.002.
499.998.
4. $5^{Kg}\ 217^{g}$.
6. 13.572+.

Art. 551.

1. 63.
2. 0.555$\frac{5}{9}$.
3. $2.17+^{Hl}$.
4. 59$\frac{7}{27}$ yd.
5. 20 %.

Art. 552.

1. 0.64+.
2. 4.69+.
3. 0.050$\frac{50}{99}$ mile.
4. $36.5+^{dm}$.

Art. 553.

1. 2.
2. L. C. M. 360.
G. C. D. 2.
3. 7.62^{m}.
4. 1879.
5. $ 0.88$\frac{8}{9}$.
6. $ 1142$\frac{6}{7}$.

Art. 554.

2. $ 12520.24.
3. $1.6093+^{Km}$.
4. $\frac{1}{2}$, or .5.
5. Spent $360.
Saved $240.
To business $180.
6. 1$\frac{3}{5}$ years.
7. 8 %.
8. 0.899.

Art. 555.

1. 2$\frac{17}{60}$.
2. $\frac{825}{1288}$.
3. 0.577+.
4. 0.024$\frac{208}{381}$.
5. 0.8314$\frac{788}{2178}$ acre.
6. $160^{sq\ dm}$.
300000.

Art. 556.

1. 7.
2. 67.2^{Dm}.
3. 5040.
4. 9 %.
5. 26$\frac{2}{3}$ days.

Art. 557.

1. 1234567.654321.
2. 1595864.
3. $\overline{\text{LXXXIV}}$DCCXCVI.
4. **Twenty millions five hundred sixty-seven thousands one hundred eighty-nine, and four million three hundred twenty-one thousand ninety-eight billionths.**

5. $330377\frac{87881}{96813}$.
6. 129.
7. 810.
8. $\begin{cases} \frac{39208}{411684}, \frac{56784}{411684}, \\ \frac{68886}{411684}, \frac{102921}{411684}. \end{cases}$
9. $0.637\frac{121}{7917}$.
10. 37.
11. $654.50.
12. 35° 30′ eastward.

13. $\frac{1}{6}$ of 1%.
14. 525.
15. 413.43\frac{3}{4}$.
16. $4\frac{28}{29}$ hours.
17. $5.87.
18. $289.76.
19. $125.03.
20. 6.

21. $\begin{cases} \text{A's \$640.} \\ \text{B's \$840.} \\ \text{C's \$840.} \end{cases}$
22. $21\frac{1}{8}$ tons.
23. 0.062.
24. 140.5 miles.
25. $19.44.
26. 19.635 sq. ft.
27. 20 sq. ft.